

- 1) The nominal rate of discount per annum convertible quarterly is 8%.
- i) Calculate the equivalent force of interest.
- ii) Calculate the equivalent effective rate of interest per annum.
- iii) Calculate the equivalent nominal rate of discount per annum convertible monthly

Ans i) $d^{(4)} = 8\%$

$$s = \ln \left(\frac{1}{1-d} \right)$$

$$s = \ln \left(\frac{1}{1 - \frac{d^{(4)}}{4}} \right)^4$$

$$s = 4 \ln \left(\frac{1}{1 - \frac{d^{(4)}}{4}} \right)$$

$$s = 4 \ln \left(\frac{1}{1 - 0.02} \right)$$

$$s = 4 \ln \left(\frac{1}{0.98} \right)$$

$$s = 4 \ln \left(\frac{1}{0.98} \right)$$

$$s = 4 \ln \left(\frac{1}{0.98} \right)$$

$$s = 0.08081082927$$

$$s = 8.081082927\%$$

ii) $d^{(4)} = 8\%$

$$1+i = \left(\frac{1}{1 - \frac{d^{(4)}}{4}} \right)^4$$

$$1+i = \left(\frac{1}{1 - 0.02} \right)^4$$

$$1+i = 1.084165785$$

$$i = 0.084165785$$

$$i = 8.4165785\%$$

iii) $d^{(12)} = ?$

$$\left(1 - \frac{d^{(4)}}{4} \right)^4 = \left(1 - \frac{d^{(12)}}{12} \right)^{12}$$

Raising both sides by $1/12$

$$\left(1 - \frac{d^{(4)}}{4} \right)^{1/3} = 1 - \frac{d^{(12)}}{12}$$

$$\left(1 - 0.02 \right)^{1/3} = 1 - \frac{d^{(12)}}{12}$$

$$\frac{d^{(12)}}{12} = 0.006711611621$$

$$d^{(12)} = 8.0539339\%$$

2. The force of interest $\delta(t) = 0.004t + 0.0002t^2$ for all t .

i) Find the Present value of sum of rupees 1000 payable at the end of 10 years from now.

ii) Find constant annual effective rate of interest over a 10yr period.

Ans) $PV = 1000$

$$\delta(t) = 0.004t + 0.0002t^2$$

$$AV = PV e^{\int_0^{10} \delta(t) dt}$$

$$AV = 1000 e^{\int_0^{10} 0.004t + 0.0002t^2 dt}$$

$$AV = 1000 e^{\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3} \right]_0^{10}}$$

$$AV = 1000 \times e^{\left(\frac{0.004(10)^2}{2} + \frac{0.0002(10)^3}{3} \right)}$$

$$= 1000 \times e^{\left[0.002(10)^2 + \frac{0.0002(10)^3}{3} \right]}$$

$$= 1000 \times e^{[0.2 + 0.066667]}$$

$$= 1305.605172$$

$PV = ? \quad AV = 1000$

$$\delta(t) = 0.004t + 0.0002t^2$$

$$AV = PV e^{\int_0^{10} \delta(t) dt}$$

$$PV = AV e^{-\int_0^{10} 0.004t + 0.0002t^2 dt}$$

$$PV = 1000 e^{-\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3} \right]_0^{10}}$$

$$PV = 1000 e^{-\left(0.002(10)^2 + \frac{0.0002(10)^3}{3} \right)}$$

$$PV = 1000 \times e^{-0.266667}$$

$$= 1000 \times 0.7659283384$$

$$= 765.9283384$$

ii) Constant annual effective rate of interest (over 10 years period).

