FINANCIAL MATHMETICS

Ques 1 =>

$$20000 = 12 \times 427.90\, a_{\overline{5}|}^{(12)} \text{ @ } i$$

$$1+\frac{i^{(12)}}{12} = (1+i)$$

$$i^{(12)} = (1+i)^{1/12-1}$$

$$\frac{20000}{12 \times 427.9} = \frac{1-(1+i)^{-5}}{i^{(12)}}$$

$$3.894991042 = \frac{1-(1+i)^{-5}}{12\left((1+i)^{1/12}-1\right)}$$

$$i = 10.8\%$$

$$\text{Flat rate} = \frac{\text{Total int}}{\text{Original loan} \times \text{Term of year}}$$

$$= \frac{\text{Total repayment} - 20000}{20000 \times 5}$$

$$= \frac{25674 - 20000}{100000}$$

$$= \frac{5674}{1000\cancel{00}} \times 1\cancel{00}$$

$$\text{Flat rate} = 5.674\%$$

$$\text{APR} = 2 \times \text{Flat rate}$$

$$= 2 \times 5.674$$

$$\boxed{\text{APR} = 11.3\%}$$

New loan.

$20000(1.108) = 12 \times 274.49\, a^{(12)}_{\overline{T}|} \quad i = 10.8\%$

$\dfrac{20000(1.108)}{274.49 \times 12} = \dfrac{1-(1.108)^{-x}}{12((1.108)^{1/12}-1)}$

$6.727628207 = \dfrac{1-(1.108)^{-x}}{0.102996083}$

$0 = 1-(1.108)^{-x} - 0.692919356$

$0 = 0.307080644 - (1.108)^{-x}$

$11.513 = x$

time for loan = 11.5 years.

Int on original loan = 5674

Restructed loan = $11.5 \times 274.49 \times 12 - 20000(1.10$

= 15719.62

more int = 10045.62

Ques 2

(i) Discounted pay back period

The discounted payback period is a capital budgeting procedure used to determine the profitability of a project. A discounted payback period gives the number of years it takes to break even from undertaking the initial expenditure by discounting future cash flows and recognising the time value of money.

Payback period

The payback period is the time you need to recover the cost of your investment. In simple terms, it is time an investment takes to reach the break even point.

(ii) The net present value method evaluates a capital projects in terms of its financial return over a specific time period, whereas the pay back method is concerned with the time that will elapse a project repays the comp. initial investment. Unlike the NPV.

(iii) PROJECT A

0	1	2	3
-170000	$+20000$ -20000	$+20000$ -10000	$+200000$

NPV $\qquad\qquad i = 6\%$

$$NPV_A = -170000 + 10000v^2 + 200000v^3$$
$$= -170000 + 10000(1.06)^{-2} + 200000(1.06)^{-3}$$
$$= ₹6823.821$$

For, IRR, $NPV_A = 0$

$$-17 + v^2 + 20v^3 = 0$$
$$\cancel{-17 + (1.06)^{-2}}$$
$$-17 + (1+i)^{-2} + 20(1+i)^{-3} = 0$$

$$IRR = 7.4\%.$$

PROJECT B

0	1	2	3	4	5	6
-200000	14000	14000	14000	14000	"	" 14000 + 200000

$$= NPV = -200000 + 14000\, a_{\overline{6}|}^{\,i=6\%} + 200000(1.06)^{-6}$$
$$= 9834.308.$$

For IRR_B

$NPV = 0$

$$-200000 + 14000\left[\frac{1-(1+i)^{-6}}{i}\right] + 200000(1+i)^{-6} = 0$$

$$-200 + 14\left[\frac{1-(1+i)^{-6}}{i}\right] + 200(1+i)^{-6} = 0$$

$IRR = 7\%$

$\Rightarrow$

maximum int rate the bank can charge us upto IRR of both the projects.

