AMALU MANU PUNNOOSE
45

MAHAGURU Page No.
Date / /

24/11/21

ASSIGNMENT - 1

1. $\lim_{h\to 0} \frac{2(3+h)^2 - 18}{h}$

$= \lim_{h\to 0} \frac{2(h^2+9-6h)-18}{h}$

$= \lim_{h\to 0} \frac{2h^2 + \cancel{18} - 12h - \cancel{18}}{h}$

$= \lim_{h\to 0} \frac{2\cancel{h}(h-6)}{\cancel{h}}$

$= \lim_{h\to 0} 2h - 12$

$= -12$

2. $g(x) = \begin{cases} 2x, & x < 6 \\ x-1, & x \geq 6 \end{cases}$

(i) $x = 4$

LHL : $\lim_{x\to 4^-} g(x)$

$= \lim_{x\to 4^-} 2x = 8$

RHL : $\lim_{x\to 4^+} 2x = 8$

$g(4) = 8$

$\lim_{x\to 4^-} g(x) = g(4) = \lim_{x\to 4^+} g(x)$

Thus, $g(x)$ is continuous at $x = 4$.

25/11/21

(ii) $x = 6$

LHL : $\lim_{x \to 6^-} 2x = 12$

RHL : $\lim_{x \to 6^+} x - 1 = 5$

LHL $\neq$ RHL

$$\lim_{x \to 6^-} g(x) \neq \lim_{x \to 6^+} g(x)$$

Thus, $g(x)$ is discontinuous at $x = 6$

3. $\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$

$= \lim_{x \to 9} \frac{(x-9)(3+\sqrt{x})}{(3-\sqrt{x})(3+\sqrt{x})}$

$= \lim_{x \to 9} \frac{3x + x\sqrt{x} - 27 - 9\sqrt{x}}{9-x}$

$= \lim_{x \to 9} \frac{\cancel{(x-9)}(3+\sqrt{x})}{-\cancel{(x-9)}}$

$= -(3+\sqrt{9})$

$= \underline{\underline{-6}}$

25/11/21

MAHAGURU Page No. Date

$a-h$

$-1-h$

$12+3h$

$3(h+4)$

$3(-1-h)$

$-3-3h$

$3+3h+9$

$12+3h$

$\frac{1-4h}{12+3h}$

$\frac{-4+4h+5}{9+3-3h}$

$\frac{1+4h}{12-3h}$

4. $f(x) = \frac{4x+5}{9-3x}$

(ia) $x = -1$

$f(-1) = \frac{4(-1)+5}{9-3(-1)} = \frac{1}{12}$

LHL: $\lim_{x\to -1^-} \frac{4x+5}{9-3x}$

$= \lim_{h\to 0} \frac{4(-1-h)+5}{9-3(-1-h)}$

$= \lim_{h\to 0} \frac{-4-4h+5}{9+3+3h}$

$= \lim_{h\to 0} \frac{1-4h}{12+3h}$

$= \frac{1}{12}$

RHL: $\lim_{x\to -1^+} \frac{4x+5}{9-3x}$

$= \lim_{h\to 0} \frac{4(-1+h)+5}{9-3(-1+h)}$

$= \lim_{h\to 0} \frac{-4+4h+5}{9+3-3h}$

$= \lim_{h\to 0} \frac{1+4h}{12-3h}$

$= \frac{1}{12}$

$\lim_{x\to -1^-} f(x) = f(-1) = \lim_{x\to -1^+} f(-1)$

Thus, $f(x)$ is continuous at $x = -1$

25/11/21

(ii) $x = 0$

$$f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$$

LHL : $\lim_{x \to 0^-} \frac{4x+5}{9-3x}$

$= \lim_{h \to 0} \frac{4(-h)+5}{9-3(0-h)}$

$= \lim_{h \to 0} \frac{-4h+5}{9+3h}$

$= \frac{5}{9}$

RHL : $\lim_{x \to 0^+} \frac{4x+5}{9-3x}$

$= \lim_{h \to 0} \frac{4(h)+5}{9-3h}$

$= \frac{5}{9}$

$\lim_{x \to 0^-} f(x \cdot 0) = f(0) = \lim_{x \to 0^+} f(x \cdot 0)$

Thus, $f(x)$ is continuous at $x = 0$

25/11/21

(iii) $x = 3$

$f(3) = \frac{4(3)+5}{9-3(3)} = \frac{17}{0}$ not defined

$\therefore$ $f(x)$ is discontinuous at $x = 3$ as it is not defined.

