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Probablity assignment - I

Q-1 $X \sim Poi(\theta)$

$$\frac{x-\theta}{\sqrt{\theta}} \stackrel{\circ}{\sim} N(0,1)$$

$$P(x=200) = P[199.5 < z < 200.5]$$
$$= P\left[\frac{199.5-\theta}{\sqrt{\theta}} < z < \frac{200.5-\theta}{\sqrt{\theta}}\right]$$

$$95\% \ CI = \theta \pm 19.6\sqrt{\theta}$$

Q-2 $X_i = 1, 2, 3 \ldots n$

$$\bar{X} = \frac{\sum x_i}{n}, \quad S^2 = \frac{1}{n-1}\sum^n (x_i - \bar{x})^2$$

i) $\bar{x} = \frac{\sum x_i}{n}$

$$E(\bar{x}) = \frac{\sum E(x_i)}{n} = \frac{\sum \mu}{n} = \frac{n\mu}{n} = \mu$$

$$var(\bar{x}) = V\left(\frac{\sum x_i}{n}\right) = \frac{1}{n^2}\sum V(x_i)$$
$$= \frac{\sum \sigma^2}{n^2} = \frac{n\sigma^2}{n^2} = \sigma^2/n$$

(ii) $S^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$

$E(S^2) = \frac{1}{n-1} \left[\sum E(x_i)^2 - nE(\bar{x})^2 \right]$

$= \frac{1}{n-1} \sum (\sigma^2 + \mu^2) - n\left(\frac{\sigma^2}{n} + \mu^2\right)$

$= \frac{1}{n-1} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2)$

$= \frac{\sigma^2 (n-1)}{n-1} = \sigma^2$

(iii) $V\left(\frac{(n-1)S^2}{\sigma^2}\right) = V\left(\chi^2_{n-1}\right)$

$= 2(n-1)$

$\frac{(n-1)^2}{\sigma^4} \cdot V(S^2) = 2(n-1)$

$V(S^2) = \frac{2(n-1)\sigma^4}{(n-1)^2}$

$= \frac{2\sigma^4}{n-1}$

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iv] $n = 10$, $\sigma^2 = 100$

Q.3] $X \sim Bin(n, p)$ $\therefore n = 180$

i) $L(P) = \prod_{i=0}^{4} {}^{180}C_0 p^0 q^{180}$
$\times {}^{180}C_1 p^1 q^{179} \times {}^{180}C_2 p^2 q^{178}$
$\times {}^{180}C_3 p^3 q^{177} \times {}^{180}C_4 p^4 q^{176}$

$= constant \times p^0 q^{180} \times p^1 q^{178}$
$\times p^2 q^{178} \times p^3 q^{177} \times p^4 q^{176}$

$Log\, L(P) = constant + 0 +$
$180 \log p + 1 \log 1 + 179 \log q$
$+ 2 \log p + 178 \log q + 3 \log p$
$+ 177 \log q + 4 \log p + 176 \log q$

$constant + 10 \log p + 90(1-p)$
$\frac{d}{dp} \log(p) = \frac{-890}{\log p}$

Q-4 $f(x) = \frac{c\beta^3}{(x+\beta)^4}$; $x > 0, \beta > 0$

1] $E(x) = \int x \cdot f(x)\, dx$

$\int_0^\infty x \cdot \frac{c\beta^3}{(x+\beta)^4}\, dx$

Q-5 $f(x) = \frac{1}{b-a}$

$E(x) = \int_a^b x \cdot f(x)\, dx$

$= \int_a^b x \cdot \frac{1}{b-a}\, dx$

$= \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b = \frac{b^2-a^2}{2(b-a)}$

$\frac{\cancel{(b-a)}(b+a)}{2\cancel{(b-a)}}$

$E(x) = \frac{a+b}{2}$

$E(x)^2 = \int_a^b x^2 (f(x))\, dx$.

