Page No.
Date

Calculus Assignment
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Q.1 $\int_{1/50}^{2} \frac{e^{2/\omega}}{\omega^2} d\omega$ — (1)

put $\frac{2}{\omega} = t$ when $\omega = 2$ $t = 1$
when $\omega = \frac{1}{50}$ $t = 100$

$\therefore \frac{dt}{d\omega} = \frac{2}{\omega^2}$

$\therefore \frac{d\omega}{\omega^2} = \frac{dt}{2}$

hence ~~equation~~ (1) becomes.

$\int_{1}^{100} \frac{e^t}{2} dt$

$\frac{1}{2} \left[e^t \right]_1^{100}$

$\frac{1}{2} \left[e^{100} - e^1 \right]$

$= 1.34405771 \times 10^{43}$

Q.2. $\int \frac{2t^3 + 1}{(t^4 + 2t)^3} dt$ — (1)

Let $t^4 + 2t = x$

$\therefore 4t^3 + 2 = \frac{dx}{dt}$

$\therefore dt \, (2t^3 + 1) = \frac{dx}{2}$

Hence ~~equat~~ ① becomes

$$\int \frac{dx}{2(x)^3}$$

$$= \frac{1}{2}\int x^{-3}\,dx$$

$$= \frac{1}{2}\,\frac{x^{-2}}{-2} + c$$

$$= \frac{-1}{4x^2} + c,$$

Resubstituting value of x we get-

$$= \frac{-1}{4(t^4+2t)^2} + c$$

Q.3. $f'(x) = x^4 + 3x - 9$

$$\therefore f(x) = \int f'(x)\,dx$$

$$= \int x^4 + 3x - 9\,dx$$

$$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + c$$

Q. 4.

$$f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \le 1 \end{cases}$$

$$\int_{-2}^{3} f(x)\, dx$$

$$= \int_{-2}^{1} 3x^2\, dx + \int_{1}^{3} 6\, dx$$

$$= \left[6x\right]_{1}^{3} + \left[\frac{3x^3}{3}\right]_{-2}^{1}$$

$$= \cancel{\left[6+12\right] + \left[27-1\right]} \quad = (1+8) + (18-6)$$

$$= 9 + 12$$

$$= 21$$

$$= \cancel{16 + 26}$$

$$= \cancel{42}$$

Q5. $\int 3(8y-1)\, e^{4y^2-y}\, dy$

Let $4y^2 - y$ be t

$\therefore \frac{dt}{dy} = 8y - 1$

$\therefore dt = (8y-1)\, dy$

$\therefore \int 3e^t\, dt$

$= 3\left[e^t\right] + c$

$= 3e^{4y^2-y} + c$

Q.6. $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$$f'(x) = \int f''(x)\, dx$$

$$= \int 15x^{1/2} + 5x^3 + 6$$

$$\cancel{B} = 2\,\frac{\cancel{15}^{5}\, x^{3/2}}{\cancel{3}_{1}} + \frac{5x^4}{4} + 6x + C_1$$

$$= 10x^{3/2} + \frac{5}{4}x^4 + 6x + C_1$$

$$f(x) = \int f'(x)\, dx$$

$$= \int 10x^{3/2} + \frac{5}{4}x^4 + 6x + C_1$$

$$= \frac{10x^{5/2} \times 2}{5} + \frac{5x^5}{4\times 5} + \frac{6x^2}{2} + C_1 x + C$$

$= 4x^{5/2} + \frac{x^5}{4} + 3x^2 + c_1x + c$

$f(1) = \frac{-5}{4}$

$\therefore 4(1) + \frac{1}{4} + 3 + c_1 + c = \frac{-5}{4}$

$\cancel{17 + 1 + 12}$

$16 + 1 + 12 + 4c_1 + 4c = -5$

$29 + 4c_1 + 4c = -5$

$4c_1 + 4c = -84 \quad 4(c_1 + c) = -5 - 29$

$c = \frac{-34 - 4c_1}{4} \quad 4(c_1 + c) = -34$

$[c_1 + c] = \frac{-34}{4}$

$f(4) = 404$

$4(4)^{5/2} + \frac{(4)^5}{4} + 3(16) + c_1 4 + c$

$128 + 256 + 48 + 4c_1 + c$

$128 + 256 + 48 + 4c_1 - \frac{34 - 4c_1}{4}$

$\frac{432 + 16c_1 - 4c_1 - 34}{4} = 404$

$4c_1 - 34 = -112$

$c_1 = -19$

$\therefore c = 10.5$

$f(x) = 4x^2\sqrt{x} + \frac{x^5}{4} + 3x^2 - 19x + 10.5$

Page No.			
Date			

(7.) $f(x+iy) + g(x-iy)$

$$f(x) + f(iy) + g(x) - g(iy) = c$$

$$\therefore f(x) + g(x) = c + g(iy) - f(iy)$$

integrating on both sides.