$$(1+i)^{10} = [AV/PV]$$

$$(1+i)^{10} = e^{\int_0^{10} 0.004t + 0.0002t^2 dt}$$

$$(1+i)^{10} = e^{\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3} \right]_0^{10}}$$

$$(1+i)^{10} = e^{\left[0.002t^2 + \frac{0.0002t^3}{3} \right]_0^{10}}$$

$$(1+i)^{10} = e^{0.2 + 0.06667}$$

$$(1+i)^{10} = e^{0.26667}$$

$$P = 0.0270254039$$

$$i = 2.70254039\%$$

or $AV = PV(1+i)^{10}$

$$1000 = 765.9283384(1+i)^{10}$$

$$(1+i)^{10} = \frac{1.305605172}{0.7659283384}$$

Raising by $^{1/10}$

$$(1+i)^{10} = 1.027025404$$

$$i = 0.027025404$$

$$i = 2.7025404\%$$

- 3) i Explain the relationship $d = iv$ by general reasoning where d is the effective annual rate of discount and i is the effective annual rate of interest.

- ii Calculate nominal rate of discount per annum convertible half yearly which is equivalent to

Q) An effective rate of discount of 2.25% per quarter.

b) A nominal rate of discount of 5% convertible every 2 years.

- iii A 91-day government bill provides the investors with an effective rate of return of 5%. Calculate the annual simple discount rate at which the bill is discounted. $1+i = \frac{1}{1-d}$ $1-d = \frac{1}{1+i}$ $d = \frac{1+i-i}{1+i}$ $d = \frac{i}{1+i}$

ii) $\frac{d^{(4)}}{4} = 2.25\% = 0.2225$ (Same as 4.1) $1 - d = 1 - 0.1 = 0.9$ $\frac{d}{2} = 0.05$ $\frac{d}{2} = 0.05$

$$d^{(2)} = 2$$

$$\left(1 - \frac{d(4)}{4}\right)^4 = \left(1 - \frac{d(2)}{2}\right)^2$$

Raising both sides by $\frac{1}{\sqrt{2}}$

$$\frac{d^2}{2} = 0.0259962537$$

$$d^{(2)} = 0.05199250715$$

$$d^{(2)} = 5.199250715$$

$$(1 - 0.0225)^{1/2} = 1 - \frac{d^2}{2}$$

iii) $i = 5\%$ $n = 91$ days.
 $sp = 2$ (annual).

$$\frac{d^2}{2} = 0.04449375$$

Let Btu amt be Rs 100

$$d^2 = 0.0889875$$

$$d^2 = 8.89875\%$$

$$\frac{PV}{AV} = AV (1+g)$$

$$= 100 (1 + 0.09)^{91/365}$$

$$= 100(1 + 0.05)^{91/365}$$

$$= 101.2238407$$

$$\left(\frac{1 - d^{(1/2)}}{1/2} \right)^{1/2} = \left(\frac{1 - d^{(2)}}{2} \right)^2$$

SD = ?

$$101.2238407 = 100 (1 + n^0)$$

$$1092238407 = 1 + \frac{919}{365}$$

$$(1 - 0.05 \times 2)^{1/2} = \left(1 - \underline{d^{(2)}}\right)^2$$

Raising both sides by $\frac{1}{2}$

$$100 = 101.2238407$$

$$C(1 - \frac{1}{2^2})^2 = 0.0490881583$$

$9 = 40908811583 //$

$$d = 4.8494619\%$$

$9 = 40908811583 //$

1) Explain the relationship $d=iV$ by general reasoning where d is the effective annual rate of discount and i is the effective annual rate of interest.