$= \int_a^b x^2 \frac{1}{b-a}\, dx = \frac{1}{b-a}\left(\frac{x^3}{3}\right]_a^b$

$= \frac{b^3-a^3}{3(b-a)}$

$b^3-a^3 = (b-a)(b^2+a^2+ab)$

$E(x^2) = \frac{a^2+b^2+ab}{3}$

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$$\bar{X} = \frac{1}{n}\sum x_i \quad (3)$$

$$x_2 = \frac{1}{n}\sum x_i^2 \quad (4)$$

$$E(x) = \bar{x} \qquad \frac{a+b}{2} = \bar{X}$$

$$E(x^2) = \bar{X}_2 \qquad \frac{a^2+b^2+ab}{3} = x_2$$

$$a+b = 2\bar{x} \Rightarrow a = 2\bar{x} - b$$

$$(2\bar{x}-b)^2 + b^2 + (2\bar{x}-b)b = 3x$$

$$b^2 + 4x^2 - 4b\bar{x} + \cancel{b^2} + \cancel{b^2} + 2b\bar{x}$$

$$= 3\bar{x}_2$$

$$b^2 - 2bx + 4x^2 - 3\bar{x} = 0$$

$$\therefore \hat{b} = \bar{x} + \sqrt{3}(x_2 - \bar{x}^2)$$

(iii) $n = 2, 4, 6, 8, 10 \rightarrow U(2, 10)$

$$x = \frac{a+b}{2} = \frac{2+10}{2} = 6$$

$$3^2 = \frac{(10-2)^2}{12} = 5.333$$

iv] $f(x) = \frac{1}{b-a}$

$$L(b) = \prod_{i=1}^{n} \frac{1}{b-a}$$

$$= \frac{1}{b^2 - a^n} = \frac{1}{(b-a)^n}$$

$$= (b-a)^n$$

$$\log(b) = n \log(b-a)$$

$$\frac{d}{db} \log L(b) = \frac{n}{b-a}(-a)$$

$$= \frac{-na}{b-a}$$

Q 7 ii) $\overline{x_1} = 30$
$\overline{x_2} = 35.0556$

$\bar{x} = 35.0556$
95% of CI $= 35.0556 \pm 1.96 \sqrt{\frac{59.91358}{9}}$

$s_1^2 = 57.1667$
$s_2^2 = 59.91358$

$\bar{x} = 35.0558$

$s_2^2 = 59.91358$

$= (29.99885, 40.11265)$

Q.5 (i) If we have a random sample
$X = (X_1, X_2 \ldots, X_n)$ from a distribution with an unknown parameter θ & $g(x)$ is an estimator of θ, it seems desirable that $E(g(x)) = \theta$

(ii) When the expected value of the true value is not equal it is called bias.

(iii) The mean square error of the estimation $\hat{\theta}$ is $MSE(\hat{\theta}) = E[(\hat{\theta} - \theta)^2]$

iv) An estimator with the lower MSE is more efficient. Therefore, $\hat{\theta}$ is more efficient the $\hat{\theta}$

v) They both be considered consistent when both MSE are

vi) we can estimate θ by using methods of moments & maximum likelihood.

(ii) The sampling distribution for x ie $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

or $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

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The t_k variable is defined by $t_k = \frac{N(0,1)}{\sqrt{\frac{\chi^2_k}{k}}}$

where the $N(0,1)$

& χ^2_k random variables are independent.

$x-\mu$ is the $N(0,1)$ &

$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$ is the

χ^2_k

(ii) t_k distribution has mean of variance $\frac{k}{k-2}$

(iii) $n=10$

$\bar{x} = 50$

$\sigma^2 = 48.667$

$\frac{\bar{x}}{\sigma/\sqrt{n}} \sim N(0,1)$

99% of CI $= \bar{x} \pm t_{\alpha/2, n-1} \sqrt{\sigma^2/n}$

$= 50 \pm 3.250 \sqrt{\frac{48.667}{10}}$

$(142.83, 57.169)$

$n = 12$

$\Sigma x_i = 21320$

$\Sigma x_i^2 = 4919860000$

$\bar{X} = 17767 \quad s^2 = 10144$

$\bar{X} = \frac{\Sigma xi}{n} \quad s^2 = \frac{1}{n-1}[(\Sigma xi - \bar{x})^2]$

$\Sigma xi = 213204$

$x = \frac{\Sigma x}{12}$

$\Sigma x_N = 213204 - 11000$
$- 48000 + 27500 + 31500$
$= 213204$

$\bar{X}_N = \frac{213204}{12} = 17767$

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$$SD^2 = \frac{1}{11}(4919860000 - 12 \times 177767)$$

$$SD^2 = 447240617.8$$

$$SD = 21148.06416.$$