1) $S = 65$, $\sigma = 25\%$ p.a, $r = 2\%$ p.a, $K = 55$, $T = 0.5$

i) $C_t = S_0 \phi(d_1) - K e^{-rT} \phi(d_2)$

$$d_1 = \frac{\ln\left(\frac{65}{55}\right) + \left(2\% + \frac{20\%^2}{2}\right) 0.5}{20\% \sqrt{0.5}} = 1.08996$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.9138$$

$$C_t = 65\,\phi(1.09) - 55 e^{-0.02 \times 0.5}\,\phi(0.93448)$$
$$= 11.4646$$

ii) Delta of call option $= \dfrac{\text{change in price of call option}}{\text{change in price of underlying}}$

iii) $\Delta_c = \phi(d_1)$, $\Delta_p = -\phi(d_1)$

$\cancel{\Delta_c = 0.86}$

iv) $\Delta_p = -0.13786$

2) i) Delta $\Rightarrow \dfrac{\text{change in Price of Derivative}}{\text{change in price of underlying}}$

Vega $\Rightarrow \dfrac{\text{Change in Price of option}}{\text{Change in volatility of underlying asset.}}$

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ii) Putcall Parity $\Rightarrow$

$$\boxed{C + K\exp(-rT) = P + S}$$

$$\Rightarrow C - P = S - Ke^{-rT}$$

$$\frac{d}{d\sigma}(C-P) = \frac{d}{d\sigma}(S-Ke^{-rT})$$

$$V_C - V_P = 0 \qquad \text{(S and K are constant)}$$

$$\boxed{V_C = V_P}$$

iii) $S_0 = 55$, $X = 50$, $\sigma = 25\%$ pa, $r = 5\%$, $T = 1$

$$d_1 = \frac{\ln\left(\frac{55}{50}\right) + \left(5\% + \frac{1}{2} \times 25\%^2\right) \times 1}{25\%} = 0.70624$$

$$d_2 = 0.70624 - 25\% = 0.4562$$

$$C_t = 55(0.76\,8115) - 50\,e^{-0.05}(0.67729)$$
$$= \$9.69$$

$P = 2.211 \leftarrow$ from Putcall parity

iv) Delta hedging → Portfolio for which overall delta = 0. The Portfolio is immune to change in the price of U.A.

Vega hedging → Portfolio where overall vega = 0. The Portfolio is immune to small changes in the assumed level of volatility.

v) call / put

	PV	Delta	Vega
Call	$C = 9.6525$	Δc	V_c
Put	$P = 2.2140$	Δp	V_p
Forward		1	0

Vega hedging → $\boxed{xV_c + yV_p + zV_F = 0}$

$[V_c = V_p, \; V_F = 0]$

$V_c(x+y) = 0$

$x + y = 0$

$\boxed{y = -x}$

Delta hedging → $\boxed{x\Delta_c + y\Delta_p + z\Delta_F = 0}$

$[y = -x, \; \Delta p = \Delta_c - 1], \; (\Delta F = 1)$

$x\Delta c - x(\Delta_c - 1) + z = 0$

$x + z = 0$

$\boxed{x = -z}$, $x = -y = -z$

Portfolio = 1000

$x \cdot C + y \cdot P + z \cdot F = 1000$

$(F = 0)$

$x \cdot C + y \cdot P = 1000$

$x \cdot 9652S + y \cdot 2.290 = 1000$

$x = 134.43$

$y = z = -134.43$

Portfolio → long posi. 134 call opn

short " 134 put opn

short " 134 forward

Q3) $\boxed{C_t = E\left(e^{-r(T-t)} C_T / F_t\right)}$

Price of Derivation in Black scholes

C_T = payoff under derivation at T

C_t = derivative value at t.

