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Calculus

Assignment - 1 -

A1). $\lim_{h\to 0} \frac{2(-3+h)^2 - 18}{h}$

$\Rightarrow \lim_{h\to 0} \frac{2(9+h^2 - 6h) - 18}{h}$

$\Rightarrow \lim_{h\to 0} \frac{\cancel{18} + 2h^2 - 12h - \cancel{18}}{h}$

$\Rightarrow \lim_{h\to 0} \frac{\cancel{h}(2h-12)}{\cancel{h}}$

$\Rightarrow \lim_{h\to 0} 2h - 12$

$= -12$ Ans

A2). $g(x) = \begin{cases} 2x & , x < 6 \\ x-1 & , x \geq 6 \end{cases}$

i) at $x = 4$.

$g(4) = 2(4) = 6$

LHS $\Rightarrow \lim_{x\to 4^-} g(x)$

$\Rightarrow \lim_{h\to 0} g(4-h)$

$\Rightarrow \lim_{h\to 0} 2(4-h)$

$\Rightarrow \lim_{h\to 0} 6 - 2h$

$= 6$

RHS $\Rightarrow \lim_{x\to 4^+} g(x)$

$= \lim_{h\to 0} g(4+h)$

$= \lim_{h\to 0}$ $2x+x-1$ ~~2x+x+1~~

$= \lim_{h\to 0} 3(4+h) - 1$

$= \lim_{h\to 0} 12+3h-1 = 12-1$

$= 11$ Ans

Since LHS $\neq$ RHS

$\Rightarrow g(x)$ is discontinuous at $x = 4$.

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(iii). at $x=6$

$$g(x) = 6-1 = \underline{5}$$

LHS $\Rightarrow \lim_{x\to 6^-} g(x)$

$= \lim_{h\to 0} g(6-h)$

$= \lim_{h\to 0} 2(6-h)$

$= \lim_{h\to 0} 12-h$

$= 12$

Since, ~~Sto~~ $g(x) \neq$ LHS,
the function is discontinuous at $x=6$.

A3). $\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}}$

$= \lim_{x\to 9} \frac{x-9\,(3+\sqrt{x})}{(3+\sqrt{x})(3-\sqrt{x})}$

$= \lim_{x\to 9} \frac{\cancel{x-9}\,(3+\sqrt{x})}{\cancel{9-x}}$

$= \lim_{x\to 9} -1(3+\sqrt{x})$

$\cancel{-3-\sqrt{9}}$

$= \lim_{x\to 9} -3-\sqrt{9}$

$= \lim_{x\to 9} -3-3$

$= -6$

A4). a).

$\Rightarrow$

$\Rightarrow$

Since

b). x=

Since

Co

c). x

Since

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A4). a). $x = -1$

$$f(-1) = \frac{4(-1)+5}{9-3(-1)}$$

$\Rightarrow$ $$f(-1) = \frac{5-4}{9+3}$$

$\Rightarrow$ $$f(-1) = \frac{1}{12}$$

$\Rightarrow$ $$\lim_{x \to -1} \left(\frac{4x+5}{9-3x}\right) = \frac{1}{12}$$

Since $f(x)$ is defined the the denominator $\neq 0$, the function is continuous.

b). $x = 0$

$$\lim_{x \to 0} \frac{4x+5}{9-3x}$$

$$= \lim_{x \to 0} \frac{4(0)+5}{9-3(0)}$$

$$= \frac{5}{9}$$

Since ~~f()~~ the denominator is $\neq 0$, the function is continuous.

c). $x = 3$.

$$\lim_{x \to 3} \frac{4x+5}{9-3x}$$

$$= \lim_{x \to 3} \frac{4(3)+5}{9-3(3)}$$

$$= \frac{17}{0}$$

Since, the denominator is 0, the f(x) is discontinuous.

A5). $\lim_{x \to \infty} \dfrac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$= \lim_{x \to \infty} \dfrac{\dfrac{6e^{4x} - e^{-2x}}{\cancel{8}e^{4x}}}{\dfrac{8e^{4x} - e^{2x} + 3e^{-x}}{\cancel{8}e^{4x}}}$

$= \lim_{x \to \infty} \dfrac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}}$

Since $x \to \infty$, $-e^{-6x}$, $-e^{-2x}$ & $+3e^{-5x}$ tends to 0.

