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Roll. no - 11

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Assignment-1

Q1.) actual radius = 12.5 cm
measured radius = 12.65 cm.

Actual area = $\pi \times (12.5)^2$
$= 156.25\pi \ cm^2$

measured area $= \pi \times (12.65)^2$
$= 160.0225\pi \ cm^2$

(I) Absolute error -

Absolute error = $160.0225\pi - 156.25\pi$
$= 3.7725\pi$

$\boxed{\text{Absolute error} = 11.8517 \ cm^2}$

(II) Relative error -

Relative error = $\dfrac{\text{Absolute error}}{\text{real value}}$

Relative error = $\dfrac{3.7725\pi}{156.25\pi}$

$\boxed{\text{Relative error} = 0.024144}$

(III) % error -

% error = Relative error × 100
% error = 0.024144 × 100

$\boxed{\%\ \text{error} = 2.4144\%}$

Q2.)
(a) using $x = 4$ and $x = 5$ -

$$\log 5 = \frac{\log 4 + \log 6}{2}$$

$$\log 5 = \frac{0.60206 + 0.778153}{2} = \frac{1.3802113}{2}$$

$$\boxed{\log 5 = 0.69010565}$$

(b) using $x = 4.5$ & $x = 5.5$ -

$$\log 5 = \frac{\log 4.5 + \log 5.5}{2}$$

$$\log 5 = \frac{0.6532125 + 0.7403627}{2} = \frac{1.3935752}{2}$$

$$\boxed{\log 5 = 0.6967876}$$

Q3.) $y = x^4 - x - 10$

$x_1 = 1.5$, $y_1 = -6.4375$
$x_2 = 2.$, $y_2 = 4$

(1)
$$x_3 = \frac{1.5 + 2}{2} = 1.75$$

$\therefore$ ~~y_3~~ corresponding y -

$$y_3 = -2.37109375$$

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(2)

$$x_4 = \frac{x_2 + x_3}{2} = \frac{2+1.75}{2}$$

$x_4 = 1.875$

corresponding y -

$y_4 = 0.484619$

(3)

$$x_5 = \frac{x_3 + x_4}{2} = \frac{1.75+1.875}{2}$$

$x_5 = 1.8125$

corresponding y -

$y_5 = -1.020248$

(4)

$$x_6 = \frac{x_4 + x_5}{2} = \frac{1.875+1.8125}{2}$$

$x_6 = 1.84375$

corresponding y -

$y_6 = -0.287734$

(5)

$$x_7 = \frac{x_4 + x_6}{2} = \frac{1.875 + 1.84375}{2}$$

$\boxed{x_7 = 1.859375}$

corresponding y -

$\boxed{y_7 = 0.0933781}$

Q4) $f(x) = x^3 - x - 1$

$\therefore f'(x) = 3x^2 - 1$

x	$f(x)$	$f'(x)$
0	-1	-1
1	-1	2
2	5	11

taking $x = 1$

$\therefore x_1 = 1$, $f(1) = -1$, $f'(1) = 2$

(1)

$$x_2 = x_1 - \frac{f(1)}{f'(1)}$$

$$x_2 = 1 - \frac{-1}{2} = 1 + \frac{1}{2}$$

$$x_2 = 1.5$$

$\therefore f(1.5) = 0.875$

$f'(1.5) = 5.75$

(2)

$$x_3 = x_2 - \frac{f(1.5)}{f'(1.5)}$$

$$x_3 = 1.5 - \frac{0.875}{5.75}$$

$$x_3 = 1.347826$$

$f(x_3) = 0.100682$

$f'(x_3) = 4.4499$

(3)

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$x_4 = 1.347826 - 0.022626$

$x_4 = 1.3252$

$f(x_4) = 0.0025666$

$f'(x_4) = 4.268465$

(4)

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}$$

$x_5 = 1.324599$

$f(x_5) = -0.000507252$

$f'(x_5) = 4.2636875$

(5)

$$x_6 = x_5 - \frac{f(x_5)}{f'(x_5)}$$

$x_6 = 1.32471797$

$f(x_6) = 0.0000000054396\overline{4}7$

$\therefore$ root of given eqn -

$$\boxed{x = 1.32471797}$$

Q5) $A^2 = A$

$7A - (I+A)^3 = ?$

$A^2 = A$

$\therefore A^2 - A = 0$

$\therefore A(A-I) = 0$

$\therefore A = 0$ or $A = I$

$\therefore$ A is a null matrix or an identity matrix.