2) Given $i = 0.08$ Calculate

a) $d^{(12)}$

b) $i^{(365)}$

c) S

d) $i^{(0.5)}$

e) $S = \ln(1+i)$

$S = \ln(1+0.08)$

$S = 0.07696104114$

$S = 7.69610414\%$

3) $i = 0.08$

$d^{(12)} = ?$

$(1 - \frac{d^{(12)}}{12})^{12} = \frac{1}{1+0.08}$

Raising both sides by $1/12$

$1 - \frac{d^{(12)}}{12} = (\frac{1}{1.08})^{1/12}$

$1 - \frac{d^{(12)}}{12} = 0.993607102$

$d^{(12)} = 0.07671477614$

$d^{(12)} = 7.671477614\%$

d) $i^{(0.5)} = ?$ $i = 0.08$

$(1 + \frac{i^{(0.5)}}{0.5})^{0.5} = (1+0.08)$

Squaring both sides

$(1 + \frac{i^{(0.5)}}{0.5}) = (1+0.08)^2$

$\frac{i^{(0.5)}}{0.5} = 0.1664$

$i^{(0.5)} = 0.0832$

$i^{(0.5)} = 8.32\%$

b) $i^{(365)} = ?$ $i = 0.08$

$(1 + \frac{i^{(365)}}{365})^{365} = 1 + 0.08$

Raising both sides by $1/365$

$(1 + \frac{i^{(365)}}{365}) = (1.08)^{1/365}$

$i^{(365)} = 0.07696915541$

$i^{(365)} = 7.696915541\%$

9) $1-d = \frac{1}{1+i}$

$d = 1 - \frac{1}{1+i}$

$d = \frac{1+i-1}{1+i}$

$d = \frac{i}{1+i}$

d is the amt of money invested at the beginning w/ interest earned during the period when interest paid at the start of the

$v = \frac{1}{1+i}$ period (1) So d must

$d = iV$ be = to PV at the beginning of the year payments interest paid at end so $d = iV$

5) i) The rate of discount per annum convertible quarterly is 6%. calculate:

- The equivalent rate of interest per annum convertible $\frac{1}{2}$ yearly.
- The equivalent rate of discount per annum convertible monthly.

ii) On 15th April, 2005 Amit borrowed Rs 2,00,000 to be repaid one year later by single payment of Rs 220,000 Amit repaid the loan early on 17th July, 2005.

- Find the sum paid by Amit to terminate the contract assuming that the interest is reduced proportionately for early settlement.

i) $d^{(4)} = 6\%$

a) $i^{(2)} = ?$

$$\left(1 + \frac{i^{(2)}}{2}\right)^2 = \frac{1}{\left(1 - \frac{d^{(4)}}{4}\right)^4}$$

Raising both sides by $\frac{1}{2}$

$$\frac{1 + \frac{i^{(2)}}{2}}{2} = \frac{1}{\left(1 - \frac{0.06}{4}\right)^2}$$

$$\frac{1 + \frac{i^{(2)}}{2}}{2} = 1.030688768$$

$$i^{(2)} = 0.06137761552$$

$$\boxed{i^{(2)} = 6.137755152\%}$$

b) $d^{(12)} = ?$ $d^{(4)} = 6\%$

$$\left(1 - \frac{d^{(4)}}{4}\right)^4 = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

Raising both sides by $\frac{1}{12}$

$$\left(1 - \frac{d^{(4)}}{4}\right)^{\frac{1}{3}} = 1 - \frac{d^{(12)}}{12}$$

$$\left(1 - \frac{0.06}{4}\right)^{\frac{1}{3}} = 1 - \frac{d^{(12)}}{12}$$

$$d^{(12)} = 0.06030252528$$

$$\boxed{d^{(12)} = 6.030252528\%}$$

ii) b) $\frac{200,000}{15^{\text{th}} \text{ April } 2005} \quad \frac{220,000}{15^{\text{th}} \text{ April } 2006}$

$\frac{200,000}{15^{\text{th}} \text{ April } 2005} \quad \frac{?}{17^{\text{th}} \text{ July } 2005}$

n April May Jun July
15 31 30 17

93 days.

If paid on 15th April 2006

$$\frac{220,000}{200,000} = (1+i)$$

$$i = 0.1 \text{ or } 10\%$$

$$1+i = \left(1 + \frac{i^{(92/365)}}{92/365}\right)^{92/365}$$

Paid early 17th July 2005 n=92 days

$$AV = PV \left(1 + \frac{i^{(92/365)}}{92/365}\right)^{92/365}$$

$$= 200,000 \left(1 + 0.1\right)^{92/365}$$

$$1+0.1 = \left(1 + \frac{i^{(92/365)}}{92/365}\right)^{92/365}$$

Raising both sides by $365/92$

$$1.00(1.01)^{365/92} = 1 + \frac{92}{365}$$

$$0.4595567279 = \frac{92}{365}$$

$$i^{(92/365)} = 0.168334$$

If paid on 17th July 2005

$$\begin{aligned} 2200 \text{ AV} &= 200,000 \times A(0.92/365) \\ &= 200,000 \times (1 + 0.1)^{92/365} \\ &= \underline{204862.8548} \end{aligned}$$

