FINANCIAL MATH. ASSIGNMENT

1. (i) $d^{(4)} = 8\%$

$\therefore \dfrac{d^{(4)}}{4} = 2\%$

$d = \left[1 - (1 - 2\%)^4\right]$

$\therefore d = 7.763184\%$

$\delta = \log \dfrac{1}{1-d} = \cancel{3.509577\%}\ 8.08108\%$

~~2~~ (ii) $\cancel{d = \dfrac{i}{1+i}}$ $\quad i = \dfrac{d}{1-d}$

$= \dfrac{7.763184}{1 - 7.763184\%}$

$\therefore i = 8.416578\%$

(iii) $d^{(12)} = 12\left[1 - (1 - 7.763184\%)^{1/12}\right]$

$\therefore d^{(12)} = 7.9125832\%$

2. $\delta(t) = 0.004t + 0.0002t^2$

(i) $PV = C \cdot e^{-S\delta t} = 1000 \cdot e^{-\int_0^{10} 0.004t + 0.0002t^2\, dt}$

$= 1000 \cdot e^{-0.26667}$

$\therefore PV = ₹765.92578$

(ii) $e^{-0.266667} = \left(\dfrac{i}{1+i}\right)^{10}$

$0.973685(1+i) = i$

$\cancel{0.973}\ 0.973685 + 0.973685i = i$

$\therefore i = 2.631\%$

3. (i) $d = iv$

Interest earned on an investment of 1 paid at the beginning of the period is 'd'. Interest earned on an investment at the end of the period is 'i'. Therefore, if we discount 'i' from the end of the period to the beginning of the period with discounting factor 'v', we obtain 'd'.

(ii) (a) $\frac{d^{(4)}}{4} = 2.25\%$

$$d = \cancel{p}\left[1 - \left(1 - \frac{d^{(P)}}{P}\right)^{P}\right]$$

$$= \left[1 - (1 - 2.25\%)^4\right]$$

$$\therefore d = 8.70078\%$$

$$\therefore d^{(2)} = P\left[1 - (1-d)^{1/P}\right]$$

$$= 2\left[1 - (1 - 8.70078\%)^{1/2}\right]$$

$$\therefore d^{(2)} = 8.89874\%$$

(b) $d^{(0.5)} = 5\%$

$$d = \left[1 - \left(1 - \frac{d^{(P)}}{P}\right)^{P}\right]$$

$$= 5.13167\%$$

$$\therefore d^{(2)} = 2\left[1 - (1 - 5.13167\%)^{\frac{1}{2}}\right]$$

$$\therefore d^{(2)} = 5.1992505\%$$

(iii) $i = 5\%$ p.a.

$$1 - \frac{91d}{365} = \left(\frac{i}{1+i}\right)^{91/365}$$

$$1 - \frac{91d}{365} = \left(\frac{5}{1+5\%}\right)^{91/365}$$

$$d = 3.819987\%$$

4.(i) already done

(ii) $i = 0.08$

(a) $d^{(12)}$

$$\therefore d = \frac{i}{1+i} = \frac{0.08}{1+0.08} = 7.4074\%$$

$$\therefore d^{(12)} = 12\left[1 - (1 - 7.4074\%)^{1/12}\right]$$

$$= 7.671469\%$$

(b) $$i^{(365)} = 365\left[(1+0.08)^{1/365} - 1\right]$$

$$= 7.696915\%$$

(c) $$\delta = \log(1+i) = \log(1.08)$$

$$\therefore \delta = 7.696104\%$$

(d) $$i^{(0.5)} = 0.5\left[(1+0.08)^{1/0.5} - 1\right]$$

$$\therefore i^{(0.5)} = 8.32\%$$

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5. (a) $d^{(4)} = 6\%$

$$d = \left[1 - \left(1 - \frac{d^{(4)}}{4}\right)^4\right]$$

$$= \left[1 - \left(1 - \frac{0.06}{4}\right)^4\right]$$

$\therefore$ $d = 5.86634\%$

$$i = \frac{0.0586634}{1-0.0586634}$$

$\therefore$ $i = \cancel{86.8412}$ 6.231926%

$$i^{(2)} = 2\left[(1 + 0.06231926)^{1/2} - 1\right]$$

$\therefore$ $i^{(2)} = 6.137746\%$

~~6~~. (b) $d^{(12)} = 12\left[1 - (1 - 5.86634\%)^{1/12}\right]$

$\therefore$ $d^{(12)} = 6.030247\%$

(ii) $n = 93$ days

$\therefore$ remaining term $= 272$ days.

