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ASSIGNMENT - 1

1. $d^{(4)} = 8\%$

(i) Equivalent force of interest $(\delta) = -\ln(1-d)$

$$\left(1-\frac{d^{(4)}}{4}\right)^4 = 1-d$$

$$\Rightarrow \left(1-\frac{0.08}{4}\right)^4 = 1-d$$

$$\Rightarrow d = 7.7613184\%$$

~~d =~~

$$\delta = -\ln(1-0.07763184)$$

$$\delta = \underline{\underline{8.08}}10829$$

(ii) Equivalent effective rate of interest (i)

$$d = \frac{i}{1+i}$$

$$\Rightarrow 0.0763184 = 0.922368160\, i$$

$$i = \underline{\underline{8.41657}}85\%$$

(iii) To find : $d^{(12)}$

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = 1-d$$

$$\Rightarrow \left(1-\frac{d^{(12)}}{12}\right)^{12} = 1-0.07763184$$

$$\Rightarrow d^{(12)} = 0.080539339$$

$$d^{(12)} = \underline{\underline{8.0539339\%}}$$

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2. $\delta(t) = 0.004t + 0.0002t^2$

(i) To find : PV $\quad c = 1000 \quad t = 10$ years.

$$P.V_{10} = c \cdot v(0,10)$$

$$= 1000 \times \frac{1}{A(0,10)}$$

$$= 1000 \times \frac{1}{e^{\int_0^{10}(0.004t + 0.0002t^2)dt}}$$

$$= \frac{1000}{e^{[0.002t^2 + 0.000066667t^3]_0^{10}}}$$

$$= \frac{1000}{e^{0.266667}}$$

$$= \frac{1000}{1.305605607}$$

$$= \underline{\underline{765.9280832}}$$

(ii) To find : i $\quad$ 10 years

$$P.V_{10} = \frac{1}{(1+i)^{10}}$$

$$0.7659280832 = (1+i)^{-10}$$

$$\Rightarrow \quad 1+i = 1.027025438$$

$$i = 0.027025438$$

$$i = \underline{\underline{2.7025438\ \%}}$$

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3. (i) $d = iv$

∵ d is the amount of money that invested at the beginning of a period will earn during the period, where interest is paid at the start of the period. Interest payable at the end of the period (a year) is i.

So, d must be equal to the present value at the beginning of the year of payment and interest paid at the end of the year (i)

Therefore, $d = iv$

(ii) $d^{(2)} = ?$

(a) $\frac{d^{(4)}}{4} = 2.25\%$

$$\left(1 - \frac{d^{(4)}}{4}\right)^4 = 1 - d$$

$$\Rightarrow (1 - 0.0225)^4 = 1 - d$$

$$\Rightarrow d = 0.08700 7806$$

$$\left(1 - \frac{d^{(2)}}{2}\right)^2 = 1 - d$$

$$\Rightarrow \left(1 - \frac{d^{(2)}}{2}\right)^2 = 1 - 0.08700 7806$$

$$\Rightarrow d^{(2)} = 0.0889875$$

$$\underline{d^{(2)} = 8.89875\%}$$

(b) $d^{1/2} = 5\%$

$$\left(1 - 2d^{(1/2)}\right)^{1/2} = 1 - d$$

$$\Rightarrow 0.948683298 = 1 - d$$

$$\Rightarrow d = 0.051316702$$

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$$\left(1-\frac{d^{(2)}}{2}\right)^{2} = 1-0.051316702$$

$$\Rightarrow \quad d^{(\cancel{1}2)} = 0.051992508$$

$$d^{(\cancel{1}2)} = \underline{\underline{5.1992508\,\%}}$$

(iii) $i = 5\%$ 91 days $d = ?$

$$\cancel{(1-d)^{n} = \frac{1}{(1+i)^{n}}}$$

$$\Rightarrow \cancel{(1-d)^{91/365} = \frac{1}{(1+0.05)^{91/365}}}$$

$$1-nd = \frac{1}{(1+i)^{n}}$$

$$\Rightarrow \quad 1-d\left(\frac{91}{365}\right) = \frac{1}{(1+0.05)^{91/365}}$$