$$\text{Interest paid for 93 days} = \frac{10}{100} \times \frac{93}{365} = 0.02547945205$$

$$\begin{aligned} \text{Amount to be paid if the contract is terminated on} \\ 17^{\text{th}} \text{ July 2005} &= 200,000 + [200,000 \times 0.02547945205] \\ &= 200,000 + [5095.8904] \\ &= \underline{205905.8904} \end{aligned}$$

6) i) The manufacturer of a certain toy sells to retailers on either the following terms :- a) cash payment : 30% below the recommended retail price.

b) Six months credit : 25% below the recommended retail price.

Find the effective annual rate of discount offered by the manufacturer to the retailers who pay cash as opposed by to those retailers who accept the credit terms

ii) Find $d^{(12)}$, $i^{(2)}$ and δ .

Let the retail price of the toy be Rs 100.

a) cash payment = 30% below the recommended retail price

$$\text{Cash payment} = 100 - \frac{30 \times 100}{100}$$

$$= 100 - 30 = 70.$$

b) Six month credit = 25% below the recommended retail price.

$$\text{Payment after 6 months} = 100 - \frac{25 \times 100}{100}$$

$$= 100 - 25 = 75.$$

Therefore the effective rate of discount is:-

$$70 = 75(1-d)^{0.5}$$

$$\frac{70}{75} = (1-d)^{0.5}$$

Squaring both sides.

$$\left(\frac{70}{75}\right)^2 = 1-d$$

$$d = 0.1288889$$

$$d = 12.88889\%$$

ii) $d^{(12)} = ?$

$$1-d = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

Raising both sides by $1/12$

$$(1 - 0.128889)^{1/2} = 1 - \frac{d^{(12)}}{12}$$

$$d^{(12)} = 0.137195439$$

$$d^{(12)} = 13.7195439\%$$

$$i^{(2)} = ?$$

$$\left(1 + \frac{i^{(2)}}{2}\right)^2 = \frac{1}{1-d}$$

Square Rooting both side

$$1 + \frac{i^{(2)}}{2} = \left(\frac{1}{1-0.128889}\right)^{1/2}$$

$$\frac{i^{(2)}}{2} = 0.07142857143$$

$$i^{(2)} = 0.1428571429$$

$$i^{(2)} = 14.28571429\%$$

$$\delta = \ln(1+i)$$

$$\delta = \ln\left(\frac{1}{1-d}\right)$$

$$\delta = \ln\left(\frac{1}{1-0.128889}\right)$$

$$\delta = 0.137985743$$

$$\delta = 13.7985743\%$$

- 7) If an investment fund offers to increase INR 20000 to INR 26000 in 17 months, calculate the following
- The nominal rate of interest per annum convertible quarterly.
 - The nominal rate of discount per annum convertible $\frac{1}{2}$ yearly.
 - Simple rate of interest per annum.
 - Comment on the results.

$n = 17 \text{ months}$

20000	26000
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$$\text{ii) } 26000 = 20000(1 + n i)$$

$$\frac{26000}{20000} = 1 + \frac{17}{12} i$$

$$9) \text{ } 26 \text{ } AV = PV \left(1 + \frac{i^{(4)}}{4} \right)^n$$

$$26000 = 20000 \left(1 + \frac{i^{(4)}}{4} \right)^{4 \times \frac{17}{12}}$$

$$0.3 = \frac{17}{12} i$$

$$\frac{26000}{20000} = \left(1 + \frac{i^{(4)}}{4} \right)^{17/3}$$

$$i = 0.2117647059$$

$$i = 21.17647059\%$$

Raising both sides by $3/17$

$$\left(\frac{13}{10} \right)^{3/17} = 1 + \frac{i^{(4)}}{4}$$

$$i^{(4)} = 0.1895525456$$

$$i^{(4)} = 18.95525456\%$$

$$\text{ii) } \left(1 - \frac{d^{(2)}}{2} \right)^2 = \left(\frac{1}{1 + \frac{i^{(4)}}{4}} \right)^4$$

