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Q1. The nominal rate of discount per annum convertible quarterly is 8%.

i) Calculate the equivalent for of interest.

$$\delta = \ln\left(\frac{1}{1-d}\right) = \ln\left(\frac{1}{\left(1-\frac{d^{(4)}}{4}\right)^4}\right) = \ln\left(\frac{1}{\left(1-\frac{8}{400}\right)^4}\right)$$

$$\delta = 8.08108\%$$

ii) Calculate the equivalent effective rate of interest per annum.

$$(1+i) = 1 \Big/ \left(1-\frac{d^{(4)}}{4}\right)^4 = 1.084165$$

$$i = 8.4165\%$$

iii) Calculate the equivalent nominal rate of ~~interest~~ discount per annum convertible monthly.

$$\left(1-\frac{d^{(4)}}{4}\right)^4 = \left(1-\frac{d^{(12)}}{12}\right)^{12}$$

$$0.922368 = \left(1-\frac{d^{(12)}}{12}\right)^{12}$$

$$d^{(12)} = 8.05395\%$$

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Q2. The force of interest $\delta(t) = 0.004t + 0.0002t^2$ for all t.

i) Find Present value of sum of rupees ₹1000 payable at the end of 10 years from now

ii) Find constant annual effective rate of interest over a 10 year period.

Sol. i)

$$AV_{10} = PV_0 \times A(0,10)$$

$$PV_0 = AV_{10} \times \frac{1}{A(0,10)}$$

$$A(0,10) = e^{\int \delta(t).dt} = e^{\int_0^{10} 0.004t + 0.0002t^2 dt}$$

$$\int_0^{10} 0.004t + 0.0002t.dt$$

$$\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3}\right]_0^{10}$$

$$0.2 + 0.06667$$

$$= 0.26667$$

$$A(0,10) = e^{0.26667} = 1.305609524$$

$$PV_0 = 1000 \times \frac{1}{1.30561}$$

$$PV_0 = ₹765.9255$$

Sol ii)

$$(1+i)^{10} = A(0,10)$$

$$(1+i)^{10} = 1.30561$$

$$1+i = 1.027026$$

$$i = 2.70258\%$$

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(Q3 i) Explain the relationship $d = iv$ by general reasoning where d is the effective annual rate of discount and i is the effective annual rate of interest.

Sol. For $t = 1$ yrs $A(0,1) = (1+i)$ & $V(0,1) = (1-d)$

As we know $A(0,1) = \frac{1}{V(0,1)}$

$$\therefore (1+i) = \frac{1}{(1-d)} \quad \text{or} \quad (1-d) = \frac{1}{(1+i)}$$

$$\cancel{\Rightarrow d = \frac{1}{(1+i)}} \Rightarrow d = 1 - \frac{1}{(1+i)} = \frac{1+i-1}{(1+i)} = \frac{i}{(1+i)}$$

Since $v(0,1) = \frac{1}{(1+i)}$

$d = i \times v$

ii) Calculate nominal rate of discount per annum convertible half yearly which is equivalent to :-

a) An effective rate of discount of 2.25% per quarter

b) A nominal rate of discount of 5% convertible every 2 years.

Sol. a) given $d = 2.25\%$ per quarter $\Rightarrow d^{(4)} = 2.25 \times 4 = 9\%$.
find $d^{(2)}$

$$\Rightarrow \left(1 - \frac{d^{(4)}}{4}\right)^4 = \left(1 - \frac{d^{(2)}}{2}\right)^2$$

$$\Rightarrow \left(1 - \frac{9}{400}\right)^4 = \left(1 - \frac{d}{200}\right)^2$$

$$\Rightarrow (0.91299)^{1/2} = 1 - \frac{d}{200}$$

$$\Rightarrow 0.955506250 - 1 = -\frac{d}{200}$$

$$\Rightarrow 0.0444938 = \frac{d}{200}$$

$$\Rightarrow d = 8.89875\ \%.$$

Sol. b) given $d = 5\%$ per 2 years $\Rightarrow d^{(0.5)} = 5 \times 2 = 10\%$
find $d^{(2)}$