F_t = filtration @ $t > 0$.

ii) $S_0 = 50$, $X = 49$, $r = 5\%$, $\sigma = 25\%$, $T = 0.5$

$$\boxed{C_t = S_0 \phi(d_1) - k e^{-rT} \phi(d_2)}$$

$$d_1 = \frac{\ln\left(\frac{50}{49}\right) + \left(5\% + \frac{1}{2} \cdot 25\%^2\right) 0.5}{25\% \sqrt{0.5}} = 0.399$$

$d_2 = 0.1673$

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$\phi(d_1) = 0.6346$

$\phi(d_2) = 0.5669$

$C_t = 50 \times 0.6346 - 49 e^{-0.05 \times 30} (0.5664)$

$= 4.66$

iii) 4.66 (Same as European call)

iv) $S_t + p = c + Ke^{-rT}$

$p = 4.66 + 49 . e^{-0.05 \times 28} - 50$

$p = 2.75$

v) Value of underlying asset will fall

Q4) $\tilde{Z}_t = Z_t + \int_0^t \gamma_s \, ds$

$\tilde{Z}_t$ = SBM under Q

If Z_t is SBM under P,

$\tilde{Z}_t = Z_t + \int_0^t \gamma_s \, ds$

Q5) i) Delta = $\Phi(d_1)$

$= \dfrac{\text{chg. in price of U.A}}{\text{chg in price of a call option}}$

ii) $d_1 \Rightarrow$

$$0.3 = \frac{\ln\left(\frac{40}{45.71}\right) + (0.02 + 0.5\sigma^2)5}{\sigma\sqrt{5}}$$

$0.3\sqrt{5} = -0.1378 + (0.1 + 2.5\sigma^2)$

$2.5\sigma^2 - 0.67083\sigma - 0.0375 = 0$

$$\sigma = \frac{0.67083 + \sqrt{0.67083^2 - 4 \times 2.5(0.0375)}}{2 \times 2.5}$$

$\sigma = 32\%$

i) $R_0 = 80$, $\sigma = \sqrt{15\%}$ p.a

$\frac{S_1}{S_0} > K_S$, $\frac{R_1}{R_0} < K_R$

Payoff $\Rightarrow$ $Ce^{-rT} Q\left(\frac{S_1}{S_0} < K_S\right) Q\left(\frac{R_1}{R_0} < K_R\right)$

$Ce^{-rT} Q(S_T/S_0 < K_S)\ Q(R_T/R_0 < K_R)$

$= 50e^{-0.02} Q(S_T/S_0 < 0.8)\ Q(R_T/R_0 < 0.6)$

$= 50e^{-0.02}\left(1 - \Phi\left(\frac{\log(1/0.8) + 0.02 - 0.5 \times 0.3}{0.32}\right.\right.$

$(1 - \Phi(\log(1/0.6) + 0.02 - 0.5 \times 0.15)/\sqrt{0.15})$

$= 50e^{-0.02}(1-0.7257)(1-0.88039)$

$= 1.61$

6)

i) $d_1 = \dfrac{\left(\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T\right)}{\sigma\sqrt{T}}$

$100000\,\Delta = -24830$

$\Delta = -0.25$

ii) $\Delta = \cancel{-2}\ -0.2483,\ d_1 = 0.68$

$\sigma = 0.68 + \sqrt{0.3709}$

$= 7.1\%$

iii) $Ke^{-rT}\phi(-d_2) = e^{-r}\phi\,\cancel{E}(-d_1 + \sigma\sqrt{T})$

$630e^{-0.03}\phi(-0.609)p = \cancel{64}\,630\cancel{\phi}\,e^{-0.03} \times 0.2712$

$= 165.806p$

$165.806 - 24830 \times \dfrac{640}{100000} = 6.894p$

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7) i) Delta $= \frac{\delta f}{\delta s}$