Raising both sides by $1/2$

$$\left(1 - \frac{d^{(2)}}{2} \right) = \left(\frac{1}{1.047388136} \right)^2$$

$$\frac{d^{(2)}}{2} = 0.0884417655$$

$$d^{(2)} = 0.1768823531$$

$$d^{(2)} = 17.68823531\%$$

iv) Interest rates keep getting lower as n when the frequency gets higher. d is the lowest numerical value & i is the highest

$$d < d^{(2)} < i^{(4)} < i$$

The simple rate of interest has to be higher than the compound discount to maintain the same amount.

Explain with the help of graph, for an effective interest rate of 8% p.a. the relationship between equivalent nominal rate of interest convertible pthly ($i^{(p)}$) and p .

The relationship between $i^{(p)}$ and p can be represented as:

$$i^{(p)} = p[(1+i)^{1/p} - 1]$$

$$i = 8\%$$

$$p = 1, 2, 3, 4, 5, 10, 50, 365$$

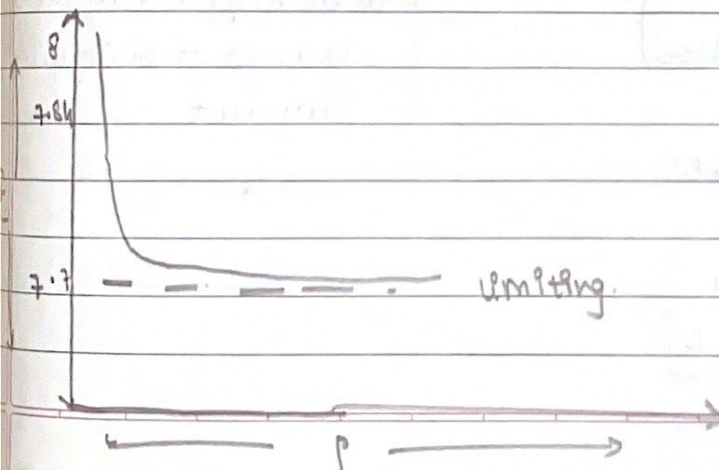
p	$i^{(p)}$
1	0.08 =
2	0.07846096908
3	0.07795670402
4	0.07770618763
5	0.07755639199
10	0.07725795243
50	0.07702030156
365	0.07696915541

As p increases, the value of i decreases.

The change is less significant, in going weekly, monthly, daily there is a limit to compounding.

The limiting case is when $p \rightarrow \infty$, there is instantaneous change and it keeps compounding the delta (δ).

Limiting interest rate
where $p \rightarrow \infty$



99) Define force of interest

ii) $i^{(2)} = 0.0775$

Calculate i , δ , $d^{(12)}$, $i^{(1/2)}$

i) Force of interest - Force of interest can be expressed as $\delta(t)$. There is a limiting factor to $i^{(p)}$ that limiting factor is δ (delta). It is the limiting interest rate where 'p' tends to infinity.