5. $\lim_{x\to\infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$= \lim_{x\to\infty} \frac{6 - e^{-2x}/e^{4x}}{8 - e^{2x}/e^{4x} + 3e^{-x}/e^{4x}}$

$x \to \infty$

$1/e^{4x} = 0$

$= \frac{6-0}{8-0} = \frac{6}{8} = \frac{3}{4}$

6. $g(y) = \begin{cases} y^2 + 5 & \text{if } y < -2 \\ 1 - 3y & \text{if } y \geq -2 \end{cases}$

(a) $\lim_{y\to 6} g(y)$

$\lim_{y\to 6} 1 - 3y$

$= 1 - 3(6)$

$= -17$

(b) $\lim_{y\to -2} g(y)$

LHL: $\lim_{y\to -2^-} y^2 + 5$

$= (-2)^2 + 5$

$= 9$

RHL: $\lim_{y\to 2^-} 1 - 3y$

$= 1 - 3(-2)$

$= 8$

25/11/21

7. $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$f'(t) = (8t-1)(t^3 - 8t^2 + 12) + (4t^2 - t)(3t^2 - 16t)$

$f'(t) = 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12 + 12t^4 - 64t^3 - 3t^3 + 16t^2$

$f'(t) = \underline{\underline{20t^4 - 132t^3 + 24t^2}} + 96t - 12$

8. $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

$R'(w) = \dfrac{(2w^2+1)(3 + 4w^3) - (3w + w^4)(4w)}{(2w^2+1)^2}$

$R'(w) = \dfrac{6w^2 + 8w^5 + 3 + 4w^3 - 12w^2 - 4w^5}{(2w^2+1)^2}$

$R'(w) = \underline{\underline{\dfrac{4w^5 + 4w^3 - 6w^2 + 3}{(2w^2+1)^2}}}$

9. $f(x) = 2e^x - 8^x$

$f'(x) = \underline{\underline{2e^x -}}\ 8^x \log 8$

11. (a) $f(x) = (6x^2 + 7x)^4$

$f'(x) = 4(6x^2 + 7x)^3 \times (12x + 7)$

$f'(x) = 4\underline{\underline{(6x^2 + 7x)}}^3 (12x + 7)$

(b) $f(t) = 5 + e^{4t + t^7}$

$f'(t) \underline{\underline{= \quad e^{4t+t^7}}} (4 + 7t^6)$

10. $y = z^5 - e^z \ln(z)$

$\dfrac{dy}{dz} = 5z^4 - [e^z \ln(z) + e^z \cdot 1/z]$

$\dfrac{dy}{dz} = 5z^4 - e^z \ln(z) + \dfrac{e^z}{z}$

25/11/21

12. (a) $z = \ln(7 - x^3)$

$$\frac{dz}{dx} = \frac{1}{7 - x^3}(-3x^2)$$

$$\frac{d^2z}{dx^2} = \frac{(7 - x^3)(-6x) - (-3x^2)(-3x^2)}{(7 - x^3)^2}$$

$$\frac{d^2z}{dx^2} = \frac{-42x + 6x^4 - 9x^4}{(7 - x^3)^2}$$

$$\frac{d^2z}{dx^2} = \frac{-3\cancel{5}x^4 - 42x}{(7 - x^3)^2}$$

$16v - 16$

16

25/11/21

(b) $$Q(v) = \frac{2}{(6+2v-v^2)^4}$$

$$Q'(v) = \frac{(6+2v-v^2)^4(0) - (2)(4)(6+2v-v^2)^3(2-2v)}{(6+2v-v^2)^8}$$

$$Q'(v) = \frac{(-16+16v)(6+2v-v^2)^3}{(6+2v-v^2)^8}$$

$$Q'(v) = \frac{16(v-1)}{(6+2v-v^2)^5}$$

$$Q''(v) = \frac{(6+2v-v^2)^5(16) - (16)(v-1)(5)(6+2v-v^2)^4(2-2v)}{(6+2v-v^2)^{10}}$$

