1. $\lim_{h \to 0} \dfrac{2(-3+h)^2 - 18}{h}$

$= \lim_{h \to 0} \dfrac{18 - 12h + 2h^2 - 18}{h}$

$= \lim_{h \to 0} \dfrac{h(2h-12)}{h} = 2h - 12 = -12$

2. $g(x) = \begin{cases} 2x & x < 6 \\ x - 1 & x \geq 6 \end{cases}$

a) $\lim_{x \to 4^-} g(x)$ $\qquad$ $\lim_{x \to 4^+} g(x)$

$= \lim_{x \to 4^-} 2x = 8$ $\qquad$ $\lim_{x \to 4^+} 2x = 8$

$\because$ LHL = RHL

$\Rightarrow$ Function is continuous at $x = 4$

b) $\lim_{x \to 6^-} g(x)$ $\qquad$ $\lim_{x \to 6^+} g(x)$

$\lim_{x \to 6^-} 2x = 12$ $\qquad$ $\lim_{x \to 6^+} (x - 1) = 5$

$\because$ LHL $\neq$ RHL

$\Rightarrow$ Function is not continuous at $x = 6$

3. $\lim_{x \to 9} \dfrac{x - 9}{3 - \sqrt{x}} = \lim_{x \to 9} \dfrac{(\sqrt{x})^2 - 3^2}{(3 - \sqrt{x})}$

$= \lim_{x \to 9} \dfrac{\cancel{(\sqrt{x} - 3)}(\sqrt{x} + 3)}{-\cancel{(\sqrt{x} - 3)}} = -6$

4. $f(x) = \dfrac{4x + 5}{9 - 3x}$

a) $\lim_{x \to -1} f(x) = \lim_{x \to -1} \dfrac{4x + 5}{9 - 3x} = \dfrac{1}{12}$ $\quad$ $\therefore$ continuous

b) $\lim_{x \to 0} f(x) = \lim_{x \to 0} \dfrac{4x + 5}{9 - 3x} = \dfrac{5}{9}$ $\quad$ $\therefore$ continuous

c) $\lim_{x \to 3} f(x) = \lim_{x \to 3} \dfrac{4x + 5}{9 - 3x} = \dfrac{17}{0}$ $\Rightarrow$ not defined

$\therefore$ Infinite discontinuity

5. $\lim_{x\to\infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$= \lim_{x\to\infty} \left(\frac{6e^{4x} - e^{-2x}}{e^{4x}}\right)\left(\frac{e^{4x}}{8e^{4x} - e^{2x} + 3e^{-x}}\right)$

$= \lim_{x\to\infty} \left(\frac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}}\right) = 3/4$

6. $g(y) = \begin{cases} y^2 + 5 & \text{if } y < -2 \\ 1 - 3y & \text{if } y \geq -2 \end{cases}$

$\lim_{y\to 6} g(y) = \lim_{y\to 6} (1-3y) = -17$

$\lim_{y\to 2^-} g(y) = \lim_{y\to 2^-} (y^2+5) = 9$

$\lim_{y\to 2^+} g(y) = \lim_{y\to 2^+} (1-3y) = -5$

7. $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$\frac{d\,f(t)}{dt} = (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$

$= 20t^4 - 132t^3 + 24t^2 - 12 + 96t$

8. $R(W) = \frac{3W + W^4}{2W^2 + 1}$

$\frac{d(R(W))}{dW} = \frac{(2W^2+1)(3+3W^3) - (3W + W^4)(4W+1)}{(2W^2+1)^2}$

9. $f(x) = 2e^x - 8^x$

$f'(x) = 2e^x - 8^x \log_e 8$

10. $y = x^5 - e^x(\ln(x))$

$\frac{dy}{dx} = 5x^4 - \left[\frac{e^x}{x} + \ln(x)e^x\right] = 5x^4 - \frac{e^x}{x} + e^x \ln(x)$