$$\lim_{p \rightarrow \infty} i^{(p)} = \delta$$

ii) $i^{(2)} = 0.0775$

$$\delta = \ln \left(1 + \frac{i^{(2)}}{2} \right)^2$$

$$\left(1 + \frac{i^{(2)}}{2} \right)^2 = 1 + i$$

$$\delta = 2 \ln \left(1 + \frac{0.0775}{2} \right)$$

$$\left(1 + \frac{0.0775}{2} \right)^2 = 1 + i$$

$$\delta = 2 \ln (1.03875)$$

$$1.079001563 = 1 + i$$

$$i = 0.079001563$$

$$\delta = 0.07603613438$$

$$i = 7.9001563\%$$

$$\delta = 7.603613438\%$$

$$d^{(12)} = ?$$

$$i^{(1/2)} = ?$$

$$\left(1 - \frac{d^{(12)}}{12} \right)^{12} = \frac{1}{\left(1 + \frac{i^{(2)}}{2} \right)^2}$$

$$\left(1 + \frac{i^{(2)}}{2} \right)^2 = \left(1 + \frac{i^{(1/2)}}{1/2} \right)^{1/2}$$

Raising both sides by $1/2$

Raising both sides by 2

$$\left(1 - \frac{d^{(12)}}{12} \right) = \left(\frac{1}{1.03875} \right)^{1/6}$$

$$\left(1 + \frac{0.0775}{2} \right)^2 = \left(1 + \frac{i^{(1/2)}}{1/2} \right)^{1/2}$$

$$1 - \frac{d^{(12)}}{12} = 0.9936836878$$

$$0.1642443719 = \frac{i^{(1/2)}}{1/2}$$

$$d^{(12)} = 0.0757957468$$

$$i^{(1/2)} = 0.08212218594$$

$$d^{(12)} = 7.57957468\%$$

$$i^{(1/2)} = 8.212218594\%$$

10 $\delta(t)$ the force of interest per annum at time t years is defined as:-

$$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \leq t \leq 5 \\ 0.06 & \text{for } 5 \leq t \leq 10 \\ 0.04 & \text{for } t \geq 10 \end{cases}$$

Using $\delta(t)$, Derive the expression for $v(t)$, the present value of 1 due at time t .

$$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \leq t \leq 5 \\ 0.06 & \text{for } 5 \leq t \leq 10 \\ 0.04 & \text{for } t \geq 10 \end{cases}$$

$$v(t) = e^{-\delta} \text{ or } \frac{1}{A(t)}$$

$$= \frac{1}{A(0,5) A(5,10) \times A(10,t)}$$

$$= e^{-\int_0^5 0.08 dt} \times e^{-\int_5^{10} 0.06 dt} \times e^{-\int_{10}^t 0.04 dt}$$

$$= e^{-(0.08t)_0^5} \times e^{-(0.06t)_5^{10}} \times e^{-(0.04t)_{10}^t}$$

$$= e^{-(0.08(5))} \times e^{-(0.06(10) - 0.06(5))} \times e^{-(0.04t - 0.4)}$$

$$= e^{-0.4} \times e^{-0.3} \times e^{-0.04t + 0.4}$$

$$= e^{-[0.3 + 0.04t]}$$

$$v(t) = e^{-(0.3 + 0.04t)}$$

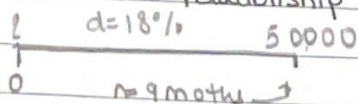
11 a) A 9-month loan, repayable by a single payment of 50000 issued at a rate of commercial discount of 18% p.a. What amt was initially lent to the borrower?

b) If $\delta = 6\%$ find $d^{(12)}$

" If $d^{(4)} = 6\%$ find $i^{(12)}$

c) Arrange the following quantities in ascending order of numerical value, giving brief reasons for the order assuming that they all corresponds to same effective interest rate $i, d, i^{(4)}, \delta$.

d) Explain in words the relationship $d = iv$ holds true.



b) ii) $d^{(4)} = 6\% = 0.06$
 $i^{(12)} = ?$

$$PV = AV (V(t))$$

$$PV = C (1-d)^n$$

$$= 50000 (1-0.18)^{9/12}$$

$$= 50000 (0.82)$$

$$PV = 43085.42623 \text{ Rs.}$$

$$(1+i) = \frac{1}{1-d}$$

$$\left(1 + \frac{i^{(12)}}{12}\right)^{12} = \left(\frac{1}{1-d^{(4)}/4}\right)^4$$

