Probability and Statistics Assignment

Ques 1

$X \sim Poi(\theta)$

$$\frac{X-\theta}{\sqrt{\theta}} \sim N(0,1) \qquad \left[\begin{array}{l} E(X) = V(X) = \mu \\ (\text{Poisson}) \end{array}\right]$$

$X = 200$

$$\therefore \quad Z = X \pm 1.96\sqrt{\frac{\theta}{n}}$$

Ques 2: $X_i \sim N(\mu, \sigma^2)$

$\bar{X} \sim N(\mu, \sigma^2/n)$

$$E(\bar{X}) = E\left(\frac{\Sigma X_i}{n}\right) = \frac{\cancel{n}\mu}{\cancel{n}} = \mu$$

$$Var(\bar{X}) = Var\left(\frac{\Sigma X_i}{n}\right) = \frac{1}{n^{\cancel{2}}}\cancel{n}\sigma^2 = \frac{\sigma^2}{n}$$

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$E(s^2) = \frac{\cancel{(n-1)}\sigma^2}{\cancel{(n-1)}} = \sigma^2$$

$$\frac{(n-1)^{\cancel{2}}}{\sigma^4} Var(s^2) = 2\cancel{(n-1)}$$

$$\boxed{Var(s^2) = 2\sigma^4/n-1}$$

$$P(50 < s^2 < 150) = \left(\frac{\cancel{50} \times 9}{\cancel{100}} < \chi^2_9 < \frac{9 \times \cancel{150}}{\cancel{100}} \right)$$

$$\text{Ans.} = (4.5 < \chi^2_9 < 13.5)$$

Ques 3: (c) $n = 4$ $\quad Bin(4, p)$ $\quad$ 180 policies.

$$L(p) = \left(P(X=0)\right)^{86} \left(P(X=1)\right)^{75} \left(P(X=2)\right)^{16} \left(P(X=3)\right)^{2} \left(P(X=4)\right)^{1}$$

$$= \left[(1-p)^4\right]^{86} \left[4p(1-p)^3\right]^{75} \left[6p^2(1-p)^2\right]^{16} \left[4p^3(1-p)\right]^{2} \left[p^4\right]^{1}$$

$$L(p) = (1-p)^{603} p^{117} \times \text{constant}.$$

$$\ln L = (603 \ln(1-p) + 117 \ln p) * \ln \text{const}.$$

$$\frac{d \ln L}{dp} = \frac{603}{1-p} \times -1 + \frac{117}{p} = 0$$

$$\frac{d \ln L}{dp} = 0$$

$$\frac{117}{p} = \frac{603}{1-p}$$

$$117(1-p) = 603p$$

$$117 - 117p = 603p$$

$$117 = \cancel{7}820p$$

$$p = \frac{117}{820}$$

$$\boxed{p = 0.1625}$$

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(ii) $P(X=0) = (1-p)^4 = (1-0.1625)^4 = 0.492$

$P(X=1) = 4p(1-p)^3 = 4(0.1625)(1-0.1625)^3 = 0.3818$

$P(X=2) = 6p^2(1-p)^2 = 6(0.1625)(1-0.1625)^2 = 0.111$

$P(X=3) = 4p^3(1-p) = 4(0.1625)^3(1-0.1625) = 0.0144$

$P(X=4) = 0.0007$

Q44

	0	1	2	3	4
O_i	86	75	16	2	1
E_i	88.56	68.72	19.98	2.59	0.126

By clubbing

	0	1	2
O_i	86	75	19
E_i	88.56	68.72	22.71

$$\boxed{C_i = \frac{\Sigma(O_i - E_i)^2}{E_i}}$$

C_i	0.0757	0.5739	0.6061

$\Sigma C_i = 1.2557 \sim \chi^2_1$

$1.2557 < 3.841$

hence do not reject.

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Ques5: $U(a,b)$ $E(x) = \frac{a+b}{2}$ $V(x) = \frac{1}{12}(b-a)^2$

$a = \bar{x} - \sqrt{3}s$ $a = 2\bar{x} - b$ $a = b - 2\sqrt{3}s$

$2\bar{x} - b = b - 2\sqrt{3}s$

$\boxed{b = \bar{x} + \sqrt{3}s}$ $\boxed{a = \bar{x} - \sqrt{3}s}$

$\bar{x} = 6$ $S = 3.16$

$a = 6 - \sqrt{3} \times 3.16$ $b = 6 + \sqrt{3} \times 3.16$

$\boxed{a = 0.527}$ $\boxed{b = 11.47}$

$U(0,b)$ $P(x) = \frac{1}{b}$

$L = \frac{1}{b^n}$

$\log L = -n\log b$

$\frac{d\ln L}{db} = -\frac{n}{b}$ as $n \neq 0$.