$$Q''(v) = \frac{[(16)(6+2v-v^2)^4][(6+2v-v^2) - (5)(v-1)(2-2v)]}{(6+2v-v^2)^{10}}$$

$$Q''(v) = \frac{16(9v^2-18v+16)}{(6+2v-v^2)^{10}}$$

25/11/21

(c) $f(t) = \ln(1+t^2)$

$f'(t) = \dfrac{1}{1+t^2}(2t)$

$f''(t) = \dfrac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}$

$f''(t) = \dfrac{2+2t^2-4t^2}{(1+t^2)^2}$

$f''(t) = \dfrac{2(1-t^2)}{(1+t^2)^2}$

13. $f(x) = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$.

Let $f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or $x = -1$

Inc Dec Inc

$-\infty$ -1 3 ∞

$f''(x) = 6x - 6$

$f''(-1) = -12$

$f''(3) = 12$

Hence $f(x)$ is maximum at ~~$x=3$~~ $x=-1$ and minimum and $x = 3$

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= 17$ (max. value of function)

$f(3) = (3)^3 - 3(3)^2 - 9(3) + 12$

$= 27 - 27 - 27 + 12$

$= -15$ (min. value of function)

14. $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

let $f'(x) = 0$

$12x^2 - 36x + 24 = 0$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x = 1$ or $x = 2$

Inc Dec Inc

$-\infty$ 1 2 ∞

$f''(x) = 24x - 36$

$f''(1) = -12$

$f''(2) = 12$

Hence $f(x)$ is maximum at $x = 1$ and minimum at $x = 2$

$f(1) = 4(1)^3 - 18(1)^2 + 24(1) - 7$

$= 4 - 18 + 24 - 7$

$= 3$

$f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$

$= 32 - 72 + 48 - 7$

$= 1$

Thus, the max. value of function is 3 and minimu value of function is 1

25/11/21

15. $f(x) = x^2(x+3)$

$f'(x) = 2x(x+3) + x^2$

Let $f'(x) = 0$

$2x^2 + \cancel{8}6x + x^2 = 0$

$x^2 + 2x = 0$

$x = 0$ or $x = -2$

Inc Dec Inc

$-\infty$ -2 0 ∞

$f''(x) = 6x + 6$

$f''(0) = 6 > 0$ [minimum value at $x = 0$]

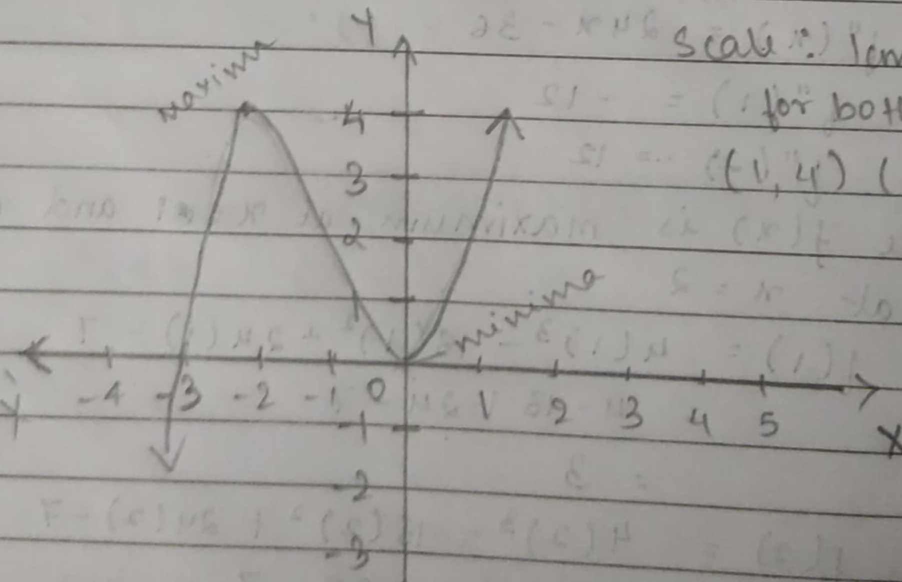
$f''(-2) = -6 < 0$ [maximum value at $x = -2$]

