Assignment 1
SRM-1

Q.1 $_uq_x = P[T_x \le u]$
$= P[K_x = 0 \text{ and } S_x \le u]$
$= P[K_x = 0] * P[S_x \le u] \Rightarrow P[K_x = 0] = q_x$
As K_x & S_x are independant
$P[S_x \le u] = \int_0^u 1\, dx = u$, $\because$ uniform dist'n.

$\therefore\ _uq_x = u * q_x$

Q.2
a) Period of exposure is 1/6/2000 to 25/10/2000
$= 30 + 31 + 31 + 30 + 25 = 147$ days
$= 147/7 = 21$ weeks

b) Period of exposure is 1/6/2000 to 31/5/2001
$\therefore$ As this is exactly a year, there are 52 weeks

Q.3
i) Left censoring :- If we dont know when the policy holder joined the company then the data will be left censored.
ii) Right censoring :- Data will be right censored if observations are lost in progress, so that we are not able to see if policy is surrendered. Also if policy is terminated in middle of investig-ation then that data will be right censored.
iii) Interval censoring :- If we only given ~~can~~ callender years of surrender ~~in y~~ then data is interval censored.
iv) ~~Informative~~ In

Q.4

i) complete expectation of life $\mathring{e}_x$.

$\mathring{e}_x = E[T_x] = \int_0^{\omega-x} {}_tP_x \, dt.$

This represent probability of survival at each future age.

ii) The curtate expectation of life.

$e_x = \sum_{K=1}^{\infty} {}_KP_0 = \sum_{K=1}^{\infty} e^{-0.0325K}$

$= \frac{e^{-0.0325}}{1-e^{-0.0325}} = 30.27$

iii) P of life aged 36 will survive to age 45.

${}_9P_{36} = \exp\left[-\int_0^9 0.0325 \, dt\right] = e^{-0.2925} = 0.7464 \approx 75\%$

iv) A exact age x representing median of life time T of new born.

i.e. ${}_xP_0 = 0.5$

$\therefore \ {}_xP_0 = 0.5 \Rightarrow \exp(x.(-0.0325)) = 0.5$

$\Rightarrow x = \frac{-\log 0.5}{0.0325} = 21.33$

Q.5

i) Gompertz law is sceitable model for human mortality for middle to older ages say 35 and over.

ii) As ${}_tP_x = \exp\left(-\int_0^t \mu_{x+s} \, ds\right)$

$\because \mu_x = BC^x$

$${}_tp_x = \exp\left(\int_0^t Bc^{x+s}\,ds\right)$$

we can write $c^{x+s} = c^x . e^{s \log c}$

$$\therefore {}_tp_x = \exp\left[-\int_0^t Bc^x . e^{s\log c}\,ds\right]$$

$$= \frac{Bc^x}{\log c}\left[e^{s\log c}\right]_0^t$$

$${}_tp_x = \frac{Bc^x}{\log c}\left[c^s\right]_0^t$$

$$= \frac{Bc^x}{\log c}\left(c^t - 1\right)$$

$$\therefore {}_tp_x = g^{c^x(c^t-1)}$$

Q.6

i) Female smoker aged 30 at entry.

ii) $\frac{h_j(t)}{h_i(t)} = \frac{\exp(-0.5)}{\exp(0.1)} = 0.866070$

j is male smoker aged 30 at entry
i is female smoker aged 40 at entry

As, $s(t) = \exp\left(-\int_0^t h(s)\,ds\right)$

$s_j(t) = [S_i(t)]^{0.86070}$

which shows $S_j(t) > S_i(t)$ for all $t > 0$

iii) $\frac{h_j(t)}{h_i(t)} = \frac{\exp(0.2)}{\exp(0.05)} = 1.161$

j is male smoker aged 30 at entry
i is female smoker aged 40 at entry

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As, $s(t) = \exp\int_0^t h(s)\,ds$

$\therefore S_j(t) = (S_i(t))^{1.161}$

$\therefore S_j(t) < S_i(t)$ for all $t > 0$.