1986 2001 22

Ques 3 :- Annuity 1

200 200 200

1st Nov 1985 — 1/2/1986 — 1/2/1987 - - - - 2007

0 0 21

$$PV = 200\, \ddot{a}_{\overline{22|}}^{\,i=6\%} \times v^{0.25}$$

$$= 200 \times a_{\overline{22|}} \times \frac{i}{d} \times (1.06)^{-0.25}$$

$$= 200(12.0416) \times (1.059995)(1.06)^{-0.25}$$

$$= 2515.8893.$$

Annuity 2

$$PV = \left(320\, \ddot{a}_{\overline{16|}}^{(4)} + 80(1.06)^{-16}\right) v^{0.25}$$

$$PV = \left(320\, a_{\overline{16|}} \times \frac{i}{d^{(4)}} + 80(1.06)^{-16}\right)(1.06)^{-0.25}$$

$$PV = ₹\, 3336.80$$

1(17)

Annuity 3.

PV

$$PV = \frac{180}{12} \ddot{a}_{\overline{225}|} \frac{i^{(12)}}{12}$$

$$PV = \frac{180}{12} \left[\frac{1 - (1.004867583)^{-225}}{0.004844005} \right]$$

$$= 2058.130188$$

$$X \ddot{a}^{(2)}_{\overline{22}|} \; i = 6\% = 2515.8893 + 3336.80 + 2058.13018$$

$$X a_{\overline{22}|} \times \frac{i}{d^{(2)}} = 7910.819480$$

$$X (12.0416)(1.044782) = 7910.819480$$

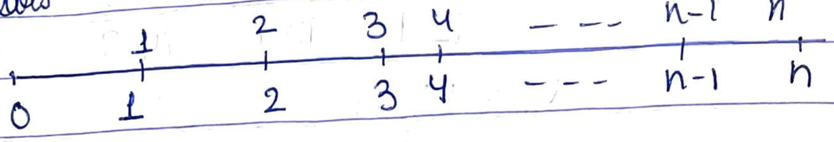
$$X = 628.798643$$

Ques 4 $\Rightarrow$

Derive from first principle

$$(I\ddot{a})_{\overline{n|}} = ?$$

Arrears



$$PV_0 = v + 2v^2 + 3v^3 + 4v^4 + \cdots + (n-1)v^{n-1} + nv^n \rightarrow (1)$$

~~NOTATION~~ = (1)

$$(2)\quad (1+i)(Ia_{\overline{n|}}) = 1 + 2v + 3v^2 + 4v^3 + \cdots + nv^{n-1}$$

Subt (2) - (1)

$$i(Ia_{\overline{n|}}) = 1 + v + v^2 + v^3 + \cdots + v^{n-1} - nv^n$$

$$(Ia)_{\overline{n|}} = \frac{\ddot{a}_{\overline{n|}} - nv^n}{i}$$

$$(1+i)(Ia)_{\overline{n|}} = (1+i) \times \frac{\ddot{a}_{\overline{n|}} - nv^n}{i}$$

$$= \frac{\ddot{a}_{\overline{n|}} - nv^n}{\frac{i}{1+i}} = (I\ddot{a})_{\overline{n|}}$$

Ques 5 ⇒

TWRR

$$\text{Property fund } (1+i) = \frac{13.1}{12.4} \times \frac{14.8}{13.1} \times \frac{15.8}{14.8} \times \frac{16.4}{15.8}$$

$$i = \frac{41}{31} - 1$$

$$i = \frac{41-31}{31}$$

$$i = \frac{10}{31} \qquad i = 32.25806\%$$

Equity fund

$$1+i = \frac{9.2}{12.1} \times \frac{10.3}{9.2} \times \frac{13.1}{10.3} \times \frac{15.5}{13.1}$$

$$= \frac{15.5}{12.1}$$

$$i = 128\%$$

(ii) Sorry Sir.

