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SRM-2

Assignment -1

1. A stochastic model is one that recognizes the random nature of the variables involved.

Advantages:

- Reflects reality better.
- Provides information on distribution of results.
- Use of Monte-Carlo Simulation.

ii) In a time-inhomogenous model, transition rates are a function of time as well as age. It is possible to improve the model.

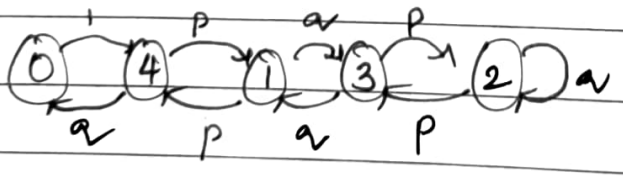
iii) $Y = P[X_t = a \mid X_{s_1} = a_1, \cdots, X_s = x]$

$\Rightarrow Y = P[X_t - X_s + x = a \mid X_{s_1} = a_1, \cdots, X_s = x]$.

$\Rightarrow Y = P[X_t - X_s + x = 1 \mid X_s = x]$.

$\Rightarrow Y = [X_t = a \mid X_s = x]$

2) i)




ii) $P = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a & p \\ 0 & 0 & a & p & 0 \\ 0 & a & p & 0 & 0 \\ a & p & 0 & 0 & 0 \end{bmatrix}$

iii) $\bar{\Pi} P = P$.

$\Pi_0 = \Pi_4 q$

$\Pi_1 = \Pi_3 q + \Pi_4 p$

$\Pi_2 = \Pi_2 q + \Pi_3 p$

$\Pi_3 = \Pi_1 q + \Pi_2 p$

$\Pi_4 = \Pi_0 q + \Pi_1 p$.

Solving we get,

$$\Pi_4 = \Pi_1 = \Pi_2 = \Pi_3 = \frac{1}{4+q}$$

$$\Pi_0 = \frac{q}{4+q}.$$

iv) $$P(\text{wet}) = \frac{q}{4+q} \cdot p$$

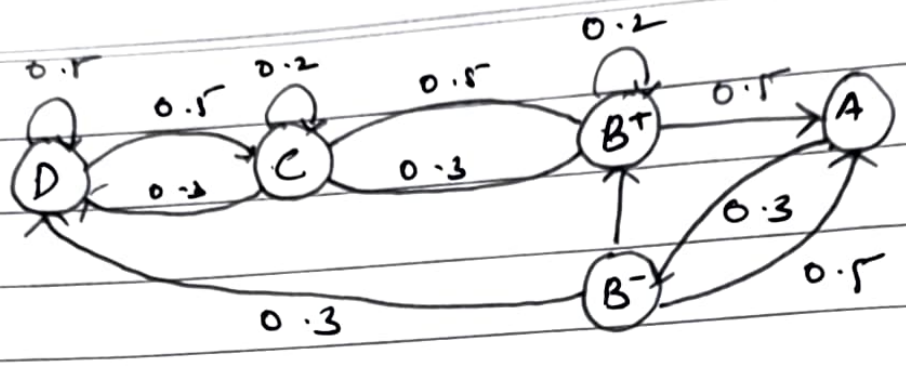
v) $P(\text{wet}) = 0.054$

$$P(\text{wet}) = \Pi_0 \cdot p = \frac{p+q}{N+q}.$$

$\Rightarrow N > 23.6.$

$\Rightarrow N \geq 24$

4. i), iii)




ii)

$$\begin{bmatrix} 0.2 & 0.5 & 0 & 0 & 0 \\ 0.3 & 0.2 & 0.5 & 0 & 0 \\ 0 & 0.3 & 0.2 & 0 & 0.5 \\ 0.3 & 0.0 & 0.2 & 0 & 0.5 \\ 0 & 0 & 0 & 0.3 & 0.7 \end{bmatrix}$$

v) $\Pi P = \Pi$.