$\therefore$ Rebate allowed $= \frac{272}{365} \times 200000$

$= ₹14904.1096.$

$\therefore$ Sum paid to settle the loan $= (22000 - 14904.1096)$

$= ₹205095.89$

OR

$2200000 = 200000(1+i)$

$\therefore i = 10\%.$

$AV = 200000(1+10\%)^{93/365}$

$\therefore AV = ₹204916.3564.$

7. $26000 = 20000(1+i)^{17/12}$

$(1.3)^{12/17} = 1+i$

$\therefore i = 20.3457\%.$

(i) $i^{(4)} = 4[(1+0.203457)^{1/4} - 1]$

$\therefore i^{(4)} = 18.95525\%.$

(ii) $d = \frac{i}{1+i} = 16.90604\%.$

$d^{(2)} = 2[1-(1-16.90604\%)^{1/2}]$

$\therefore d^{(2)} = 17.68823\%.$

(iii) $\cancel{\frac{26000}{20000} = (1+i)}$

$\cancel{i = 30\%}$

$$26000 = 20000\left(1 + \frac{17i}{12}\right)$$

$$\frac{26000}{20000} = 1 + \frac{17i}{12}$$

$$i = 21.1764\%$$

(iv) rate of interest received in compound interest is less than the interest received in simple interest.

~~9 (i) Force of interest is the instantaneous change in the fund value~~ X

9.(i) Measure of interest at individual moments of time is known as force of interest.

(ii) $i^{(2)} = 0.0775$

$$i = \left[1 + \frac{i^{p}}{p}\right]^{p} - 1 = \left(1 + \frac{0.0775}{2}\right)^{2} - 1$$

$$\therefore i = 7.90016\%$$

$$\delta = \log(1+i) = \log(1.0790016)$$

$$\therefore \delta = 7.6036\%$$

$$d = \frac{0.0790016}{1.0790016}$$

$\therefore d = 7.32173\%$

$d^{(12)} = 12[1 - (1 - 0.0732173)^{1/12}]$

$\therefore d^{(12)} = 7.57958\%$

$i^{(0.5)} = 0.5[(1 + 0.0790016)^{1/0.5} - 1]$

$\therefore i^{(0.5)} = 8.21222\%$

10. $$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \le t \le 5 \\ 0.06 & \text{for } 5 \le t \le 10 \\ 0.04 & \text{for } t \ge 10 \end{cases}$$

$v(0,5) = e^{-\int_0^5 \delta t} = e^{-\int_0^5 0.08} = e^{-0.4}$

$v(5,10) = e^{-\int_5^{10} \delta t} = e^{-\int_5^{10} 0.06} = e^{-0.3}$

$v(10,t) = e^{-\int_{10}^{t} \delta t} = e^{-\int_{10}^{t} 0.04} = e^{-(0.04t - 0.4)}$

$\therefore v(0,t) = e^{(-0.4)} \cdot e^{(-0.3)} \cdot e^{(0.4 - 0.04t)}$

$= e^{-(0.04 + 0.3)}$

$\xrightarrow{\quad}$ PTO

11.(a) $\cancel{50000 = c\left(1 + 18\% \times \frac{9}{12}\right)}$

$$\cancel{PV = 50000\left[\frac{1}{(1.18)}\right]^{9/12}}$$

$$PV = 50000\left[\frac{1}{1-0.18}\right]^{9/12}$$

$$= \underline{\underline{43085.426}}$$

(b)(i) $\delta = 6\%$

$$\therefore d = 1 - e^{-\delta}$$

$$\therefore d = \underline{\underline{5.82354\%}}$$

$$d^{(12)} = 12\left[1 - (1 - 5.82354\%)^{1/12}\right]$$

$$\therefore d^{(12)} = \underline{\underline{5.985018\%}}$$

(ii) $d^{(4)} = 6\%$

$$d = \left\{1 - \left[1 - \frac{d^{(4)}}{4}\right]^{p}\right\}$$

$$= \left[1 - (1 - 0.015)^4\right]$$

$$\therefore d = \underline{\underline{5.866345\%}}$$

$$i = \frac{d}{1-d} = 6.2319316\%$$

$$i^{(4)} = 4\left[(1+0.062319)^{1/4} - 1\right]$$

$$\therefore i^{(4)} = \underline{\underline{6.09137\%}}$$

$$i^{(12)} = 12\left[(1+0.062319)^{1/12} - 1\right]$$
$$= 6.060679\%$$

12. 7049 $\quad d^{(12)} = 6\%$

$0 \quad i^{(12)} = 6\% \quad 2y \quad i = 1.5\% \quad 4y\ 6m \quad 6y$

$C = 7049$.

Accumulated value after 2 years:

$$i^{(12)} = 6\%$$

$$\left[1 + \frac{0.06}{12}\right]^{12} = 1+i$$

$$\therefore\ i = 0.061677872.$$

$$AV_2 = 7049(1+0.61677872)^2$$
$$= 7945.349262$$

$i = 1.5\%$ for 10 quaters

$$AF = 1 + ni$$
$$= 1.15$$

Acc. value after $4\frac{1}{2}$ years $= 7945.3493 \times 1.15$
$= 9137.151651$

$d^{(12)} = 6\%$ for $1\frac{1}{2}$ years

$$\left[1 - \frac{d^{(12)}}{12}\right]^{12} = 1-d$$

$$d = 0.058377193$$

$$AF = \frac{1}{(1-d)^n} = 1.09442 1325$$

Accumulated value after 6 years

$$= 9137.157651 \times 1.094421325$$
$$= 9999.893617$$