11. a) $f(x) = (6x^2 + 7x)^4 = 4(6x^2 + 7x)^3(12x + 7)$

b) $f(t) = 5 + e^{4t + t^7}$

$f'(t) = e^{4t + t^7}(4 + 7t^6)$

12. a) $z = \ln(7 - x^3)$

$$\frac{dz}{dx} = \frac{-1}{7-x^3}(3x^2)$$

$$\frac{d^2z}{dx^2} = -\left[\frac{(7-x^3)(6x) - (3x^2)(-3x^2)}{(7-x^3)^2}\right]$$

$$\frac{d^2z}{dx^2} = -\left[\frac{42x - 6x^4 + 9x^4}{(7-x^3)^2}\right] = \left[\frac{-42x - 3x^4}{(7-x^3)^2}\right]$$

b) $$Q(v) = \frac{z}{(6+(2v)-v^2)^4} = y$$

$$\frac{dy}{dv} = \frac{2(-4)(2-2v)}{(6+2v-v^2)^5} = \frac{-16(1-v)}{(6+2v-v^2)^5}$$

$$\frac{d^2y}{dv^2} = -16\left[\frac{(6+2v-v^2)^5(-1) - (1-v)(6+2v-v^2)^4(5)(2-2v)}{(6+2v-v^2)^{10}}\right]$$

$$= -16\,\frac{(v^2-2v-6) - (1-v)(2-2v)}{(6+2v-v^2)^6}$$

$$= \frac{(2(1-v)^2 - (v^2 \cancel{x} + 2v + 6)}{(6+2v-v^2)^6}\ 16$$

c) $f(t) = \ln(1+t^2)$

$$f'(t) = \frac{1}{1+t^2}(2t)$$

$$f''(t) = \frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}$$

$$= \frac{2+2t^2-4t^2}{(1+t^2)^2} = \frac{2-2t^2}{(1+t^2)^2}$$

13. $f(x) = y = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$

$f''(x) = 6x - 6$

$f''(-1) = -12$ $\quad f''(3) = 18 - 6 = 12$

$f(-1) = -1 - 3 + 9 + 12 = 17 \rightarrow$ maximum value

$f(3) = 27 - 27 - 27 + 12 = -15 \rightarrow$ minima value

14. $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

$f''(x) = 24x - 36$

$f''(2) = 48 - 36 = 12$

$f''(1) = 24 - 36 = -12$

$f(1) = 4 - 18 + 24 - 7 = 3 \rightarrow$ maxima

$f(2) = 80 - 119 = 1 \rightarrow$ minima

15. $f(x) = x^2(x+3) = x^3 + 3x^2$

$f'(x) = 3x^2 + 6x = 0$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0, -2$

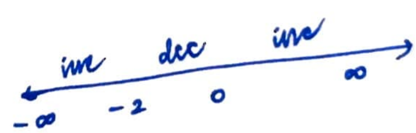
$f''(x) = 6x + 6$

$f''(0) = 6 \qquad f''(-2) = -6$

$f(0) = 0 \qquad f(-2) = 4$

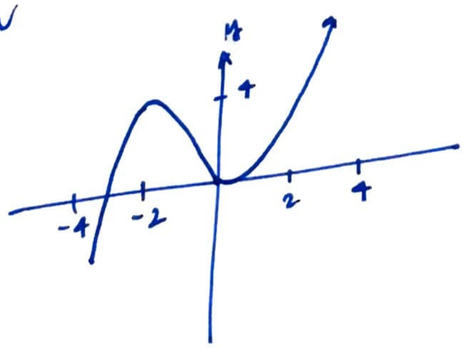
$\downarrow \qquad\qquad \downarrow$

minima $\qquad$ maxima



$f'(1) = 9$

$f(-3) = 9$



16. $f(x) = 3x^2 - 6x \qquad f'(x) = 6x - 6 \qquad f''(x) = 6$

$f(1) = -3 \qquad f'(0) = -6 \qquad f'(2) = 6$