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Ques 6 ⇒

(i) $160000 = X\, a_{\overline{10}|}^{(12)} \quad i = 8\%$

$160000 = X\, a_{\overline{10}|} \times \frac{i}{i^{(12)}}$

$160000 = X\,(6.7101)(1.038)$

$\boxed{24916.74 = X}$

monthly ~~investmen~~ installment = 2076.3957

ii

loan O/S at the 4

$24916.74 \times a_{\overline{6}|}^{(12)} = X\, a_{\overline{6}|} \quad i = 10\%$

$24916.74 \times \cancel{4.3553} \times (1.045045) = X\, \cancel{(4.3553)}\, a_{\overline{6}|}$

$\boxed{26039.11455 = X}$

iii)

~~24916~~ -

$26039.11 \times a_{\overline{3}|}\ i = 10\% = X\, a_{\overline{3}|}\ i = 9\%$

$\frac{26039.11\ (2.4869)}{2.5313} = X$

$25582.37822 = X$

$$(1+i)^{10} = (1.08)^4 (1.1)^3 (1.09)^3$$

$$i = 8.896834\%$$

Ques 7 ⇒

Disadvantages of investment in property

a) Low liquidity ⇒ Govt securities are highly liquid and can be brought sold for a profit in a fraction of second. Real investments are comparably illiquid because property cant be easily and quickly sold without a substantial loss in value.

b) Requires management and maintainance.
Financing payments > Real estate, insurance, taxes and manage-ment fees and maintanance costs can add up quickly, especially if the property sits empty for extended period of time.

c) Creates liabilities
Real estate investing involves taking on a great deal of financial and legal liability. All the disadvantages mentioned above add to the liability a real estate investor takes on when purchasing, maintaing a property

(ii) (a) ⇒ Tender usually refers to the process whereby governments invite bids for large projects that must be submitted with in a finite deadline. A tender loan is a loan issued to a bidder under special conditions. This type of loan is issued to ensure the fulfilment of obligations based on results of the tenders

b) Advantages of Bond tenders offers:

1. Decreasing the cost of capital
⇒ The interest payments or coupon payments made to the bond holders represent a cost to the debtor company. For companies with a highly leveraged capital structure it is favourable to eliminate or decrease the cost of repurchasing debt.

2. Retiring existing bonds at less than face value. On the open market, many debt securities trade below their face values thus making the repurchasing of a debt attractive to a company. In the case of a bond tender offer the company offers to buy the bonds above their market value

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Disadvantage
Tender offers for buyback of any security can be made without the consent of members of a company's board of directors. Such offer can represent the threat of a hostile takeover.

C. 3 securities which have uncertain income :-

1 Stocks/Equity : The returns on equity/shares are variable almost everyday as the markets rise and fall continuously. The investment income from stocks is very variable.

2 Index linked bonds : In index linked bonds the amount is never certain because it varies with inflation and hence cannot be predicted.

3 Contingent annuities : In contingent annuities payment are made on a contingency such as survival or death. Hence payment is dependent on happening of the event.

Ques 9 (i) TWRR

$$(1+i)^3 = \frac{33}{30.5} \times \frac{38.5}{39} \times \frac{37}{38.5} \times \frac{45}{41.05} \times \frac{45.6}{45} \times \frac{47}{43.1}$$

$$i = 7.53285\%$$

MWRR:

$$30.5(1+i)^3 + 6(1+i)^{26/12} = 47 + 4.5(1+i)^{1.5} - 2.5(1+i)^{0.5}$$

$$i = 7.23\%$$

ii LIRR

$$(1+i)^3 = \left(\frac{33-30.5}{30.5}\right) \times \left(\frac{39-38.5}{38.5}\right)$$

iii MWRR is sensitive to amounts and timing of the net cashflows. If, we are assessing the detail of the fund manager does not control the timing or amount of the cash flows, he or she is merely responsible for investing the positive net cashflows and realising cash to meet the negative cash flows

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(ii) LIRR

$$(1+i)^3 = \left(1 + \frac{38.5-30.5}{30.5}\right)\left(1+\frac{45-38.5}{38.5}\right)\left(1+\frac{47-45}{45}\right)$$

LIRR = 30.567%

Ques 10 P = 200000

Q = 20000

P − Q = 180000

Let loan amt = X

$$X = 180000\, a_{\overline{15}|} + 20000\, Ia_{\overline{15}|} \;@\, 12\%$$

$$= (180000 \times 6.8109) + 20000(40.731)$$

$$X = ₹\ 2040582$$

Let cost of house = C

$$0.8(C) = 2040582$$

$$C = ₹\ 2550727.5$$

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(ii) 9^{th} installment = 200000 + (20000 × 8)
= 360000