$= \dfrac{6}{8}$

$\boxed{= \dfrac{3}{4}}$

A6). $g(y) = \begin{cases} y^2+5 & , y < -2 \\ 1-3y & , y \geq -2 \end{cases}$

i) $\lim_{y \to 6} g(y)$

$= \lim_{y \to 6} 1 - 3y$

$= \lim_{y \to 6} 1 - 3(6)$

$\boxed{= -17}$

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iii) $\lim_{y \to -2} g(y)$

$= \lim_{y \to -2} 1 - 3y$

$= \lim_{y \to -2} 1 - 3(-2)$

$\boxed{= 7}$

*7) let $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$
be $\cancel{y} = u.v.$

$\frac{du}{dt} = \frac{d(4t^2 - t)}{dt} = 8t - 1$

$\frac{dv}{dt} = \frac{d(t^3 - 8t^2 + 12)}{dt} = 3t^2 - 16t$

$\frac{dy}{dt} = u\frac{dv}{dt} + v\frac{du}{dt}$

$= (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$

$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - t^3 - 64t^3 + 8t^2 + 96t - 12$

$\boxed{= 20t^4 - 132t^3 + 24t^2 + 96t - 12.}$

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A8) Let $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

$$\frac{dy}{dw} = \frac{(2w^2+1)(4w^3+3) - (3w+w^4)4w}{(2w^2+1)^2}$$

$$= \frac{8w^5 + 6w^2 + 4w^3 + 3 - 12w^2 - 4w^5}{(2w^2+1)^2}$$

$$= \frac{8w^5 - 4w^5 + 4w^3 - 6w^2 + 3}{(2w^2+1)^2}$$

$$\boxed{= \frac{4w^5 + 4w^3 - 6w^2 + 3}{(2w^2+1)^2}}$$

Ans

A10) $y = z^5 - e^z \ln(z)$

$$\frac{dy}{dz} = 5z^4 - \left[e^z\left(\frac{1}{z}\right) + \ln(z)\, e^z\right]$$

$$\boxed{= -e^z \ln(z) - \frac{e^z}{z} + 5z^4}$$

Ans

A9) $f(x) = 2e^x - 8^x$

$$\frac{dy}{dx} = \boxed{2e^x - 8^x \log(8).}$$

~~A11) a)~~ $\cancel{f(x) = (6x^2+7x)^4}$

$$\cancel{\frac{dy}{dx} = 4(6x^2+7x)^3(12x+7)}$$

$$\cancel{\frac{dy}{dx} = 4x^3(6x+7)^3(12x+7)}$$

A11) a) $f(x) = (6x^2+7x)^4$

$$\frac{dy}{dx} \boxed{= 4(6x^2+7x)^3(12x+7)}$$

$$\cancel{= (24x^2+28x)^3(12x+7)}$$

b) $f(t) = 5 + e^{4t+t^7} = e^{t^7+4t} + 5$

$$\frac{dy}{dt} \boxed{= (7t^6+4)\, e^{t^7+4t}}$$

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A12) a)
$$z = \ln(7 - x^3)$$

$$\frac{dy}{dz} = \frac{1}{7-x^3} \times (-3x^2)$$

$$\Rightarrow \frac{dy}{dz} = \frac{-3x^2}{7-x^3}$$

$$\Rightarrow \frac{dy}{dz} = \frac{3x^2}{x^3-7}$$

$$\Rightarrow \frac{d^2y}{dz^2} = \frac{(x^3-7)(6x) - (3x^2)(3x^2)}{(x^3-7)^2}$$

$$= \frac{6x^4 - 42x - 9x^4}{(x^3-7)^2}$$

$$= \frac{-3x^4 - 42x}{(x^3-7)^2}$$

$$\boxed{= \frac{-3x(x^3+14)}{(x^3-7)^2}}$$

Ans

b)