$$\int f(x) + g(x)\, dx = c\int g(iy) - f(iy)\, dy$$

(8) $\dfrac{dr}{d\theta} = \dfrac{r^2}{\theta}$ $\quad r(1) = 2$

$$\frac{dr}{r^2} = \frac{d\theta}{\theta}$$

Integrating on both sides.

$$\int r^{-2}\, dr = \int \theta^{-1}\, d\theta$$

$$\frac{-1}{r^{-1}} = \log(\theta) + c$$

$$-\frac{1}{r} = \log(\theta) + c$$

~~when $r = 2$~~

~~$\log(\theta) = 2$~~

~~$\therefore \theta = e^2$~~

~~$= 7.389056027$~~

$r(1) = 2$

$$\therefore \frac{d(1)}{d\theta} = \frac{(1)^2}{\theta} = 2$$

$$\therefore \frac{1}{\theta} = 2$$

$$\therefore \theta = \frac{1}{2}$$

Page No.
Date

9. $\frac{dv}{dt} = 9.8 - 0.196v$

$\frac{dv}{9.8 - 0.196v} = dt$

Integrating on both sides

$$\int \frac{1}{9.8 - 0.196v} dv = \int 1\, dt$$

$$\int \frac{1}{50 \times 0.196 - 0.196v} dv = t + c$$

$$\int \frac{1}{0.196(50 - v)}$$

Let $\frac{1}{0.196}$

$$\frac{1}{0.196} \times \int \frac{1}{(50 - v)} dv = t + c$$

$$\frac{1}{0.196} \times \frac{\log(50 - v)}{-1} = t + c$$

$$\frac{-\log[50 - v]}{0.196} = t + c$$

$$-\log|50 - v| = 0.196(t) + C$$

Q.10. $ty' + 2y = t^2 - t + 1$ $\quad y(1) = \frac{1}{2}$

$t\frac{dy}{dt} + 2y = t^2 - t + 1$ $\quad$ when $t=1$, $y = 1/2$

$1\frac{dy}{dt} + \cancel{2} \times \frac{1}{\cancel{2}} = 1 - 1 + 1$

$\frac{dy}{dt} + 1 = 1$

$\frac{dy}{dt} = 0.$

Q.11. $y' = \frac{xy^3}{\sqrt{1+x^2}}$

$y(0) = -1$, when $x = 0$, $y = -1$

$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$

$y^{-3}\, dy = \frac{x}{\sqrt{1+x^2}}\, dx$

Integrating both side.

$\frac{y^{-2}}{-2} + C_1 = \frac{1}{2}\int \frac{2x}{\sqrt{1+x^2}}\, dx$

$\frac{-1}{2y^2} + C_1 = \cancel{\frac{1}{2}}\, 2\sqrt{1+x^2} + C_2$

$\frac{-1}{2(-1)^2} + C_1 =$

$$-\frac{1}{2} + C_1 \;\xi\; \therefore\; C_1 = \frac{1}{2}$$

$$2\sqrt{1+x^2} = -C_2$$

$$2\sqrt{1+0} = -C_2$$

$$\therefore\; 2 = -C_2$$

$$\therefore\; C_2 = -2.$$

$$\therefore\; -\frac{1}{2y^2} + \frac{1}{2} = 2\sqrt{1+x^2} - 2$$

Q.12. $f(x) = x^3 - 10x^2 + 6$ $\quad x = a = 3 \quad f(a) = -57$

$f'(x) = 3x^2 - 10x \quad f'(3) = \cancel{9} - 3$

$f''(x) = 6x - 10 \quad f''(3) = 8$

$f'''(x) = 6 \quad f'''(3) = 6$

According to taylor series

$f(x) = x^3 - 10x^2 + 6 =$

$$-57 + \frac{(x-3)}{1} \times (-3) + \frac{(x-3)^2}{\cancel{2}\times 1} \times (\cancel{8}) + \frac{(x-3)^3}{3\times 2\times 1} \times 6 + \cdots$$

