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Prob. & stat. Assignment 1.

Q1) Calculate the mean, median, mode, std dev. and IQR of the foll data:-

i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25.

$\rightarrow$ Mean $= \frac{134}{10} = \underline{13.4}$

Median $= \underline{13}$

Mode $= \underline{18}$

$$\text{std. dev} = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{374.4}{9}}$$

$$= \sqrt{41.6}$$

$$= \underline{6.449}$$

IQR $\rightarrow$ 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

$\therefore Q_3 = 3\left(\frac{n+1}{4}\right)$

$= 3\left(\frac{11}{4}\right) = 8^{th}$ obsr $\Rightarrow$ 18

$\therefore Q_1 = \left(\frac{n+1}{4}\right) = 3^{rd}$ obsr $\rightarrow$ 9

$\therefore IQR = Q_3 - Q_1$

$= 18 - 9$

$= \underline{9}$

ii)

No. of households	0	1	2	3	4
No. of people	5	11	26	15	3

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$\therefore$

x_i	f_i	$x_i f_i$	c.f	$\lvert x_i - \bar{x} \rvert$	f_i (1)
0	5	0	5	2	10
1	11	11	16	1	11
2	26	52	42	0	0
3	15	45	57	1	15
4	3	12	60	2	6
	60	120	N		42

$$\text{Mean} = \frac{120}{60} = \underline{\underline{2}}$$

$$\text{Median} \Rightarrow \frac{N}{2} = \frac{60}{2} = 30 \quad \therefore \text{Median} = \underline{\underline{42}}.$$

Mode = $\underline{\underline{2}}$ since with maximum frequency.

$$\text{Std-dev} \Rightarrow \frac{\Sigma f_i \lvert x_i - \bar{x} \rvert}{\Sigma f_i} = \frac{42}{60} = \underline{0.7}$$

$$\text{IQR} \rightarrow \text{For } Q_1 \Rightarrow \frac{N}{4} = \frac{60}{4} = 15^{th} \text{ term}$$

$$\text{For } Q_3 \rightarrow \frac{3N}{4} = 45^{th} \text{ term}$$

$$\therefore Q_1 = 1 \quad ; \quad Q_3 = 3$$

$$\therefore \text{IQR} = 3 - 1 = \underline{\underline{2}}$$

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Q2) It is assumed that claims arising on an industrial policy can be modelled as a Poisson process at a rate of 0.5 / yr.

i) Determine the probability that no claims arise in a single year.

→ $\therefore P(X=0) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$

where $\lambda = 0.5$ & $x = 0$

$\therefore P(X=0) = \frac{e^{0.5} \times 0.5^{0}}{0!} = e^{-0.5}$

$= \underline{0.60653}$

ii) Determine the probability that, in three consecutive yrs, there is one / more claims in one of the yrs and no claims in each of the other 2 yrs.

→ Let X = no of years with a claim.

$\therefore X \sim \text{binomial}(3, 0.3935)$

$\therefore P(X=1) = 3(0.3935)(0.60653)^2$

$= \underline{0.434}$.

iii) Suppose a claim has just occurred. Determine the probability that more than 2 yrs will elapse before the next claim occurs.

→ Let T = time until next claim

then $T \sim \exp(0.5)$

$\therefore P(T>2) = e^{-0.5(2)} = e^{-1}$

$= \underline{0.368}$.

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Q3] An analyst is interested in using gamma distribution with parameters $\alpha = 2$ and $\lambda = 1/2$. The range of x is defined as $0 < x < \infty$.

i) State the mean and S.D of this distribution.

→ Gamma $(2, 1/2)_\lambda$ and $0 < x < \infty$

α ↳ rate

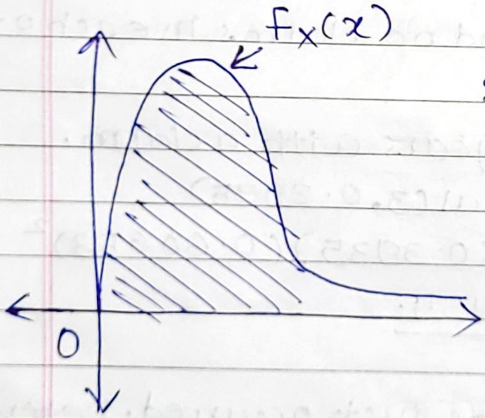
$$\therefore \text{Mean} = \frac{\alpha}{\lambda} = \frac{2}{1/2} = 4$$

$$\therefore S.D = \sqrt{var}$$

$$= \sqrt{\frac{\alpha}{\lambda^2}} = \sqrt{\frac{2}{1/4}} = \sqrt{8} \text{ or } 2\sqrt{2} \text{ or } 2.828$$

ii) Hence comment briefly on its shape.

→



Since variance is positive then skewness will also be positive or right skewed thus positive peakedness.

iii) Calculate the cumulative distribution function:-

$$\to C.D.F = \frac{1}{\Gamma(\alpha)} \cdot \lambda$$

$$= \frac{1}{(\alpha-1)!} \times \lambda$$

$$= \frac{1}{(2-1)!} \times 1/2$$

$$= 0.5$$

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Q4) In an actuarial office, there are 10 male and 8 female staff. 6 of the males and 7 of the females are fully qualified actuaries. Calculate the probability that a team of two workers randomly selected consists of two qualified females given that it contains at least one female.