Q.7

i) The most appropriate rate interval to use is policy year rate interval starting on policy aniversary where lives are aged x - next bday.
The average age at start of rate interval is $x - 1/2$ assuming that birthday are uniformaly distributed.

ii) $P_x(t) = 1/2\,(P_{x,t} + P_{x+1,t})$
we assume that policy anniversary are uniformly distributed over the calendar year.

$$E_x^c = \int_0^{10} P_x(t)\,dt = \frac{1}{2}\sum_{t=0}^{9} P_x(t) + P_x(t+1)$$

Now assuming population varies linearly bet'n dates of investigation.

$$E_x = E_x^c + 1/2 \sum_{t=0}^{10} O_{x,t}$$

Q.8

i) a) Type I censoring
b) Type II censoring
c) Random censoring

ii) The censoring will be informative.
The patients who died were probably recovering less well that patient who discharged from hospital.

iii) Keplan-Meirer estimate of survival function is estim-ated as follows.

T	n	d	c	d/n	(1-d)/n	S(t)
0	13					
5	13	1	0	0.0769	0.9231	0.92
7	12	1	0	0.0833	0.9167	0.85
14	11	1	2	0.0909	0.9091	0.77
28	8	1	2	0.1250	0.8750	0.67
35	5	1		0.2	0.8000	0.54

∴ the value ~~of~~ ~~so~~ survival function at end of inves-tigation period is 0.54.

Q.9

a) we have

$$\int_0^1 {}_tP_x \, dt = \int_0^1 (1 - {}_tP_x)\,dt = [t - 0.5t^2 q_x]_0^1$$

$$= 1 - 0.5 q_x$$

As $q_x = 0.3$, $m_x = \frac{0.3}{1-0.15} = 0.353$

b) $q_x = 1 - e^{-\mu}$

$$\int_0^1 {}_tP_x \, dt = \int_0^1 e^{-\mu t} dt = \frac{1}{\mu}(1-e^{-\mu}) = q_x/\mu$$

$\therefore m_x = \mu = -\ln(1-q_x) = -\ln 0.7 = 0.3566675.$

Q.10

i) In cox model, it is proportional to baseline hazard and constant of proportionality depending on certain measurable quantities called as co-variates. Hence is also called as cox proportional hazzard.

ii) Baseline hazzard is anual policy taken through online channel & whos premiums are paid by ~~credit~~ debit.

iii) we have

$\exp[(\beta_D \cdot 1)] / \exp[(\beta_D \cdot 1) + \beta_f \cdot 1 + \beta_M \cdot 1] = 0.75$

$\therefore \exp[\beta_f + \beta_M] = 4/3$ --- ①

$\exp[\beta_D \cdot 1] / \exp[\beta_f \cdot 1] = 1$ --- ②

$\exp[\beta_M \cdot 1] / \exp[\beta_D \cdot 2] = 0.75$ --- ③

from ② & ①

$\exp(\beta_D + \beta_M) = 4/3$

$\exp(\beta_D) * \exp(\beta_M) = 4/3$

from ③

$\exp(\beta_D)^2 \cdot 0.75 = \exp(\beta_M)$

$\therefore \exp(\beta_D) \cdot \exp(\beta_D)^2 \times 0.75 = 4/3$

$\exp(\beta_D)^3 = 1.7778$

$\exp(\beta_D) = 1.2114$

$\beta_D = 0.19179$

$\beta_F = 0.19179$

$\beta_M = 0.0959.$

Q.11

t	S(t)	$\lambda(t)$	nt	dt	ct
0	1	0	12	0	
1	0.9167	0.0833	12	1	2
3	0.7130	0.22	9	2	2
6	0.4278	0.4	5	2	3

∴ 2 insects died at duration 3 weeks
& 2 insects dies at duration 6 weeks

ii) Suming all deaths we have = 5
∵ we started with 12, remaining 7 are right censored.