$$\Rightarrow \left(1 - \frac{d^{(0.5)}}{0.5}\right)^{0.5} = \left(1 - \frac{d^{(2)}}{2}\right)^2$$

$$\Rightarrow \left(1 - \frac{10}{50}\right)^{0.5} = \left(1 - \frac{d}{200}\right)^2 \Rightarrow 0.8944272 = \left(1 - \frac{d}{200}\right)^2$$

$$\Rightarrow 0.945741614 - 1 = -\frac{d}{200} \Rightarrow 0.0542584 = \frac{d}{200}$$

$$\Rightarrow d = 10.8516\ \%.$$

iii) A 91-day government bill provides the investor with an annual effective rate of return of 5%. Calculate the annual simple discount rate at which the bill is discounted)

$$(1-d) = \frac{1}{(1+i)}$$

$$\left(1 - \frac{91}{365} d\right) = 1 \Big/ \left(1 + \frac{91}{365} \times \frac{5}{100}\right)$$

$$\left(1 - \frac{91}{365} d\right) = 0.9877682728$$

$$d = 4.9384\%$$

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Q4 i) Repeated Question

ii) Given $i = 0.08$, calculate

a) $d^{(12)}$; $d^{(p)} = p[1 - (1+i)^{-1/p}]$

$d^{(12)} = 12[1-(1+0.08)^{-1/12}]$

$d^{(12)} = 7.6715\%$

b) $i^{(365)}$; $i^{(p)} = p[(1+i)^{1/p} - 1]$

$i^{(365)} = 365[(1+0.08)^{1/365} - 1]$

$i^{(365)} = 7.69692\%$

c) δ ; $\delta =$ ~~to~~ $\ln(1+i)$

$\delta = \ln(1+0.08)$

$\delta = 7.6961\%$

d) $i^{(0.5)}$; $i^{(p)} = p[(1+i)^{1/p} - 1]$

$i^{(0.5)} = 0.5[(1+0.08)^{1/0.5} - 1]$

$i^{(0.5)} = 8.32\%$

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Q5) i) The rate of discount per annum convertible quaterly is 6%. Calculate:-

a) The equivalent rate of interest per annum convertible half yearly.

sol. given : $d^{(4)} = 6\%$, find : $i^{(2)}$

$$\left(1+\frac{i^{(p)}}{p}\right)^p = \frac{1}{\left(1-\frac{d^{p}}{p}\right)^p} \Rightarrow i^{(p)} = p\left[\left(\frac{1}{\left(1-\frac{0.06}{p}\right)^p}\right)^{1/p}-1\right]$$

$$i^{(2)} = 2\left[\left(\frac{1}{\left(1-\frac{0.06}{4}\right)^4}\right)^{1/2}-1\right]$$

$$i^{(2)} = 6.1378\%$$

b) The equivalent rate of discount per annum convertible monthly

sol. given : $d^{(4)} = 6\%$, find : $d^{(12)}$

$$d^{(12)} = p\left[1-\left(\left(1-\frac{d^{(4)}}{4}\right)^4\right)^{1/12}\right] = 12\left[1-\left[\left(1-\frac{d^{(4)}}{4}\right)^4\right]^{1/12}\right]$$

$$d^{(12)} = 6.0303\%$$

ii) On 15th April, 2005 Amit borrowed ₹200000 to be repaid one year later by single payment of ₹ 220000. Amit repaid the loan on 17th July, 2005.

a) Find the sum paid by Amit to terminate the contract assuming that the interest is reduced proportionately for early settlement

Sol.

$AV(0,1) = PV_0 \times A(0,1)$

$220000 = 200000 \times (1+i)$

$i = 10\%$

15th April 2005 to 17th July 2005 = 93 days = 365 yrs

$AV\left(0,\frac{93}{365}\right) = PV_0 \times A\left(0,\frac{93}{365}\right)$

$AV = 200000 \times \left(1+\frac{10}{100}\right)^{93/365}$

$AV = ₹204916.3564$

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Q7. If an investment fund offers to increase ₹20000 to ₹26000 in 17 months, calculate the following:

i) The nominal rate of interest per annum convertible quarterly.