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b) $Q(v) = \dfrac{2}{(6+2v-v^2)^4}$

$$\frac{dy}{dv} = \frac{\cancel{2}\ 0 - 2\times[4(6+2v-v^2)^3(-2v+2)}{(6+2v-v^2)^8}$$

$$\cancel{= 0 - 2[4(6}$$

$$= \frac{-2\times 4\times(-2v+2)}{(6+2v-v^2)^5}$$

$$= \frac{-8(2-2v)}{(-v^2+2v+6)^5}$$

$$= \frac{-16+16v}{(-v^2+2v+6)^5} = \frac{-16(v-1)}{(-v^2+2v+6)^5}$$

$$\frac{d^2y}{dv^2} = \frac{[(-v^2+2v+6)^5(-16)] - [-16(v+1)]\ 5(-v^2+2v+6)^4(-2v+2)}{(-v^2+2v+6)^{10}}$$

$$\cancel{= (16v^2-32v-96)^5 - [32v-32v}$$

$$= \frac{\cancel{(-v^2+2v+6)^4}\ [(-16)(-v^2+2v+6) - (-16v+16)(-2v+2)5]}{(-v^2+2v+6)^{\cancel{10}}}$$

$$= \frac{+16v^2-32v-96-5(32v^2-32v-32v+32)}{(-v^2+2v+6)^6}$$

$$= \frac{144v^2+288v-256}{(-v^2+2v+6)^6}$$

$$\boxed{= \frac{16(9v^2+18v-16)}{(-v^2+2v+6)^6}}$$

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A c) $f(t) = \ln(1+t^2)$

$$\cancel{\frac{d^2y}{dt}} \frac{dy}{dt} = \frac{2t}{t^2+1}$$

$$\frac{d^2y}{dt^2} = \frac{(t^2+1)(2) - 2t(2t)}{(t^2+1)^2}$$

$$= \frac{2t^2+2 - 4t^2}{(t^2+1)^2}$$

$$\boxed{= \frac{-2(t^2-1)}{(t^2+1)^2}}$$

A13). $f(x) = x^3 - 3x^2 - 9x + 12$

$$\frac{dy}{dx} = 3x^2 - 6x - 9 = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow x^2 - 3x + x - 3 = 0$$

$$\Rightarrow x(x-3) + 1(x-3) = 0$$

$$\Rightarrow (x+1)(x-3) = 0$$

$$\Rightarrow x = -1 \quad \text{or} \quad x = 3.$$

$$\frac{d^2y}{dx^2} = 6x - 6$$

for $x = -1$	for $x = 3$
$6(-1) - 6$	$6(3) - 6$
$= -12$	$= 12$

$\therefore$ The max. value for $f(x)$ is 12 & min. value is -12

$\frac{)(2) - 2t(2t)}{(t^2+1)^2}$

$\frac{- 4t^2}{+1)^2}$

$\boxed{\frac{^2-1)}{1)^2}} >$

$+12$

$= 0$

$= 0$

$3 = 0$

$x-3) = 0$

$(x-3) = 0$

or $x = 3$.

for $x = 3$

$6(3) - 6$

$= 12$

12 & min. value is -12

A14). $f(x) = 4x^3 - 18x^2 + 24x - 7$

$\frac{dy}{dx} = 12x^2 - 36x + 24 = 0$

$\Rightarrow x^2 - 3x + 2 = 0$.

$\Rightarrow \cancel{x^2 - 3x + 3x + 2 = 0}$

$\Rightarrow \cancel{x(x-3)}$

$\Rightarrow x^2 - x - 2x + 2 = 0$

$\Rightarrow x(x-1) - 2(x-1) = 0$

$\Rightarrow (x-1)(x-2) = 0$

$x = 1 \quad | \quad x = 2$.