Raising both sides by $1/12$

$\delta = 6\%$ $d^{(12)} = ?$

$$\delta = \ln \left(\frac{1}{1-d} \right)$$

$$1 + \frac{i^{(12)}}{12} = \left(\frac{1}{1-0.06/4} \right)^{1/3}$$

$$0.06 = \ln \left(\frac{1}{1-d^{(12)}/12} \right)^{12}$$

$$e^{0.06} = \left(\frac{1}{1-d^{(12)}/12} \right)^{12}$$

$$\frac{i^{(12)}}{12} = 0.0050590721$$

$$i^{(12)} = 0.06060708865$$

$$i^{(12)} = 6.060708865\%$$

Raising both sides by $1/12$

$$\frac{1-d^{(12)}/12}{1} = \frac{1}{e^{0.06/12}}$$

$$\frac{d^{(12)}}{12} = 0.004987620807$$

$$d^{(12)} = 0.05985024969$$

$$d^{(12)} = 5.985024969\%$$

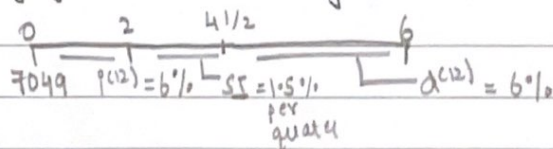
Q) $i, d, i^{(4)}, s$

$$d < s < i^{(4)} < i$$

The order is purely based on the payment of interest d is payable at the beginning of the year, immediate payment. Hence the numeric percentage would be less/smaller than i which is interest payable at end. Similarly s and $i^{(4)}$ is also less than i as s (force of interest) is at least due to force of interest and $i^{(4)}$ is nominal interest convertible quarterly.

a) Same question (answer in 4(i))

- 12 A deposit of 7049 is accumulated at the following rates of interest: 6% per annum nominal convertible month for the 1st 2 years followed by 1.5% per quarter simple interest next 2 ^{1/2} years followed by rate of discount of 6% per annum convertible monthly for next 1 & 1/2 years. Calculate amt after 6 years.



$$\begin{aligned} \text{Accumulated value} &= 7049 \times A(0, 2) \times A(2, 4\frac{1}{2}) \times A(4\frac{1}{2}, 6) \\ &= 7049 \times \left(1 + \frac{i^{(12)}}{12}\right)^{24} \times (1 + 10 \times s) \times \frac{1}{\left(1 - \frac{d^{(12)}}{12}\right)^{\frac{3 \times 12}{2}}} \\ &= 7049 \times \left(1 + \frac{0.06}{12}\right)^{24} \times (1 + 0.15) \times \left(\frac{1}{1 - \frac{0.06}{12}}\right)^{18} \end{aligned}$$

$$= 7049 \times (1.127159776) \times (1.15) \times 1.0904421325$$

$$\boxed{AV = 9999.893613}$$

13 Calculate the time, in years for an investment to double at

- i) An effective rate of interest of 10% p.a
- ii) A nominal discount rate of 10% p.a convertible monthly.

Let the initial investment be x Rs

$$AV = 2(x) = 2x$$

$$\begin{array}{ccc} x & & 2x \\ | & & | \\ 0 & & ? \end{array} \quad i = 10\%$$

$$2x = x(1+i)^n$$

$$2 = (1 + 0.1)^n$$

$$2 = (1.1)^n$$

taking log on both sides

$$n \log 1.1 = \log 2$$

$$n = 69.66071689$$

$$n = 7.0272540902 \text{ years}$$

$$ii) d^{(12)} = 10\%$$

$$2x = x(1-d)^n$$

$$2 = (1 - \frac{d^{(12)}}{12})^{12n}$$

$$\frac{1}{2} = (1 - \frac{d^{(12)}}{12})^{12n}$$

taking log on both sides

$$12n \ln(1 - \frac{0.1}{12}) = \ln \frac{1}{2}$$

$$12n \ln 0.99166667 = \ln \frac{1}{2}$$

$$12n = 82.83060471$$

$$n = 6.902550392 \text{ years}$$