$AV = PV(1+i)$

$26000 = 20000(1+i)^{17/12}$

$1.3 = (1+i)^{17/12}$

$i = 20.3457\%$

$\cancel{i^{(4)} = \left(1+\frac{i^{(4)}}{4}\right)^4}$

$i^{(4)} = 4\left[(1+i)^{1/4} - 1\right]$

$i^{(4)} = 4\left[\left(1+\frac{20.3457}{100}\right)^{1/4} - 1\right]$

$i^{(4)} = 18.9553\%$

ii) $d^{(2)}$

$d = \frac{i}{1+i} = \frac{20.3457\%}{1+20.3457\%} = 16.9060\%$

$d^{(2)} = 2\left[1-(1-d)^{1/2}\right] = 2\left[1-\left(1-\frac{16.9060}{100}\right)^{1/2}\right]$

$d^{(2)} = 17.6882\%$

iii) i (simple interest)

$$AV = PV(1+i)$$

$$26000 = 20000\left(1+\frac{17}{12}\times i\right)$$

$$1.3 = 1+\frac{17i}{12}$$

$$i = 21.1765\%$$

Q9: i) Define force of interest

Force of interest is the measure of the intensity with which interest operates at each individual moment of time

ii) given $i^{(2)} = 0.0775$, calculate :-

a. i ; $i = \left(1 + \frac{i^{(2)}}{2}\right)^2 - 1$

$i = \left(1 + \frac{0.0775}{2\cancel{\times 100}}\right)^2 - 1$

$i = 7.9\%$

b. δ ; $\delta = \ln(1+i)$

$\delta = \ln(1 + 7.9\%) = \ln(1 + 0.079)$

$\delta = 7.60347\%$

c. $d^{(12)} =$

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c. $d^{(12)}$; $d^{(p)} = p[1-(1+i)^{-1/p}]$

$$d^{(12)} = 12\left[1-\left(1+\frac{7.9}{100}\right)^{-1/12}\right]$$

$$d^{(12)} = 7.5794\ \%$$

d. $i^{(1/2)}$; $i^{(p)} = p[(1+i)^{1/p} - 1]$

$$i^{(0.5)} = 0.5\left[\left(1+\frac{7.9}{100}\right)^{1/0.5} - 1\right]$$

$$i^{(0.5)} = 8.2121\ \%$$

Q10. $\delta(t)$ the force of interest per annum at time t years is defined as :-

$$\delta(t) = \begin{cases} 0.08 & 0 \leq t \leq 5 \\ 0.06 & 5 \leq t \leq 10 \\ 0.04 & t \geq 10 \end{cases}$$

Using $\delta(t)$, Derive the expression for $V(t)$, the present value of 1 due at Time t.

$$v(t) = v(0,5) \times v(5,10) \times v(10,t)$$

$$v(t_1,t_2) = e^{-\int_{t_1}^{t_2} \delta(t)\cdot dt}$$

$$v(t) = e^{-\int_0^5 0.08\cdot dt} \times e^{-\int_5^{10} 0.06\cdot dt} \times e^{-\int_{10}^{t} 0.04\cdot dt}$$

$$v(t) = e^{-[0.08t]_0^5} \times e^{-[0.06t]_5^{10}} \times e^{-[0.04t]_{10}^{t}}$$

$$v(t) = e^{-0.4} \times e^{-[0.6-0.3]} \times e^{-[0.04t-0.4]}$$

$$v(t) = e^{-0.4} \times e^{-0.3} \times e^{-0.04t} \times e^{0.4}$$

$$v(t) = e^{-0.3} \times e^{-0.04t}$$

$$\therefore \; v(t) = 0.74082 \times e^{-0.04t}$$

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Q11.a) A 9 month loan, repayable by a single payment of ₹50000 is issued at a rate of commercial discount of 18% pa. What amount was initially lent to the borrower?