(1) <u>If A is a null matrix -</u>

$$7\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right)^3$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)^3$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

(2) <u>If A is an identity matrix -</u>

$$7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)^3$$

$$7A - (I+A)^3 = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^3$$

$$7A - (I+A)^3 = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} \; \\ \; \end{bmatrix}$$

$$7A - (I+A)^3 = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$$

$$7A - (I+A)^3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Hence we can see that, whether A is a null matrix or identity matrix,

Ans- $7A - (I+A)^3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

~~Q6~~

Q6.) let $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$|A| = 1(-6) + 1(-4) + 2(4)$

$|A| = -6 - 4 + 8 = -2.$

minors-

$M_{11} = -6$, $M_{21} = -1$, $M_{31} = -6$

$M_{12} = -4$, $M_{22} = -1$, $M_{32} = -2$

$M_{13} = 4$, $M_{23} = 1$, $M_{33} = 4$

co-factors-

$a_{11} = -6$, $a_{21} = 1$, $a_{31} = -6$

$a_{12} = 4$, $a_{22} = -1$, $a_{32} = 2$

$a_{13} = 4$, $a_{23} = -1$, $a_{33} = 4$

matrix of co-factors $= \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

Ans- $\therefore$ Inverse of $A = \frac{-1}{2}\begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

Q7) $\bar{A} = 8\hat{i} - 9\hat{j}$
$\bar{B} = 7\hat{i} - 9\hat{j}$

$\bar{A}\cdot\bar{B} = (8\hat{i} - 9\hat{j})(7\hat{i} - 9\hat{j})$
$\bar{A}\cdot\bar{B} = 56 + 81$
$\bar{A}\cdot\bar{B} = 137$

$|\bar{A}| = \sqrt{8^2 + (9)^2} = \sqrt{64+81} = \sqrt{145}$
$|\bar{B}| = \sqrt{7^2 + (-9)^2} = \sqrt{49+81} = \sqrt{130}$

we know -

$\cos\theta = \frac{\bar{A}\cdot\bar{B}}{|\bar{A}||\bar{B}|}$

$\therefore \theta = \cos^{-1}\left(\frac{\bar{A}\cdot\bar{B}}{|\bar{A}||\bar{B}|}\right)$

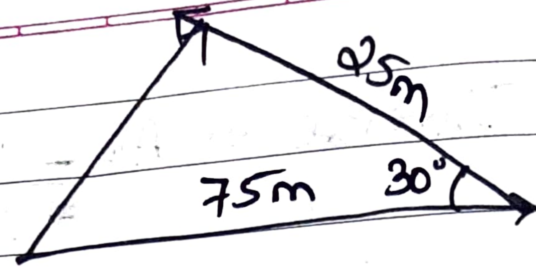
$\theta = \cos^{-1}\left(\frac{137}{137.29}\right)$

rounding off 134.29 to 137.

$\theta = \cos^{-1}\left(\frac{137}{137}\right) = \cos^{-1}(1)$

$\boxed{\theta = 0°}$

Q8.)



Q9)

least 3 digit no. divisible by 3 = 102

largest 3 digit no. divisible by 3 = 999.