Minimum value $= (0)^2(0+3)$

$= 3$

Maximum value $= (-2)^2(-2+3)$

$= 4$



Scale: 1cm = 1unit for both axis

(1, 4) (-3, 10)

25/11/21

16. $f(x) = 3x^2 - 6x$

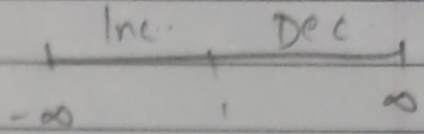
$f'(x) = 6x - 6$

Let $f'(x) = 0$

$6x - 6 = 0$

$x - 1 = 0$

$x = 1$



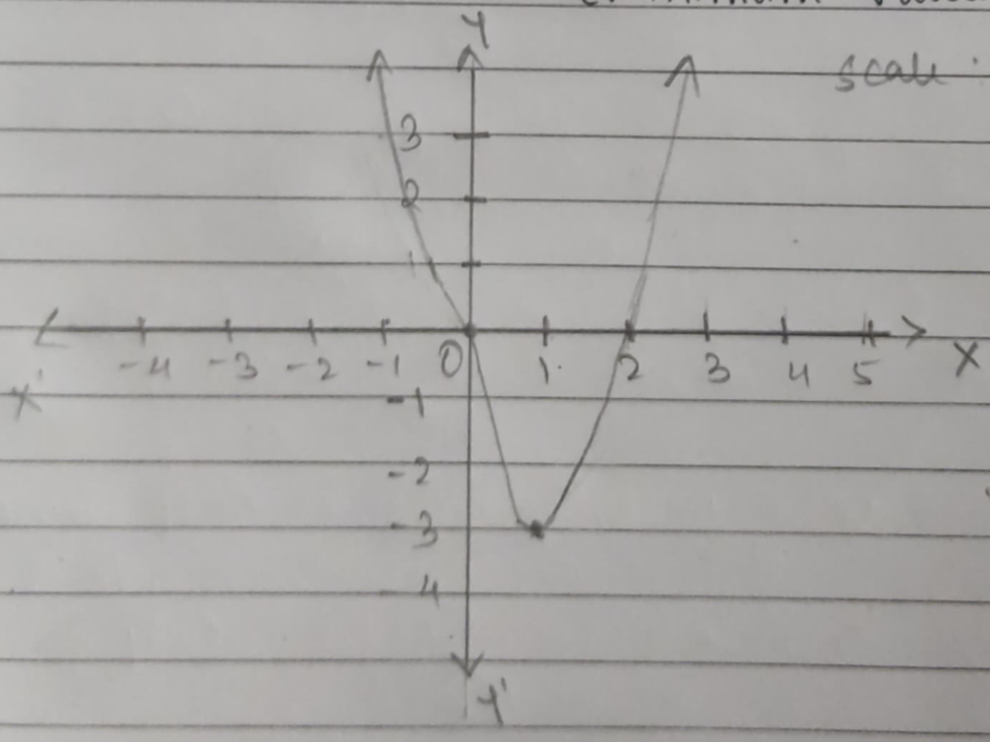
$f''(x) = 6$

$f''(1) = 6 > 0$ [minimum value at $x = 1$]

$f(1) = 3(1)^2 - 6(1)$

$= 3 - 6$

$= -3$ [minimum value



scale : 1cm = 1 unit for both axis

(-1, 9)

(2, 0)

(3, 9)