Loan O/S after 8th installment

PV_8 = 340000 $a_{\overline{7}|}$ + 20000 $Ia_{\overline{7}|}$

= 340000(4.5638) + 20000 × 16.208

PV_8 = 1875852

Loan Schedule

Year	Loan at Start	Installment	Int	Capital	Loan O/S
9	1875852	360000	225102.24	134897.76	1740954.24
10	1740954.24	380000	208914.5088	171085.412	1569868.749

(iii) 11^{th} installment = 200000 + (20000 × 10)
= 400000

Loan O/S after 10^{th} installment

PV_{10} = 380000 $a_{\overline{5}|}$ + 20000 $(Ia)_{\overline{5}|}$
= 380000(3.6048) + 20000(10.0018)
= 1569860

New loan amt = 1569860 + (1% of 15,69,860)
= 1585558.6

Let revised annual payment be X.

$$15,85,558.6 = X\, a_{\overline{5}|} @ 10\%$$

$$X = 418264.9045$$

→ Total amt. paid in Senerio I $= 200000 + 220000 + \text{---}$
$\text{---} + 480000$
$= \frac{15}{2}(680000)$
$= 5100000$

Total amt paid in senerio II =
$200000 + 220000 + \text{---} + 380000 + 4$

$$= \frac{10}{2}(200000 + 380000) + 4(418264.9045)$$

$$= 45,73,059.618$$

Thus, it is advisible for Sam to transpher the loan to new bank as he payes less in installments i.e. total value paid by him is senerio 2 is less than that of senerio 1.

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Ques 11 (i) TWRR = 20%

$$(1+i)^2 = \frac{500}{460} \times \frac{550}{500} \times \frac{600}{550} \times \frac{x}{600}$$

$$\frac{(1.2)^2 \times 460 \times 500 \times 550 \times 600}{500 \times 550 \times 600} = x$$

$$\boxed{662.4 = x}$$

(ii) $460(1+i)^2 + 40(1+i)^{21/24} - 50(1+i) = 662.4$

$i = 21.19\%$

(iii) $(1+i)^2 = \left(1 + \frac{550-460}{460}\right)\left(1 + \frac{662.4-550}{580}\right)$

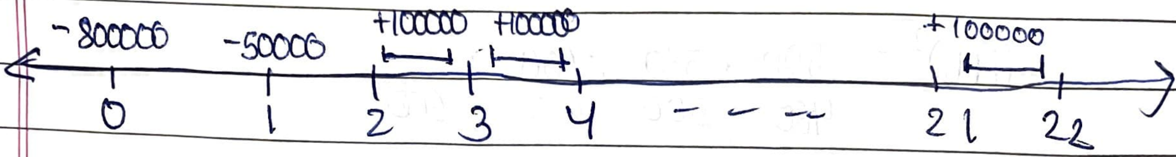
$i = 20\%$

$LRR = 20\%$

(iv) If the fund performance is reasonably stable in the period of assesment, then TWRR and MWRR will be give similar results

(v) TWRR is a better measure of investment as MWRR is sensetive to amount and timing of cashflows whereas TWRR eliminates this effect.

Ques 12 Cashflows



(i) The discounted payback period (DPP) tells us how long it takes for the project to move into position of profit. In many practical problems, the net cashflow changes sign only once, this change being from negative to positive. In these circumstances the balance in the investors accounts will change from negative to positive at a unique time $> t_1$ or it will always be negative, in which case the project is not viable. t_1 is referred to as the DPP. It is the smallest value of t such that $A(t) \geq 0$, where

$$A(t) = \sum_{s \leq t} c_s (1+i)^{t-s} + \int_0^t \rho(s)(1+i)^{t-s}\,ds$$

(ii) $i = 7\%$

Let $DPP = t$

$AV_t \Rightarrow 0$

$$-800000(1+i)^t - 50000(1+i)^{t-1} + 100000\left(\bar{s}_{\overline{t-2}|}\right) = 0$$

$$-80(1.07)^t - \frac{5(1.07)^t}{1.07} + 10\left(\frac{(1.07)^{t-2} - 1}{\ln(1.07)}\right) = 0$$