$\therefore a = 102,$

$d = 3$

$t_n = 999.$

$T_n = a + (n-1)d$

$999 = 102 + (n-1)3$

$\therefore 3n = \cancel{\quad} 897 - 3$

$3n = 894$

$n = 298$

$S_n = \frac{n}{2}\left[a + t_n\right]$

$S_n = \frac{298}{2}\left[102 + 999\right]$

$\boxed{S_n = 164049}$

Q10)

$t_6 = 32$

$t_8 = 128$

$\therefore ar^5 = 32$ & $ar^7 = 128$

$a = \frac{32}{r^5}$

$\therefore$

$\therefore \frac{32}{r^5} \cdot r^7 = 128$

$r^2 = \frac{128}{32}$

$r^2 = 4$

$\therefore \boxed{r = \pm 2}$

Q11) $(4+3x)^5$

$$(a+b)^n = a^n + n\,a^{n-1}b + \frac{n(n-1)a^{n-2}b^2}{2!} + \frac{n(n-1)(n-2)a^{n-3}\cdot b^3}{3!} + \dots$$

$$(4+3x)^5 = 4^5 + 5(4^4)(3x) + 10(4^3)(3x)^2 + 10(4^2)(3x)^3 + 5(4)(3x)^4 + (3x)^5$$

Ans- $\therefore (4+3x)^5 = 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$

Q12) $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

<u>eigen values-</u>

$\lambda^3 - (2-5+3)\lambda^2 + (-15+6-4)\lambda - [2(-15)+3(6)] =$

$\lambda^3 - 13\lambda + 12 = 0$

$\lambda = 1$ satisfies given eqn.

$$\begin{array}{c|cccc} 1 & 1 & 0 & -13 & 12 \\ \hline & & & & \\ & 1 & 1 & -12 & 0 \end{array}$$

$\therefore \lambda^2 + \lambda - 12 = 0$

$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$

$$(\lambda-3)(\lambda+4)=0$$

$\therefore$ ~~eig~~ $\boxed{\lambda = 3, 1, \cancel{-4}}$ → eigen values.

<u>eigen vectors</u> —

① $\lambda = 3$,

$0, \frac{1}{2}, 3$ → eigen vectors

② $\lambda = -\cancel{4}$

$0, 1, 1$ → eigen vectors

③ $\lambda = 1$

$1, 0, 1$ → eigen vectors.

Q13) $f(x) = x^3 - 7x^2 + 8x - 3$

$x_0 = 5$

$f(x_0) = -13$

$\therefore f'(x) = 3x^2 - 14x + 8$

$f'(x_0) = 13$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 5 - \left(\frac{-13}{13}\right)$$

$$x_1 = 6$$

$$f(x_1) = 9$$
$$f'(x_1) = 32$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 6 - \frac{9}{32}$$

$$\boxed{x_2 = 5.71875}$$

Q14.) $(1-x+x^2)^4$

$$(1-x+x^2)^4 = 1(1-x)^4 + 4(1-x)^3 x^2 + 6(1-x)^2 x^4 + 4(1-x)(x^2)^3 + (x^2)^4$$

$$(1-x+x^2)^4 = (1-x)^4 + 4x^2(1-x)^3 + 6x^4(1-x)^2 + 4x^6(1-x) + x^8$$

Ans $\rightarrow$ $(1-x+x^2)^4 = x^8 - 4x^7 + 10x^6 - 16x^5 + 19x^4 - 16x^3 + 10x^2 - 4x + 1$

Q15.) $e^{-x} = 3\log x$

taking natural log on both sides -

$$-x\log e = \log(3\log x)$$

$$-x = \log 3 + \log(\log x)$$

$$\therefore \log(\log x) + x + \log 3 = 0$$

$$\therefore y = \log(\log x) + x + 1.098612$$

$x_0 = e$, $y_0 = 3.816894$

$x_1 = 3$, $y_1 = 4.1926598$

$x_2 = 2$, $y_2 = 2.732099$

(1)

$$x_3 = \frac{x_0 + x_2}{2} = \frac{e+2}{2} = 2.359141$$

corresponding y -

$y_3 = 3.304948$

(2)

$$x_4 = \frac{x_2 + x_3}{2} = \frac{2+2.359141}{2}$$

$x_4 = 2.1795705$

corresponding y -

$y_4 = 3.028602$

(3)

$$x_5 = \frac{x_2 + x_4}{2} = 2.089785$$

$y_5 = 2.883313$

~~(4) x_6 =~~