$\boxed{b \to \infty}$

Ques6: H_0 $P_1 = 0.1$ H_1 $P > 0.1$

$P \to$ Prob. of missiles hitted.

12 missiles fired

H_0: 3 or more hits observed

otherwise

24 missiles fired total H_0 = 500 more hits observed.

Q7 $\bar{X}_{pub} = 30$ $\bar{X}_{poi} = 35.058$

(i) $S^2_{pub} = 64.3125$ $S^2_{poi} = 67.403$ $S^2_p = 58.123$

$$\frac{(\bar{X}_{pub} - \bar{X}_{poi}) - (\mu_{pub} - \mu_{poi})}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$$

(ii) $H_0 : \mu_{pub} = \mu_{poi}$ $H_1 : \mu_{pub} \neq \mu_{poi}$.

$$\frac{30 - 35.058}{\sqrt{8.123}\sqrt{\frac{2}{9}}} = -1.697$$

$\therefore$ Accept H.

(iii) Private hospital $\boxed{\text{do not}}$ result in higher claim.

Qus 8 (i) Unbias when $E(\hat{\theta}) = \theta$

(ii) bias when $E(\hat{\theta}) - \theta \neq 0$

(iii) MSE $\boxed{\hat{\theta} = \text{var} + \text{bias}^2}$.

Qu 9: (i) $t_k = \frac{N(0,1)}{\sqrt{\frac{t_k}{k}}}$; $\frac{\bar{x} - \mu}{S/\sqrt{n}} \sim t_{n-1}$

(i) $\mu(t_k) = 0$ $\text{var}(t_k) = \frac{k}{k-2}$

(iii) $n = 10$ $\bar{x} = 50$ $S^2 = 48.667$ C.I $= 99\%$.

(a) $t_{0.005, 9} = 3.25$

$$\mu = \bar{x} \pm t_{0.005,9} \frac{S}{\sqrt{n}} = 50 \pm 3.25\sqrt{\frac{48.667}{10}}$$

$\boxed{\mu = 57.17, 42.83}$

(b) $t_{0.005,9} = 2.5758$

$$\mu = 50 \pm 2.5758 \sqrt{\frac{48.667}{10}}$$

$\boxed{\mu = 55.68, 44.32}$

Ques10: $n = 1000$ $\quad X \sim Poi(3)$

$X \sim N(3000, 3000)$

if $P(X < 3100)$ man $X \sim Poi(3)$

else X do not follow $Poi(3)$

(i) $P(H_0 \text{ is reject} \mid H_0 \text{ is true})$

$= P(X > 3100,\ \lambda = 3)$

$= P\left(Z > \frac{3100 - 3000}{\sqrt{3000}}\right) \quad P(Z > 1.826)$

$\boxed{\text{Type 1 Error} = 3.39\%}$

(ii) Type 2 Error =

$P(H_0 \text{ accept} \mid H_0 \text{ is false})$

(iii) Power = $P(\text{reject } H_0 \mid H_0 \text{ false})$

$= 1 - \left(Z < \frac{3100 - n\mu}{\sqrt{n\mu}}\right)$

$= 1 - \left(Z < \frac{3100 - 3000}{\sqrt{3000}}\right) = 1 - \left(Z < \frac{100}{10\sqrt{30}}\right)$

$= 1 - (Z < 1.82)$

$\boxed{= 0.03436}$

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Ans

$n = 12 \qquad \Sigma x = 213200 \qquad \Sigma x^2 = 4919860000$

$\bar{x} = 17767 \qquad s = 10149$

$x_1 = 11000 \qquad x_2 = 48000 \qquad x_1' = 27500 \quad x_2' = 31500$

$\Sigma x = 213200 = A + 11000 + 48000$

$\boxed{A = 154200}$

$\Sigma x' = A + x_1' + x_2' = 213200$

$\boxed{\bar{x}' = 17767}$

$\Sigma x' = A + x_1' + x_2' = 213200$

$\Sigma x = 4919860000 = B + 11000^2 + 48000^2$

$\boxed{B = 2494860000}$

$\Sigma' x^2 = B + x_1' + x_2' = 424360000$

$s = \frac{1}{\sqrt{11}} \sqrt{4243360000 - 12(17767)^2}$

$\boxed{s = 6434.03}$