$$\Rightarrow \quad 1-d\left(\frac{91}{365}\right) = 0.987909561$$

$$\Rightarrow \quad d = 0.048494619$$

$$d = \underline{\underline{4.8494619\,\%}}$$

4. (i) same question

(ii) $i = 0.08$

(a) $d^{(12)} = ?$

$$d = \frac{i}{1+i}$$

$$d = 7.4074074\,\%$$

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = 1-d$$

$$d^{(12)} = \underline{\underline{7.6714776\,\%}}$$

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(b) $i^{(365)}$

$$\left(1+\frac{i^{(365)}}{365}\right)^{365} = 1+0.08$$

$$i^{(365)} = \underline{\underline{7.69691555\%}}$$

(c) δ

$$\delta = \ln(1+i)$$

$$\delta = \ln(1+0.08)$$

$$\delta = \underline{\underline{0.076961041}}$$

$$\delta = 7.6961041\%$$

(d) $i^{(0.5)}$

$$\left[1 + 2\, i^{(1/2)}\right]^{1/2} = 1+0.08$$

$$\Rightarrow i^{(0.5)} = \underline{\underline{8.32\%}}$$

5.(i) $d^{(4)} = 6\%$

(a) $i^{(2)} = ?$

$$\left(1-\frac{d^{(4)}}{4}\right)^{4} = 1-d$$

$$\Rightarrow \left(1-\frac{0.06}{4}\right)^{4} = 1-d$$

$$\Rightarrow d = 0.058663449$$

$$d = \frac{i}{1+i}$$

$$i = 0.062319315$$

$$\left(1+\frac{i^{(2)}}{2}\right)^{2} = 1+0.062319315$$

$$\Rightarrow i^{(2)} = \underline{\underline{6.1377515\%}}$$

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(b) $d^{(12)} = ?$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = 1 - 0.05866 3449$$

$$d^{(12)} =$$

$$d^{(12)} = 6.0302525\ \%$$

(ii) Amt. amit borrowed = ₹ 2,00,000

Interest for 1 year (365 days) $= \frac{20,000}{2,00,000} \times 100 = 10\%$

Interest for 93 days (As early payment was done) $= \frac{10}{100} \times \frac{93}{365}$

$= 0.02547 9452$

$= 2.5479452\%$

Amt. to be paid to terminate the contract

$[2,00,000 + (2,00,000 \times 0.02547 9452)]$

$= ₹\ 205095.8904$

6. (i) Let the retail price be ₹ 100

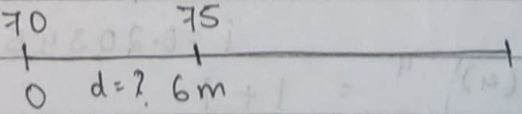
Cash price = ~~30~~ 100 − 30% of 100

= ₹ 70

Credit price (6 months) = 100 − 25% of 100

= ₹ 75

$d = ?$



$P.V_{6m} = C \cdot (1-d)^n$

$\Rightarrow 70 = 75\,(1-d)^{0.5}$

$\Rightarrow \frac{70}{75} = (1-d)^{0.5}$

$\Rightarrow d = 12.8888889\ \%$

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(ii) → $d^{(12)}$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = 1 - 0.128888889$$

$$d^{(12)} = 13.7195439\,\%$$

→ $i^{(2)}$

~~$d = \frac{i}{1+i}$~~ $\quad i = \frac{d}{1-d}$

~~i = [illegible]~~

$$i = 0.147959184$$

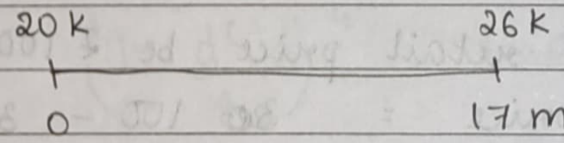
$$\left(1 + \frac{i^{(2)}}{2}\right)^{2} = 1 + 0.147959184$$

$$i^{(2)} = 14.2857143\,\%$$

→ $\delta = \ln(1+i)$

$\delta =$ 13.7985743 %

7.



& $C = 20{,}000$ $\quad AV_{17m} = 26{,}000$

(i) $i^{(4)} = ?$

$$26{,}000 = 20{,}000\,(1+i)^{17/12}$$

$$i = 0.203457067$$

$$\left(1 + \frac{i^{(4)}}{4}\right)^{4} = 1 + i$$

⇒ $i^{(4)} = 18.9552545\,\%$

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(ii) $d^{(2)} = ?$

~~$i = \frac{d}{1-d}$~~

$$d = \frac{i}{1+i}$$

$$d = 0.169060511$$

$$\left(1 - \frac{d^{(2)}}{2}\right)^2 = 1 - 0.169060511$$

$$\underline{\underline{d^{(12)} = 17.6882353\%}}$$

(iii) $C = 20,000$ $AV_{17m} = 26,000$

$$26000 = 2000\left(1 + \frac{17}{12} \times i\right)$$

$$\underline{\underline{i = 21.1764706\%}}$$

(iv) The interest rates keep getting lower as and when the frequency gets higher. It also shows that d has the lowest numerical value and i is the highest numerical value.

$$d < d^{(12)} < i^{(4)} < i$$

The simple rate of interest is higher than the compound interest. This is to maintain same amount overall.