Gamma $= \frac{\delta^2 f}{\delta s^2}$

Vega $= \frac{\delta f}{\delta \sigma}$

ii) Delta $= 0.801$

iii) hedge delta $= 17.91 - 0.801 \cancel{80} \times 60$
$= 20.15$

iv) $f(S, \sigma + \delta) \approx f(s + \sigma) + \delta\, df/d\sigma$

option price $= 17.91 + 2900 \times 0.02$
$= 18.49$

8) i) Δ first partial derivative of option price w.r. U.A

ii) $\sigma = -0.0600 + \frac{1}{2} \sigma^2$

9) i) PDE of Black scholes is

$$\frac{1}{2} \sigma^2 x^2 g_{xx} + (r-q) x g_x - rg + g_t = 0$$

ii) $g(t, x) = (x^n / S_0^{\,n-1})\, e^{u(T-t)}$

$$\boxed{\text{PDE} \Rightarrow \quad u = \frac{1}{2} \sigma^2 n(n-1) + (n-1)r - nq}$$

10) i) Put call parity for a stock paying no dividend. =>

$$\boxed{C_t + Ke^{-r(T-t)} - P_t = S_t}$$

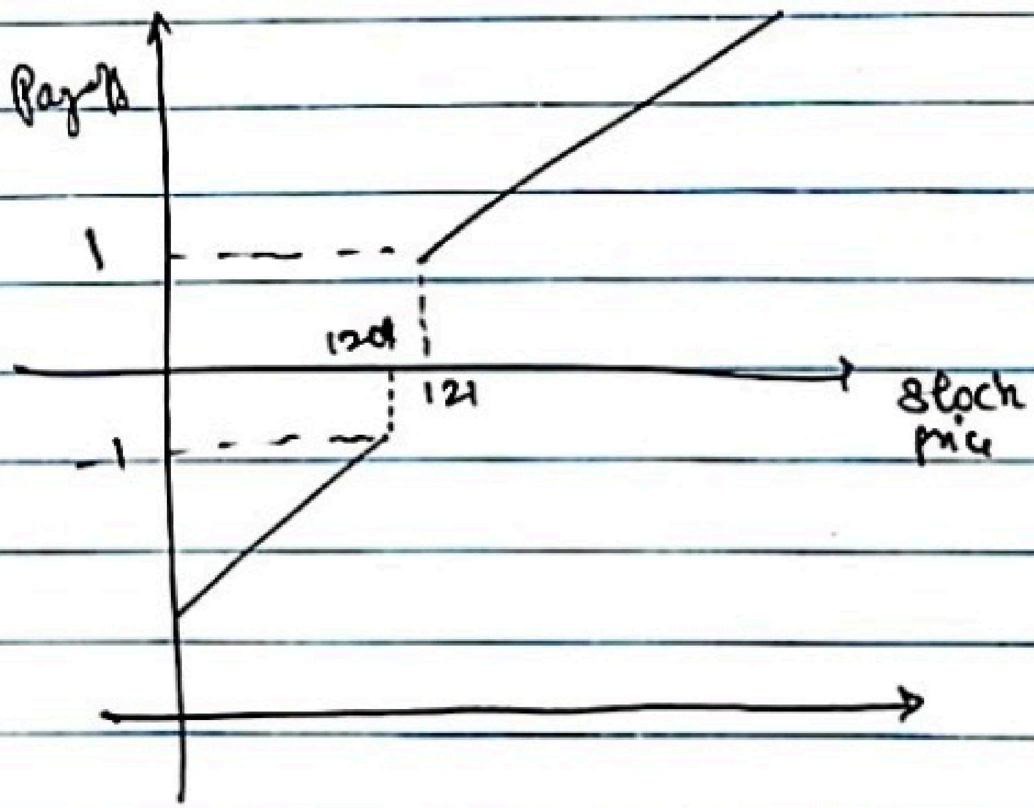
ii) $S_0 \phi(d_1) Ke^{-rT} \phi d_2$

$S_0 = 110, \; K = 120, \; r = 0.02, \; T = 1$

$$d_1 = \frac{\left(\log\left(\frac{11}{12}\right) + 0.02 + 1/2\sigma^2\right)}{\sigma}$$

$$d_2 = d_1 - \sigma$$

iii)



iv) $S_1 - 121 \leq D \leq S_1 - 120$

$$S_0 - 121e^{-r} \leq V \leq S_0 - 120e^{-r}$$