→ Total Actuaries = 10M + 8F

= 18

Fully Qualified = 6M + 7F

= 13

(of 2)

Total no. of combination of qualified people = ${}^{13}C_2$

Total pairs of two qualified female = ${}^{7}C_2$

∴ Probability = $\frac{{}^{7}C_2}{{}^{13}C_2}$

$= \frac{7!}{2!\,5!} \times \frac{2!\,11!}{13!}$

$= \frac{7 \times \cancel{6}\,3}{\cancel{2} \times 1} \times \frac{\cancel{2} \times 1}{13 \times \cancel{12}\,6}$

$= \frac{21 \times 1}{78} = \frac{\cancel{21}\,7}{\cancel{78}\,26}$

$= \frac{7}{26}$

Conditional Probability → $\frac{7/26}{{}^{8}C_1} = \frac{7}{26} \times \frac{1}{8} = \frac{7}{208}$

½

n.

28

e

d

s.

Q5] The supply chain service in a country has three levels of service: Level 1 (next day delivery), Level 2 (2-day delivery) and Level 3 (3-day delivery). 40% of packages are delivered via level 1, 50% level 2 ~~and~~ The probability of a package arriving late using each level is 20%, 40% & 10% resp. Calculate the prob. that a late package was sent via Level 2 service.

→ Given:- $P(L/A) = \frac{20}{100}$; $P(L/B) = \frac{40}{100}$; $P(L/C) = \frac{10}{100}$

$P(A) = \frac{40}{100}$; $P(B) = \frac{50\%}{100}$; $P(C) = \frac{10}{100}$

∴ To find:- $P[B/L]$

$$\therefore P(B/L) = \frac{P(B \cap L)}{P(L)}$$

$$= \frac{P(B) \cdot P(L/B)}{P(B) \cdot P(L/B) + P(A) \cdot P(L/A) + P(C) \cdot P(L/C)}$$

$$= \frac{\frac{50}{100} \times \frac{40}{100}}{\frac{50}{100} \times \frac{40}{100} + \frac{40}{100} \times \frac{20}{100} + \frac{10}{100} \times \frac{10}{100}}$$

$$= \frac{1/5}{1/5 + 4/50 + 1/100}$$

$$= \frac{1/5}{\frac{20}{100} + \frac{8}{100} + \frac{1}{100}}$$

$$= \frac{1/5}{29/100}$$

$$= \frac{1}{5} \times \frac{100}{29} = \frac{100}{145} = 0.689655$$

$$= \underline{68.965\%}$$

Q6] If $X \sim U(1,2)$ and $Y = \frac{3}{6-x}$ determine the PDF of Y.

→ $X \sim U(1,2)$

$\quad\quad\quad\quad\ \ \downarrow\ \ \downarrow$

$\quad\quad\quad\quad\ \ a\ \ b$

$\therefore$ P.D.F of $Y = \int_1^2 \frac{3}{6-x}$

$= \left[-3 \ln(6-x)\right]_1^2$

$= -3\ln(4) - [-3\ln(5)]$

$= -3\ln(4) + 3\ln(5)$

$= \ln 4^{-3} + \ln 5^3$

$= -4.1588 + 4.8283$

$= \underline{0.6695}$

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Q7] A discrete R.V 'x' has:-

X	1	2	3	4	5
$P(X=x)$	0.05	0.15	0.35	0.4	0.05

calculate:-

i) $F(3)$

ii) $E(X)$

iii) $V(X)$

iv) coefficient of skewness

→ i) $F(3) = \sum_{1}^{3} P(X=x)$

$= \sum_{1}^{3} P(X=1,2,3)$

$= 0.05 + 0.15 + 0.35$

$= \underline{0.55}$

ii) $E(X) = \frac{\sum f_i \cdot x_i}{\sum f_i}$

$= \frac{0.05 + 0.30 + 1.05 + 1.6 + 0.25}{1}$

$= \underline{3.25}$

iii) $V(X) = [S.D(X)]^2$

$= E(X^2) - [E(X)]^2$

$= \sum_{\text{all } x} x^2 \cdot P(X=x) - (3.25)^2$

$= (0.05 + 0.6 + 3.15 + 6.4 + 1.25) - 3.25^2$

$= 11.45 - 10.5625 = \underline{0.8875}$

iv) Coefficient of skewness:-

We know that $V(X) = E(X_i - \bar{X})^2 = \sigma^2$

and skewness $= E(X_i - \bar{X})^3 \Rightarrow \tilde{\mu}_3$

$\therefore$ skewness $= \dfrac{\sum_{\text{all } x} (X_i - \bar{X})^3}{(N-1) \times \sigma^3}$

$\therefore$ skewness $= \dfrac{-11.390 - 1.9531 - 0.015 + 0.4218 + 5.3593}{(5-1) \times (0.942072)^3}$

$= \dfrac{-7.577}{4 \times 0.83608}$

$= \underline{-2.26560}$

i.e. left skewed or negatively skewed :)

Q8) Insurance claim amounts are independent and exceed 1000 with probability 0.2. In a sample of 15 claims, calculate the prob. that fewer than 3 of them exceed 1000.