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8. $i = 8\%$

The relationship between $i^{(p)}$ and p can be represented as :

$$i^{(p)} = p\left[(1+i)^{1/p} - 1\right]$$

$i = 8\%$

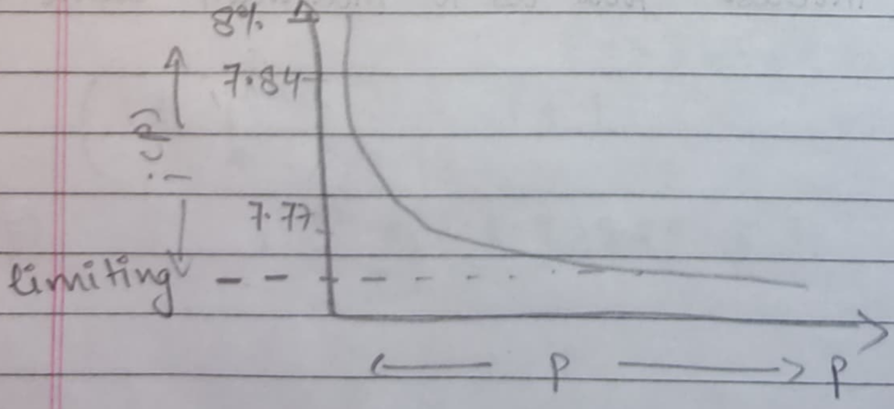
Let $p = 1, 2, 3, 4, 8, 40, 365$

p	$i^{(p)}$
1	0·08
2	0·07846096
3	[illegible] 0·077956704
4	~~1·038848~~ [illegible] 0·077706188
8	0·077332419
40	0·077035126
365	0·076969155

→ As p increases, the value of i decreases.

→ The change is less significant, in going from monthly to weekly or even daily compounding, so there is a limit in compounding.

→ The limiting case is when $p \to \infty$, there is instataneous change and it keeps continously compounding till delta (δ)



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9\. (i) The force of interest is the instantaneous change in the fund value expressed as an annualised percent of the current fund value.

(ii) $i^{(2)} = 0.0775$

→ $i = ?$

$$\left(1 + \frac{i^{(2)}}{2}\right)^2 = 1 + i$$

$$\Rightarrow \left(1 + \frac{0.0775}{2}\right)^2 = 1 + i$$

$$\underline{\underline{i = 7.9001563\%}}$$

→ $d^{(12)} = ?$

$$d = \frac{i}{1+i}$$

$$d = 0.073217283$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = 1 - \cancel{d}\,0.073217283$$

$$\Rightarrow \underline{\underline{d^{(12)} = 7.5795747\%}}$$

→ $i^{(1/2)} = ?$

$$\left[1 + 2i^{(1/2)}\right]^{1/2} = 1 + 0.07900 1563$$

$$\underline{\underline{i^{(1/2)} = 8.2122186\%}}$$

10.

$$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \leqslant t \leqslant 5 \\ 0.06 & \text{for } 5 \leqslant t \leqslant 10 \\ 0.04 & \text{for } t \geqslant 10 \end{cases}$$

$$v(t) = e^{-\delta} \text{ or } \frac{1}{At}$$

$$= \frac{\cancel{a} \quad 1}{A(0,5) \times A(5,10) \times A(10,t)}$$

$$v(t) = e^{-\int_0^5 0.08\,dt} \times e^{-\int_5^{10} 0.06\,dt} \times e^{-\int_{10}^{t} 0.04\,dt}$$

$$= e^{-[0.08t]_0^5} \times e^{-[0.06t]_5^{10}} \times e^{-[0.04]_{10}^{t}}$$

$$= e^{-0.4} \times e^{-0.3} \times e^{+0.4 - 0.04t}$$

$$= e^{-0.4-0.3+0.4-0.04t}$$

$$\cancel{= e^{-0.11+0.04t}}$$

$$= \underline{\underline{e^{-[0.3+0.04t]}}}$$

$$\underline{\underline{v(t) = e^{-(0.3+0.04t)}}}$$

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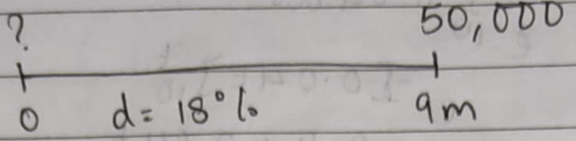
11\. (a)



$$P.V = C\cdot(1-d)^n$$
$$= 50{,}000 \times (1-0.18)^{9/12}$$
$$= \underline{\underline{43085.42623}}$$

