Assignment

1) $I = \int_{1/50}^{2} \frac{e^{2/\omega}}{\omega^2} d\omega \rightarrow (1)$

sub $\frac{2}{\omega} = t \rightarrow (2)$

$\therefore \omega = \frac{2}{t} \rightarrow (3)$

diff (1) w.r.t ω

$2(-1\,\omega^{-2}) = \frac{dt}{d\omega}$

$\frac{-2}{\omega^2} d\omega = dt$

$\frac{1}{\omega^2} d\omega = -\frac{dt}{2} \rightarrow (4)$

$I = -\frac{1}{2}\int_{1/50}^{2} e^t \, dt \rightarrow$ from (1), 3, 4

$I = -\frac{1}{2}(e^t)_{1/50}^{2}$

$\cancel{-\frac{1}{2}(e^{1/50} - e^2)}$

$I = -\frac{1}{2}(e^2 - e^{1/50})$

$\boxed{I = -\frac{1}{2} e^2 - \frac{e^{1/50}}{2}}$

2. $I = \int \frac{2t^3+1}{(t^4+2t)^3}\,dt \rightarrow (1)$

Sut $(t^4+2t) = x \rightarrow (2)$

diff w.r.t t.

$4t^3+2 = \frac{dx}{dt}$

$2(2t^3+1) = \frac{dx}{dt}$

$(2t^3+1)\,dt = \frac{dx}{2} \rightarrow (3)$

from (1), (2), (3)

Now,

$I = \frac{1}{2}\int \frac{dx}{(x)^3}$

$I = \frac{1}{2}\int x^{-3}\,dx$

Ingrating both side

$I = \frac{1}{2}\left(-\frac{x^{-2}}{2}\right) + c$

$I = \frac{-1}{4}x^{-2} + c$

$I = \frac{-1}{4x^2} + c$

re substitute $x = (t^4+2t)$

$I = \frac{-1}{4(t^4+2t)^2} + c$

3 If $f'(x) = x^4 + 3x - 9$.

Solution

$f'x = x^4 + 3x - 9$

Integrate both side w.r.t x

$$\int f'x \, dx = \int x^4 + 3x - 9 \, dx$$

$$\boxed{F(x) = \frac{x^5}{5} + 3\frac{x^2}{2} - 9x + C}$$

4. $f(x) = \begin{cases} 6 & x > 1 \\ 3x^2 & x \leq 1 \end{cases}$

Evaluate.

$$I = \int_{-2}^{3} f(x)\,dx$$

I) Let $f(x) = 3x^2$

$$I = \int_{-2}^{3} 3x^2\,dx$$

Integrating both side

$$I = \left[3x^2\right]_{-2}^{3} dx$$

$I = 3(-2)^2 \rightarrow \because$ for $f(x) = 3x^2$, $x \leq 1$ so $x \neq 3$

$\boxed{I = 12}$

II) Let $f(x) = 6$

$$\therefore I = \int_{-2}^{3} 6\,dx$$

$$I = \left[6x\right]_{-2}^{3}$$

$I = 6(3) \rightarrow \because$ for $f(x) = 6$, $x > 3$ so $x \neq -2$

$\boxed{I = 18}$

$\therefore I = \int_{-2}^{3} f(x)\,dx$ has two value

$I = 12$ for $f(x) = 3x^2$ and $I = 18$ for $f(x) = 6$

5) $\int 3(8y-1)\, e^{4y^2-y}\, dy$

Solution

Let $I = \int 3(8y-1)\, e^{(4y^2-y)}\, dy \rightarrow (1)$

Substitute $4y^2-y = t \rightarrow (2)$

diff (2) w.r.t y

$$(8y-1) = \frac{dt}{dy}$$

$$(8y-1)\, dy = dt \rightarrow (3)$$

$$I = \int 3e^{t}\, dt$$

Integrate both side

$$I = 3e^{t} + C \rightarrow (4)$$

resubstitute $t = 4y^2-y$ in (4)