$$= -57 - 3(x-3) + 4(x-3)^2 + (x-3)^3$$

13. $f(x) = (1+x)^n$ $\quad \cancel{f(0) = (0)^n = 0}$

$f'(x) = n(1+x)^{n-1}$ $\quad \cancel{f'(0) = n(}$

$f''(x) = n(n-1)(1+x)^{(n-1)-1}$

$f'''(x) = (n-2)(n-1)n\,(1+x)^{n-3}$

$f(0) = (1)^n = 1$

$f'(0) = n(1)^{n-1} = n$

$f''(0) = n(n-1)(1)^{n-2} = n(n-1)$

$f'''(0) = n(n-1)(n-2)(1)^{n-3} = n(n-1)(n-2)$

Maclaurin Series

$$f(x) = f(0) + \frac{(x)'}{1!} \times f'(0) + \frac{x^2}{2!} \times f''(0) + \frac{x^3}{3!} \times f'''(0) + \dots$$

$$\therefore (1+x)^n = 1 + (x) \times n + \frac{x^2}{2\times 1} \times n^2 - n + \frac{x^3}{3\times 2\times 1} \times n(n-1)(n-2) \dots$$

$$= 1 + xn + \frac{x^2}{2!} n(n-1) + \frac{x^3}{3!} \times n(n-1)(n-2) + \dots$$

(14) $f(x) = \sqrt{1+x}$ $\qquad f(0) = 1$

$f'(x) = \dfrac{1}{2\sqrt{1+x}}$ $\qquad f'(0) = \dfrac{1}{2}$

$f''(x) = \dfrac{-\sqrt{1+x}}{4}$ $\qquad f''(0) = -\dfrac{1}{4}$

$f'''(x) = \dfrac{-1}{8\sqrt{1+x}}$ $\qquad f'''(0) = -\dfrac{1}{8}$

Mac lawrin Series

$$f(x) = f(0) + \frac{(x)^1}{1!} f'(0) + \frac{(x)^2}{2!} f''(0) + \frac{x^3}{3!} \times f'''(0) + \cdots$$

$$\sqrt{1+x} = 1 + \frac{(x)}{2} + \frac{x^2}{2\times1} \times \left(\frac{-1}{4}\right) - \frac{x^3}{3\times2\times1\times8}$$

$$= 1 + \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{48}$$

(15) $f(x) = e^{-6x}$ $(a = -4)$ $\qquad f(-4) = e^{24}$

$f'(x) = (e^{-6x}) \times (-6)$ $\qquad f'(-4) = e^{24} \times (-6)$

$f''(x) = e^{-6x} \times 36$ $\qquad f''(-4) = e^{24} \times 36$

$f'''(x) = e^{-6x} \times (-216)$ $\qquad f'''(-4) = e^{24} \times (-216)$

Taylor Series:

$$f(x) = f(a) + \frac{(x-a)^1}{1!} \times f'(a) + \frac{(x-a)^2}{2!} \times f''(a) + \frac{(x-a)^3}{3!} \times f'''(a) \ldots$$

$$\therefore e^{-6x} = e^{24} + \frac{(x+4)}{} \times e^{24} \times (-6) + \frac{(x+4)^2}{2 \times 1} \times e^{24} \times 36 +$$

$$\frac{(x+4)^3}{3 \times 2 \times 1} \times e^{24} \times (216) + \ldots$$

$$= e^{24} - 6(x+4)e^{24} + 18(x+4)^2 e^{24} - 36(x+4)^3 e^{24} + \ldots$$

Q.16. $f(x) = \ln(3+4x) \quad (a=0) \qquad f(0) = \ln(3)$

$$f'(x) = \frac{4}{3+4x} \qquad f'(0) = \frac{4}{3}$$

$$f''(x) = \frac{-1}{(3+4x)^2} \times 4 \times 4 \qquad f''(0) = -\frac{16}{9}$$

$$f'''(x) = \frac{-2 \times -1 \times 4 \times 4 \times 4}{(3+4x)^3} \qquad f'''(0) = \frac{32}{27} =$$

$$= \frac{2 \times 4 \times 4}{(3+4x)^3} = \frac{32}{(3+4x)^3}$$

Taylor Series

$$f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} \times f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \cdots$$

$$\ln(3+4x) =$$

$$\ln(3) + \frac{x}{3} + \frac{x^2}{2!} \times \left(-\frac{4}{9}\right) + \frac{x^3}{3 \times 2 \times 1} \times \frac{\overset{16}{\cancel{32}}}{27} + \cdots$$

$$= \ln(3) + \frac{x}{3} - \frac{2x^2}{9} + \frac{16x^3}{81} + \cdots$$