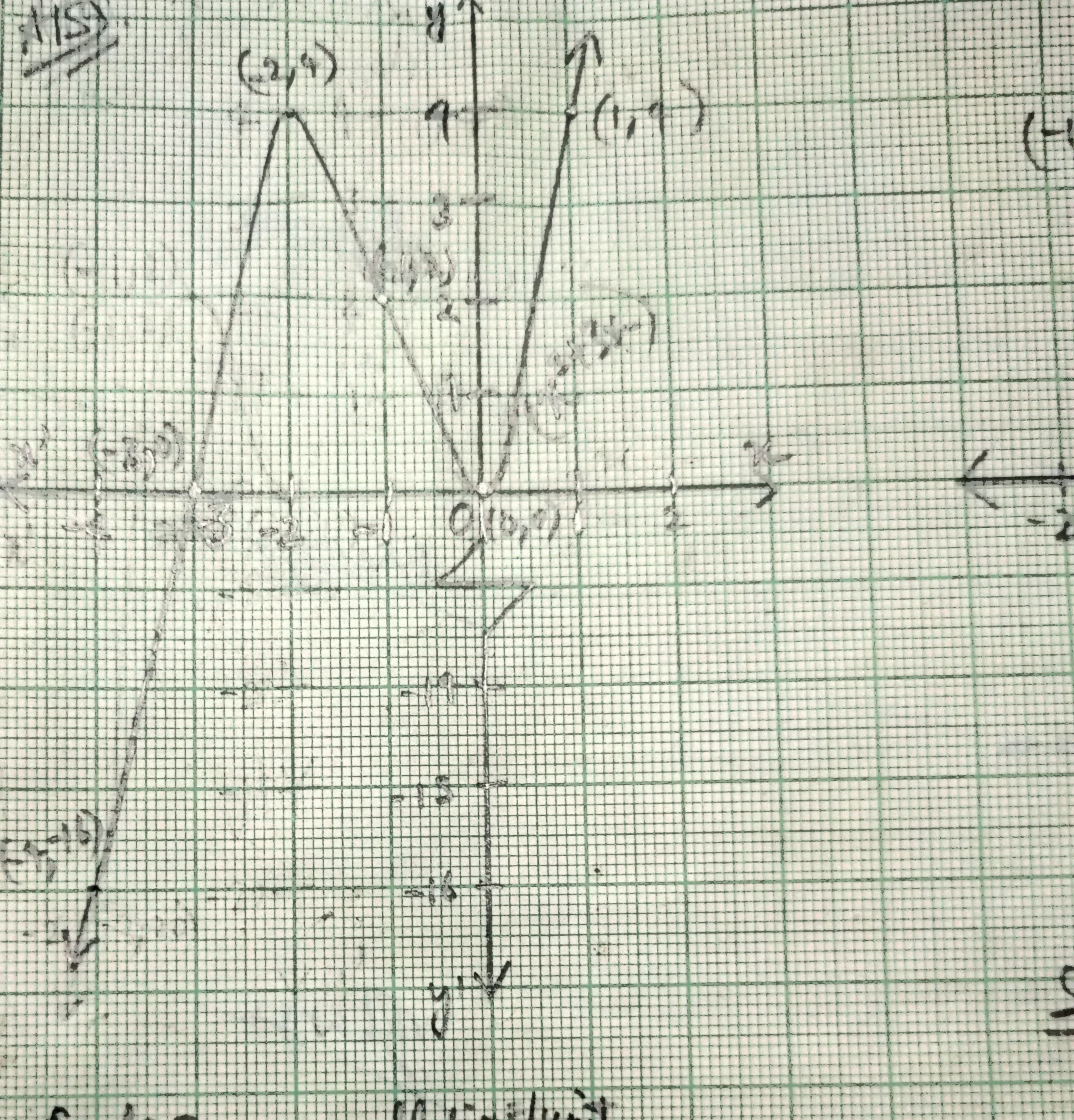
$\frac{d^2y}{dx^2} = 24x - 36$.

for $x = 1$	for $x = 2$
$24(1) - 36$	$24(2) - 36$
$= -12$	$= 12$

$\therefore$ Ther max. value of $f(x)$ is 12 & min. value of $f(x)$ is -12.

A15). $f(x) = x^2(x+3) \Rightarrow y = x^3 + 3x^2$

x	0	1	−1	−2	−3	−4
y	0	4	2	+4	0	−16



Scale –
x axis – 10 small div = 1 unit
y axis – 10 small div. = 1 unit

A1(b): $f(x) = 3x^2 - 6x = y$

x	0	1	-1	2	3
y	0	-3	9	0	9