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Assignment - 1

Q1 (i) Arbitrage Opportunity is when we can make profit with no risk.

(ii) Law of one price : This states that any 2 portfolios that behave in exactly the same way must have the same price.

(iii) Rt → 3 month put $S_t = 125$

₹30 , rf = 5% $K = 120$

a) Put Call Parity $q = 1.52\%$

$P_t = C_t + K e^{-r(T-t)} - S_t e^{-q(T-t)}$

$0.25 \to \frac{3}{12}$

$= 30 + 120 e^{-5\%(0.25)} - 125 e^{-0.015(0.25)}$

$= 28.11$

b) Put options are only 23

Q2 $$dX_t = \alpha\mu(T-t)\,dt + \sigma\sqrt{T-t}\,dZ_t$$

Let $A_t \to \alpha\mu(T-t)$

$B_t \to \sigma\sqrt{T-t}$

$$f(x,t) = e^{M(T-t)-x}$$

By Itôs lemma.

$\cancel{df = \frac{dy}{dx}B}$ $\quad dX_t \to A_t\,dt + B_t\,dZ_t$

$$df = \frac{dy}{dx}B_t\,dZ_t + \left(\frac{df}{dt} + \frac{df}{dx}A_t + \frac{1}{2}\frac{d^2y}{dx^2}\right)dt$$

$$= -f B_t \, dZ_t + f \frac{dm}{dt}(T-t)\,dt - f A t \, dt + \frac{1}{2} f B t^2 \, dt$$

So,

$$dF = f\left[\frac{dm}{dt}(T-t) - At + \frac{1}{2}Bt^2\right]dt - f B_t \, dZ_t$$

Q5 Stock Price $\sim$ Geo BM

$$S_t = S_0 e^{(\mu - \sigma^2/2) + \sigma W_t}$$

So log normal of it.

$\ln(S_t)$ follows normal distⁿ $(\ln(S_0) + (\mu + v^2/2), \sigma^2 t)$

mean variance

$$\ln(S_t) \sim \phi\left[\ln 254 + (0.16 - 0.35^{2}/2) \times 0.5,\ 0.35 \times 0.5\right)$$

$$\ln(S_t) \sim \phi(5.59, 0.247)$$

$$= \frac{\ln S_t - \mu(S_t)}{\sigma t^{(1/2)}} \sim \xi \omega.$$

~~Hence Put Option~~

Stock Price is less than 258 in 6 months

Prob $\rightarrow 1 - 0.5542$

$= 0.4457$

Q9

(i) $13.334 + 70 \times e^{(-0.25 r)} = 0.12075$
$8.869 + 75 \times e^{(-0.25 r)} = 0.56875$

$S = 82$

Substitution
$\rightarrow r = 7\%$

forward price $f = 82 \cdot e^{(0.07 \times 0.25)}$
$= 83.45$

(ii) $e^{(r_6 \times 0.5)} = e^{(r_3 \times 0.25)} \times e^{(r_f + 0.25)}$
$= r_3 + r_f$

Using $r_3 = 7.1$

$2.569 + 90 \times e^{(0.5 \times r_6)} = 7.909 + 82$

$r_6 = 6.2$, $r_f = 5.2$

(iii) Using Put Call Parity
$6.899 + a \times e^{(-0.07 \times 0.25)} = 1.055 + 82$
$a = 77.5$

$6 + 80 \times e^{(-0.07 \times 0.25)} = 1.789 + 82$
$b = 5.177$

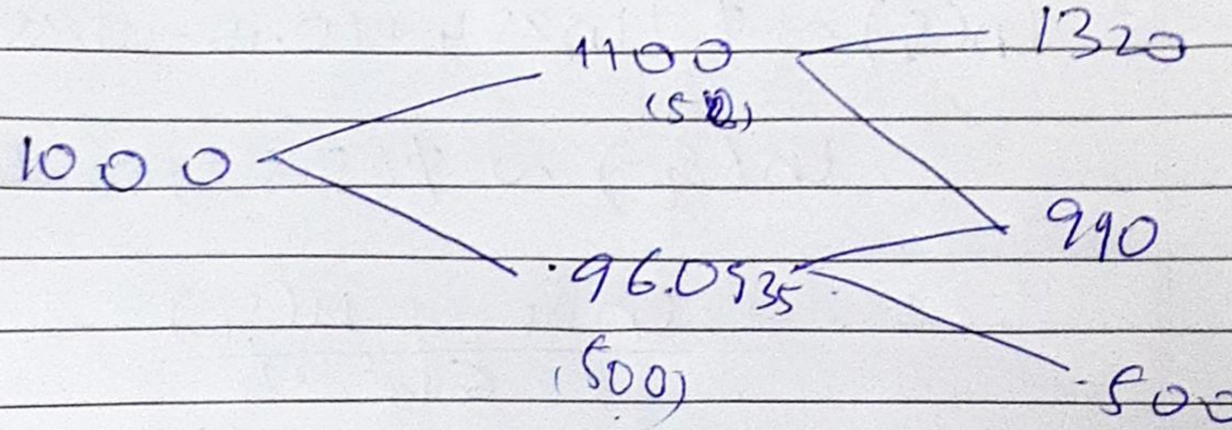
$2.594 + 85 \times e^{(-0.07 \times 0.05)} = c + 82$
$c = 5.119$

Q 11

a) A recombining binomial tree has upstep & downstep assumed to be same under all states.

It has a risk neutral probality of q which is constant

(ii)



a) $q = \frac{e^{r} - d}{u - d}$ = $\frac{e^{0.0775} - 0.5}{1.1 - 0.5}$

= 0.80442.

$C(d) = e^{-r}[q(0) + (1-q)(500)]$

= 96.0935

$C_1 = e^{-0.0775}[0 + (1-q)(96.0935)]$

= 18.4675

Q14

500 rf → 5% 3 month period

500 → 530 → 561.8, 503.5

500 → 475 → 503.5, 451.25

$$p = \frac{e^{0.05 \times 3/12} - 0.95}{1.06 - 0.95} = 0.5689 \quad \frac{e^{(r-T)} - d}{u - d}$$

~~S.78 56.18 > 5~~ $q = 1 - p = 0.4311$

Value of European call

$\Rightarrow e^{(0.5 \times 0.05)} \times 51.8 \times 0.5689$

$= 16.351$

500 → 530 → 56.18, 50.35

500 → 475 → 50.35, 45.125