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$$(1.07)^t = 3.327196325$$
$$t \geq 17.77 \text{ years}$$
$$t = 18 \text{ years}$$

b ⇒ $AV_{22} = \left(-800000(1.07)^{18} - 50000(1.07)^{17} + 100000\, \bar{S}_{\overline{16}|}\right)$ @ 7% $\times (1.06)^4 + 100000\, \bar{S}_{\overline{4}|}$ @ 6%

$$= (-2703945.821 - 157940.7605 + 2885312.953) \times (1.06)^4$$
$$= 29575.25488 + 450458.6032$$
$$AV_{22} = 480033.258$$

c ⇒ DPP

$AV_t = 0$

$$-800000(1+i)^t - 50000(1+i)^{t-1} + 100000\, S_{\overline{t-2}|} = 0$$
$$-80(1.07)^t - 5\frac{(1.07)^t}{1.07} + 10\left(\frac{(1.07)^{t-2} - 1}{0.07}\right) = 0$$

$$40.104064(1.07)^t = 142.8571429$$
$$t \geq 18.77 \text{ year}$$

DPP or $t = 19$ years.

$AV_{22} = \left(-800000(1.07)^{19} - 50000(1.07)^{18} + 100000\, S_{\overline{17}|}\right)$ @ 7% $\times (1.06)^3 + 100000\, S_{\overline{3}|}$ @ 6%

$$AV_{22} = 344327.827$$

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Ques 13. (i) Let annual repayment $= X$

$$988000 = X\, a_{\overline{25}|}^{(12)} \ @7\%$$

$$988000 = X \left(\frac{1 - v^{25}}{i^{(12)}} \right)$$

$$X = 82176.89243$$

Monthly installment $= 6848.07437$

(ii) Loan o/s given by $PV_{10/03/29}$

$$PV_{10/03/29} = 6848.07437\, a_{\overline{130}|} \ @ \frac{i^{(12)}}{12}$$

$$\frac{i^{(12)}}{12} = 0.005654167$$

Loan o/s $= 648577.7278$

(iii) Loan o/s after 10/09/2026

$$= 6848.07437\, a_{\overline{166}|} \ @ \frac{i^{(12)}}{12}$$

$$= 736123.4973$$

Loan o/s after 10/10/2026

$$= 6848.07437\, a_{\overline{165}|}$$

$$= 733437.5845$$

capital repaid on 10th oct 2026 $=$ ① $-$ ② $= 2685.909224$

(iv) Loan O/S after 10^{th} April 2033

$$PV_{10/04/33} = 6848.0743\ a_{\overline{87}|} \ @ \ \frac{i^{(12)}}{12}$$

$$= 469560.5748$$

LOAN SCHEDULE

Date	Loan at start	Installment	Interest	Capital Repaid	Loan O/S
10/4/33	469560.5748	6848.07437	2654.9739	4193.100463	462712.5
10/5/33	462712.5004	"	2616.2537	4231.82062	455864.42
10/6/33	455864.4261	"	2577.5335	4270.540776	449016.35
10/7/33	449016.351	"	2538.8134	4309.260932	442168.277
10/8/33	442168.2773	"	2500.09328	4347.981088	4135320.203
10/9/33	435320.203	"	2461.373126	4386.701244	428472.1286
10/10/33	428472.1286	"	2422.65297	4425.4214	421624.0542
10/11/33	421624.0542	"	2383.932814	4464.141556	414775.9798
10/12/33	414775.9198	"	2345.212658	4502.861712	407927.90
10/01/34	407927.9055	"	2306.492501	4541.581869	401079.83
10/02/34	401079.8311	"	2267.7723	4580.3020	394231.7567
10/03/34	394231.7567	"	2229.05218	4619.022181	387383.682