~~$AV = PV(1-d)$~~

~~$50000 = PV\left(1-\frac{18}{100}\right)^{9/12}$~~

~~$PV = \frac{50000}{0.86170}$~~

$PV = AV(1-d)^{9/12}$

$PV = 50000\left(1-\frac{18}{100}\right)^{9/12}$

$PV = 50000 \times 0.86170$

$PV = ₹43085$

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6. i. If $\delta = 6\%$, find $d^{(12)}$

$$\left(1 - \frac{d^{(p)}}{p}\right)^p = e^{-\delta}$$

$$\left(1 - \frac{d}{12}\right)^{12} = e^{-0.06}$$

$$1 - \frac{d}{12} = (0.941765)^{1/12}$$

$$-\frac{d}{12} = 0.9950125$$

$$d = 5.985\%$$

ii. If $d^{(4)} = 6\%$, find $i^{(12)}$

$$\left(1 + \frac{i^{(12)}}{12}\right)^{12} = 1 \Big/ \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$i^{(12)} = 12\left[\left(\frac{1}{0.94134}\right)^{1/12} - 1\right]$$

$$i^{(12)} = 6.0603\%$$

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c. Arrange the following quantities in ascending order of numerical value, giving brief reasons for the order assuming that they all correspond to same effective interest rate

$d < \delta < i^{(4)} < i$

Since nominal rate of interest ($i^{(4)}$) gets compounded 4 times a year and generates a higher value than annual compounding, the value of effective (i) is greater as it gets compounded only once per year. The value (d) is further smaller as it is compounded on AV which is a larger value and % required to accumulate the same value is lesser.

d. Explain in words why the relationship $d = iv$ holds true.

For a time period 1, Present value is discounting factor multiplied by Accumulated value.

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Q12. A deposite of ₹7049 is accumulated at the following rates of interest : 6% per annum nominal convertible monthly for first two years followed by 1.5% per quater simple for next two and a half years followed by a rate of discount of 6% per annum convertible monthly for next one and a half years. Calculate the accumulated amount after 6 years.

$$\cancel{AV_6 = PV_0 \left(1 + \frac{1.5\%}{24}\right)^{-24}}$$

$$\cancel{AV_2 = PV_0 \left(1 + \frac{i^{(p)}}{p}\right)^{pn} = PV_0 \left(1 + \frac{i^{(12)}}{12}\right)^{12n}}$$

$$\cancel{AV_2 = 7049 \left(1 + \frac{1.5}{1200}\right)^{24} = 7263.5379 = PV_2}$$

~~AV~~

$$AV_2 = PV_0 \left(1 + \frac{i^{(12)}}{12}\right)^{12n} = 7049 \left(1 + \frac{6}{1200}\right)^{24}$$

$$AV_2 = PV_2 = ₹\,7945.3493$$

$$AV_{4\frac{1}{2}} = PV_2 \left(1 + \frac{i^{(4)}}{4}\right)^{4n} = 7945.3493 \left(1 + \frac{1.5}{400}\right)^{10}$$

$$AV_{4\frac{1}{2}} = PV_{4\frac{1}{2}} = ₹\,8248.3784$$

$$AV_6 = PV_{4\frac{1}{2}}\left(1+\frac{i^{(12)}}{12}\right)^{12} \quad \cancel{PV_{4\frac{1}{2}}[1+}$$

$$= 8248.3784\left(1+\frac{6}{1200}\right)^{30}$$

$$AV_6 = ₹\,9579.6674$$

13Q Calculate the time in years for an investment to double at :

i) An effective rate of interest of 10% pa.
Let PV = 1 & AV = 2

$$AV = PV \times A(0,n)$$

$$2 = 1\left(1+\frac{10}{100}\right)^n$$

Taking ln on both sides

$$\ln 2 = n \cdot \ln\left(1+\frac{10}{100}\right)$$

$$0.69315 = n \cdot 0.09531$$

$n = 7.27\cancel{2584+}$ years

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ii) A nominal discount rate of 10% pa convertible monthly.

$$2 = \left(1+\frac{10}{1200}\right)^{12n}$$

taking ln on both sides

$$\ln 2 = 12n \cdot \ln\left(1+\frac{10}{1200}\right)$$

$0.6932 = 12n \cdot 1.00833$

$12n = 0.68742$

$n = 0.0573$ years