$$\boxed{I = 3e^{4y^2-y} + C}$$

6 $f''(x) = 5\sqrt{x} + 5x^3 + 6$

Integrate both side by dx

$$\int f''(x)\, dx = \int (5\sqrt{x} + 5x^3 + 6)\, dx$$

$$f'(x) = \frac{2\times 15\, x^{3/2}}{3} + \frac{5x^4}{4} + 6x + C$$

$$f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + C_1$$

again Integrate both side by x

$$\int f'(x)\, dx = \int \left(10x^{3/2} + \frac{5}{4}x^4 + 6x + C_1\right) dx$$

$$f(x) = \frac{2\times 10\, x^{5/2}}{5} + \frac{5}{4}\frac{x^5}{5} + \frac{6x^2}{2} + C$$

$+ C_1 x$ $C_1 = 6$

$f(x) = 4x^{3/2} + \frac{x^5}{4} + 3x^2 + C$ ---(1)

$f(1) = \frac{5}{4}$, (given that)

Put sub $x = 1$ in eq(1)

$f(1) = 4(1)^{3/2} + \frac{(1)^5}{4} + 3(1)^2 + C$

$\frac{5}{4} = 4 + \frac{1}{4} + 3 + C$

$\frac{5}{4}$ = multiply both side by 4

$5 = 16 + 1 + 12 + C$

$5 = 29 + C$

$\boxed{-24 = C}$ ---(2)

Sub $x = 4$ in (1)

$f(4) = 4(4)^{3/2} + \frac{(4)^5}{4} + 3(4)^2 + C$

$f(4) = 4 \times 8 + 1024 + 3 \times 16 + C$

$\boxed{-766 = C}$ ---(3)

When In General,

$f(x) = 4x^{3/2} + \frac{x^5}{4} + 3x^2 + C$

Where $x \in N$ and ~~$x \neq f(1)$~~

for $f(x) = 4x^2$ value of $C = -34$

for $x = 1$ value of $C = -766$

for $x = 5$ value of C

8) $r = \frac{dr}{d\theta} = \frac{r^2}{\theta}$ $\quad r(1) = 2$

$$\left(\frac{1}{r^2}\right) dr = \left(\frac{1}{\theta}\right) d\theta$$

Integrate Bothside.

$$-r^{-1} = \log\theta + C$$

$$-\frac{1}{r} = \log\theta + C \to ①$$

$\therefore r(1) = 2$

Where $\theta = 1$, $r = 2$

$$-\frac{1}{2} = \log 1 + C$$

$$\boxed{-\frac{1}{2} = C}$$

$$-r = \frac{1}{\log\theta + C}$$

$$r = -\frac{1}{\log\theta - \frac{1}{2}}$$

$$\boxed{r = \frac{2}{2\log\theta - 1}}$$

2) $\frac{dv}{dt} = 9.8 - 0.196v$

$\left(\frac{1}{9.8 - 0.196v}\right) dv = dt$

Integrate both side.

$$\boxed{-\frac{\log(9.8 - 0.196v)}{0.196} = t}$$

3) $\frac{dV}{dt} = 9.8 - 0.196V$

$\frac{1}{(9.8 - 0.196V)} dV = dt$

Integrate both side

$$\boxed{\frac{-\log(9.8 - 0.196V)}{0.196} = t}$$

10) $t y' + 2y = t^2 - t + 1$

$y(t)$, $y(1) = \frac{1}{2}$

Sub both $t = 1$

$y'(1) + 2y = 1 - 1 + 1$

10) $y'(t) + 2y = t^2 - t + 1$

$y't = t^2 - t + 1 - 2y$

Integrating both side

$\int y'(t)\,dt \int t^2 - t + 1 - 2y\,dt$

$y(t) = \frac{t^3}{3} - \frac{t^2}{2} + t - 2yt + C$

Sub $t = 1$

$\cancel{y(1) = \frac{1}{24} - \frac{1}{6} + \frac{1}{2} - 2y + C}$

$y(1) = \frac{1}{3} - \frac{1}{2} - 2y + C$

$\frac{1}{2} = \frac{1}{3} - \frac{1}{2} + 2y + C$