Q.12

i) Gompertz law is,
$\lambda x = Bc^{x}$
where λx = force of mortality.

ii) Substituting $B = \exp(\beta_0 + \beta_1 x_1 + \beta_2 x_2)$
$\lambda x = \exp(\beta_0 + \beta_1 x_1 + \beta_2 x_2) \cdot c^{x}$ ⇒ x is duration since 50'th bday

∴ c^{x} is baseline which depend on time and $\exp(\beta_0 + \beta_1 x_1 + \beta_2 x_2)$ is depend on covariates. So ratio does not depend on time hence its proportional hazzard model.

iii) Baseline hazard of model is related to non smoker female.

iv) for female smoker, we have
$x_1 = 0$ & $x_2 = 1$, $x = 4$

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$\therefore$ hazard is given by

$\lambda x = \exp(\beta_0 + \beta_1 .0 + \beta_2 .1)\, c^4$

$= \exp(-4 + 0.65) \times 1.05^4$

$= 0.0351 \times 1.2155$

$\lambda x = 0.04266.$

v) hazard for non smoker is

$\lambda s = \exp(\beta_0 + \beta_1 x_1)\, c^s$

hazard for smoker at duration t is,

$\lambda t = \exp(\beta_0 + \beta_1 x_1 + 0.65)\, c^t$

if smoker & non hazard are ~~sme~~ same then

$\lambda s = \lambda t$

$\exp(\beta_0 + \beta_1 x_1) = \exp(\beta_0 + \beta_1 x_1 + 0.65) c^t$

$c^s = \exp(0.65)\, c^t$

$c^{s-t} = \exp(0.65) = 1.9155$

As $c = 1.05$

$1.05^{s-t} = 1.9155$

$s-t = \frac{\ln(1.9155)}{\ln(1.05)} = \frac{0.65}{0.04879}$

$s-t = 13.32$

$\therefore$ when two hazard are equal non-smoker is approx 13.32 yrs older than smoker.

Q.13

i) E_x^c is central exposed to risk at age label x

$E_x^c = \int_{t=0}^{2} P'(x)\,dt$, $P'(x)$ is nearest bday at time t

Assuming $P'_{56}(t)$ is linear.

$E^c_{56} = \frac{1}{2}(P'_{56}(2015) + P'_{56}(2016) + 1/2(P'_{56}(2016) + P'_{56}(20..$

$= 1/2 \cancel{P'(56)}\ P'_{56}(2015) + P'_{56}(2016) + 1/2\ P'_{56}(2017)$

$\because$ no. of policyholder aged label 56 nearest bday will be bet'n 5.5 & 56.5 i.e bet'n age label 55 last bday & 56 last bday.

Assuming that bday are uniformly dist'n.

$P'_{56}(2015) = 1/2\ (P_{55}(2015) + P_{56}(2015))$

$= 20050$

Also, $P'_{56}(2016) = 1/2\ (P_{55}(2016) + P_{56}(2016))$

$= 20800$

$P'_{56}(2017) = 1/2\ (P_{55}(2017) + P_{56}(2016))$

$= 19250$

$E^c_{56} = 1/2 \times 20050 + 20800 + 1/2\ 19250$

$= 40450$

$\mu_{56} = d_{56} / E^c_{56} = 1380/40450$

$= 0.0341$

ii) we can estimate initial rates of mortality using estimated values of μ from i)

$q_{55.5} = 1 - \exp(-\mu_{56}) = 0.0335$

And $q_{56.5} = 1 - \exp(-\mu_{57}) = 0.0352$.

Q.14

i) Hazard is given by,

$\lambda(t,z) = \lambda_0(t)\exp[0.3z_1 + 0.22z_2 + 0.3z_3 + 0.5z_4 - 0.1z_5 + 0.7z_6 + 0.15z_7 + 0.4z_8]$

ii) The prob of person who has been looking for life partner for one year will stay single for next 2 years is,

$\exp(-\text{integral}\,^3_1 \lambda(t,z)\,dt)$

If the person is female & 37 then

$P_F = \exp(-e^{0.37_1 - 0.12_5 - 0.42_8}\ \text{integral}|_1^3 \lambda_0(t)\,dt]$

Q.15

i) Advantages :-

- compared to initial exposure to risk, it is easy to calculate.
- It is difficult to model exposed risk in terms of underlying process being modelled.

ii) Rita → 3 months exposed to risk

Sita → 1 yr exposed to risk

Nita → ~~1 yr~~ 9 months exposed to risk

Gita → None exposed to risk

iii) Total exposed to risk

centra → $0.25 + 1 + 0.75 + 0 = 2$ yr.

Initial → $1 + 1 + 0.75 + 0 = 2.75$ yr.