Solving equations we get,

$\Pi_A = 0.42445$

$\Pi_{B^+} = 0.12773$

$\Pi_{B^-} = 0.12773$

$\Pi_C = 0.15280$

$\Pi_D = 0.16808$.

vi) \$1.4762m

5) i) There is a dependence on past behaviour in the distribution of Y. Hence, Markov Property does not hold.

ii) Can't form a Markov Chain.

iii)
$$\begin{bmatrix} p & (1-p) & 0 & \cdots & 0 & \cdots & 0 \\ 0 & pe^{-\lambda} & 1-pe^{-\lambda} & 0 & \cdots & \cdots & 0 \\ \vdots & 0 & \ddots & p^{e^{-2\lambda}} & 1-p^{e^{-2\lambda}} & \cdots & 0 \\ \vdots & \vdots & & \ddots & & \ddots & 0 \end{bmatrix}$$

iv) a) Time Homogenous.
b) Irreducible
c) No stationary dist
d) No recurrent states.

v) $P[\text{No Error}) = pe^{-j\lambda} = p^n e^{-nj\lambda}$

6) i) The probability that a life is in a state depend on age and current state.
$q_{x+t} = \mu_{x+t}\, dt + o(h)$
$\mu_{x+t}$ is constant for each $x$.

ii) a) Exponential.
b) $\hat{\rho} = \frac{n_{SH}}{t_S}$, $\hat{\sigma} = \frac{n_{HS}}{t_H}$,

c) We could use $\chi^2$ test.

iii) a) We can use time-inhomogenous model to improve fit.

b) $\sigma = \int f(a,s)\,\sigma(a)\,da$ ; age profile $= f(a,s)$

7) i) Refer to Q1 (i).

b) $\sigma_{ij}(s) = \frac{d}{dt} p_{ij}(s,t)$

ii) $\sum \sigma_{ij}(s) = \sum \frac{d}{dt} p_{ij} = \frac{d}{dt} \sum p_{ij} = 0$

8) i) State:

1 - Not defeated.
2 - Defeated last game
3 - Defeated last 2 games.
4 - Defeated last 3 games.
5 - Sacked.

$$P = \begin{bmatrix} 0.7 & 0.3 & 0 & 0 & 0 \\ 0.7 & 0 & 0.3 & 0 & 0 \\ 0.7 & 0 & 0 & 0.3 & 0 \\ 0.7 & 0 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

ii) Chain is not irreducible.

iii) a) 0.0081

b) 0.00567

c) 0.00567

d) 0.0056241

iv) $N_4 = 123.46$

$N_3 = 160.49$

$N_2 = 171.6$

$N_1 = 174.94 = E[N]$

9) i) State Space = {T, FL, NL, FW, NW}.

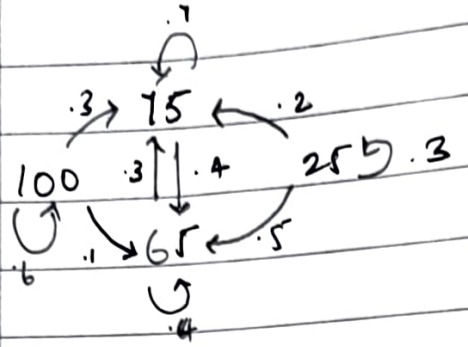
ii) $P = \begin{bmatrix} 0 & 0.55 & 0 & 0.45 & 0 \\ 0.45 & 0 & 0.55 & 0 & 0 \\ 0.55 & 0 & 0 & 0 & 0.45 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

iii) Chain is not reducible or aperiodic.

iv) Prob = $0.55 \times 0.45 + 0.45 \times 0.55 = 0.495$

v) $N = 4.26 = 5$

10) i)




ii) $\Pi_n = \Pi_{n-1} P$.

$\therefore \Pi_2 = [0.017, 0.5005, 0.3615, 0.12]$

$\Pi P = \Pi$

Solving we get:-

$\Pi = [0, 0.54, 0.35, 0.1]$

11. $P = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 0 & 1/4 & 1/4 \\ 0 & 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 & 1/2 & 0 \end{bmatrix}$

ii) The chain is irreducible.

iii) $\Pi P = \Pi$

Solving, $\Pi_1 = .1$

12.

a) $P_{rr} = 3/7$, $P_{rs} = 4/7$, $P_{sr} = 8/15$, $P_{ss} = 7/15$

1st dwl :

b) $P_{rs} = 8/14$, $P_{ss} = 7/15$, $P_{sr} = 8/15 = 0.142\ldots$

2nd dwl :

$P_{rs} = 8/14$, $P_{sr} = 8/15$, $P_{rr} = 6/14$, $= 0.16$

3rd

$P_{rr} = 6/14$, $P_{rr} = 6/14$, $P_{sr} = 6/14 = 0.018$.

Total $= 0.43$.

ii) a) $P[X] = \frac{e^{-\mu} \mu^x}{x!}$

$P[X_0] = \frac{e^{-3} 3^0}{0!} = 0.0491$.

$\therefore P[X \geq 0] = 1 - P[X_0) = 0.95021$.

b) $P[2 \leq X \leq 5] = 0.6161$

c) Avg $= 0.6$.

$P[X] = 0.32929$.