Single payment → simple int hai kya

(b) (i) $\delta = 6\%$ $\quad d^{(12)} = ?$

$$\delta = \log\left(\frac{1}{1-d}\right)$$
$$\Rightarrow e^{0.06} = \frac{1}{1-d}$$
$$\Rightarrow 1.0618365547(1-d) = 1$$
$$d = 0.058235466$$
$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = 1-0.058235466$$
$$d^{(12)} = \underline{\underline{5.985025\%}}$$

(ii) $d^{(4)} = 6\%$ $\quad i^{(12)} = ?$ $\quad i^{(4)} = ?$

$$\left(1-\frac{d^{(4)}}{4}\right)^4 = 1-d$$
$$d = 0.058663449$$
$$i = \frac{d}{1-d} \qquad i = 0.062319315$$

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$i^{(12)}$

$$\left(1+\frac{i^{(12)}}{12}\right)^{12} = 1+i$$

$$\Rightarrow \quad i^{(12)} = 6.06\,0607088\,\%$$

$i^{(4)}$

$$\left(1+\frac{i^{(4)}}{4}\right)^{4} = 1+i$$

$$i^{(4)} = 6.0913705\,\%$$

~~(d)~~ (c) $i, d, i^{(4)}, \delta$

Ascending order : $d < \delta < i^{(4)} < i$

The order is ~~decided~~ based on the payment of interest. d is the interest payable at the beginning of the term, that is immediate payment. Hence, the amount would be less / smaller than i which is interest payable at end. Similarly δ and $i^{(4)}$ is also less than i as δ is an interest at exact time due to force of interest and $i^{(4)}$ is the nominal interest rate convertible quarterly.

(d) same question

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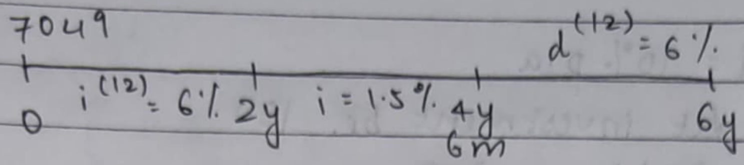
12.



$c = 7049$

Acc. value after 2 years:

$i^{(12)} = 6\%$

$$\left(1+\frac{i^{(12)}}{12}\right)^{12} = 1+i$$

$$i = 0.061677872$$

$$AV_2 = 7049\,(1+0.061677812)^2$$
$$= 7945.349262$$

$i = 1.5\%$ for 10 quarters SI

~~A.V~~ $AF = 1+ni$
$= 1+(10 \times 0.015)$
$= 1.15$

Acc. value after 4 1/2 years = 7945.349262×1.5
$= 9137.151651$

$d^{(12)} = 6\%$ for 1 1/2 years

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = 1-d$$

$$d = 0.058377193$$

$$A.F = \frac{1}{(1-d)^n}$$
$$= 1.094421325$$

Acc. value after 6 years = $9137.151651 \times 1.094421325$
$= \underline{\underline{9999.893617}}$

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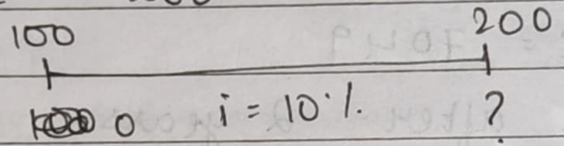
13. (i) $i = 10\% \text{ p.a}$

Let investment be 100

Double = 200



$C = 100 \quad AV = 200 \quad i = 10\%$

$$200 = 100(1 + 10\%)^n$$

$$2 = (1.1)^n$$

$$\ln 2 = n \ln(1.1)$$

$$n = 7.272540902 \text{ years}$$

$$\approx 7 \text{ years}$$

(ii) $d^{(12)} = 10\%$

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(ii) $d^{(12)} = 10\%$

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = 1-d$$

$$d = 0\cdot095541626$$

$$200 = \frac{100}{(1-d)^{n}}$$

~~0·0~~ $(0\cdot904458374)^{n} = 1/2$

$$n \ln(0\cdot904458374) = \ln(1/2)$$

$$n = \underline{\underline{6\cdot902550396}} \text{ yrs}$$