$\rightarrow$ $P(A) = 0.2 = p$ $\therefore q = 0.8$

where A = event where claim amt. exceeds 1000.

Let B = event when fewer than 3 claims exceed 1000

$P(B) = P(B=0) + P(B=1) + P(B=2)$

$= {}^{15}C_0 p^0 q^{15} + {}^{15}C_1 \cdot p^1 q^{14} + {}^{15}C_2 \cdot p^2 q^{13}$

$= 1(0.2)^0 (0.8)^{15} + 15(0.2)(0.8)^{14} + 105(0.2)^2(0.8)^{13}$

$= 0.03518 + 0.1319 + 0.2308$

$= \underline{\underline{0.3979}}$

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Q9] A man enters a raffle each week where the probability of winning a prize is 0·01. calculate the probability that he takes seven weeks upto and including the week where he wins his third prize.

→ $p = 0·01$

$\therefore q = 0·99$

$\therefore P(X=3) \Rightarrow {}^6C_2 \cdot p^2 \cdot q^4$

$= {}^6C_2 \cdot (0·01)^2 \cdot (0·99)^4$

$= \frac{6!}{2!\,4!} \times 100 \times 10^{-6} \times 0·960596$

$= \frac{\cancel{6}^{3} \times 5}{\cancel{2} \times 1} \times 1 \times 10^{-3} \times 0·960596$

$= 0·01440$

Q10) The no. of claims submitted to an insurance office averages two per working week (of 5 days). Calculate the probability that more than 2 claims arrive in one day.

→ Here $\lambda = 2$

Avg. no. of policies sold perday $= \frac{2}{5}$

$= 0.4$

So on a given day,

$$P(X) = \frac{e^{-0.4}(0.4)^2}{2!}$$

$$= \frac{0.6703 \times 0.160}{2}$$

$$= \underline{0.053624}$$

Q11) If the probability of having a male and female child is equal and woman has 5 boys and 2 girls. calculate prob. that her next three children are boys.

→ Possible outcomes ⇒ GGG

BBB → required.

GBG

GGB

BGG

BGB

BBG

GBB

∴ Prob. that next 3 children are Boys $= \frac{1}{8}$

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Q12) Claims follow a Poisson process with rate 10/day. Calculate the prob. that the time betn two claims is less than 0.1 days.

→ Let $X \sim Poi(10)$ or $T \sim expo(10)$

$$P(T<0.1) = e^{-10(0.1)}$$
$$= e^{-1}$$
$$= 0.367879$$

Q13) If X is following uniform distribution with parameters a=75 & b=120 calculate the prob. that X lies betn 80 and 100.

→ $X \sim U(75,120)$

$P(80 < X < 100) = ?$

$\therefore P(80 < X < 100) = P(X < 100) - P(X > 80)$

$$\therefore F_X(100) = \frac{100-75}{120-75} = \frac{25}{45} \quad -① $$

$$F_X(80) = \frac{80-75}{120-75} = \frac{5}{45} \quad -②$$

$$\therefore P(80 < X < 100) = \frac{25}{45} - \frac{5}{45}$$
$$= \frac{20}{45}$$
$$= \frac{4}{9}$$

Q14) If $X \sim$ Gamma $(\overset{\alpha}{5}, \overset{\lambda}{0.4})$. Calculate $P(X < 22.89)$

→ If $X \sim$ Gamma$(5, 0.4)$

$$\therefore 2(0.4)X \sim \chi^2_{2\alpha}$$

$$\therefore 0.8X \sim \chi^2_{10}$$

$$\therefore P(X < 22.89) = P(0.8X < 18.312)$$
$$= P(\chi^2_{10} < 18.312)$$

Since table also gives less than probability,

we have $\nu = 2\alpha = 10$ & $\alpha = 5$

$P(X < 22.89) = \underline{0.1088}$

Q15) A continuous R.V 'X' has the foll. prob. density func: $\dfrac{k}{(x+5)^4}$, $x > 0$

Calculate:

i) k

ii) F(10)

iii) E(X)

→ We have $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\therefore \int_0^{\infty} \frac{k}{(x+5)^4}\,dx = 1$$

$$\therefore \frac{5k}{(x+5)^4} = 1$$

$$5k = (x+5)^4$$

$$k = \frac{(x+5)^4}{5}$$

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ii) $F(10) = \int_0^{10} \frac{k}{(x+5)^4} \cdot dx$

$= \left[\frac{50}{(x+5)^4}\right]_0^{10}$

$= \frac{50}{(15)^4}$

$= \underline{0.000987654}$

iii) $E(X) = \int_0^{10} x^2 \cdot \frac{k}{(x+5)^4} \cdot dx$

$= \left[\frac{x^3}{3} \cdot \frac{50}{15^4}\right]_0^{10}$

$= \frac{1000}{3} \times \frac{50}{50625}$

$= \underline{0.329218}$

———x———o———x———