

Date ____ / ____ / ____

①

X = claim size on a home insurance

$$X \sim N(800, 100^2)$$

Y = claim size on a car insurance

$$Y \sim N(1200, 300^2)$$

$$P(y_1 + y_2 + y_3 > x_1 + x_2 + x_3 + x_4 + 800)$$

$$P(y_1 + y_2 + y_3 - x_1 - x_2 - x_3 - x_4 > 800)$$

$$y_1 + y_2 + y_3 - x_1 - x_2 - x_3 - x_4 \sim N(3(1200) - 4(800), 3 \times 300^2 + 4 \times 100^2)$$

$$\therefore \sim N(400, 310000)$$

$$P\left(Z > \frac{800 - 400}{\sqrt{310000}}\right) = P(Z > 0.718)$$

$$= 1 - P(Z \leq 0.718) = 0.23576$$

②

N = Number of claims

$N \sim$ negative binomial dist $(3, 0.9)$

X = Claim Amount

$$X \sim \text{Gamma}(6, 2)$$

Y = total claim amount

$$M_Y(t) = M_N \log(M_X(t))$$

$$M_X(t) = \left(1 - \frac{t}{2}\right)^{-6}$$

Date ___/___/___

Saathi

$$M_y(t) = M_u \left(\log \left(1 - \frac{t}{2} \right)^{-1} \right)$$

$$M_u(t) = \left(\frac{p e^t}{1 - q e^t} \right)^K$$

$$M_y(t) = \left(\frac{p e^{\log(1-t/2)^{-1}}}{1 - q e^{\log(1-t/2)^{-1}}} \right)^3$$

$$M_y(t) = \left(\frac{(0.9) \left(1 - \frac{t}{2} \right)^{-1}}{1 - (0.1) \left(1 - \frac{t}{2} \right)^{-1}} \right)^3$$

$$E(N) = \frac{0.3}{0.9} = \frac{10}{3} = 3.33$$

$$V(N) = \frac{K(0.1)}{(0.9)^2} = 3.33 \times \frac{1}{9} = 0.37$$

$$E(X) = \frac{\lambda}{2} = \frac{6}{2} = 3 \quad V(X) = \frac{\lambda}{1^2} = \frac{6}{1} = 1.5$$

$$\begin{aligned} V(Y) &= E(N)V(X) + E(X)^2 V(N) \\ &= 5 + \frac{10}{3} = \frac{25}{3} = 8.333 \end{aligned}$$

$$SD(Y) = \sqrt{\frac{25}{3}} = 2.88$$

Date ___/___/___

③ $f_x(x) = \lambda e^{-\lambda(x-\alpha)}$

$$M_x(t) = E(e^{tx})$$

$$= \int_{\alpha}^{\infty} e^{tx} \lambda e^{-\lambda(x-\alpha)} dx$$

$$= \lambda e^{\lambda\alpha} \int_{\alpha}^{\infty} e^{x(t-\lambda)} dx$$

$$= \lambda e^{\lambda\alpha} \left[\frac{e^{(t-\lambda)x}}{t-\lambda} \right]_{\alpha}^{\infty} = \lambda e^{\lambda\alpha} \left[\frac{e^{-\lambda(t-\lambda)x}}{t-\lambda} \right]_{\alpha}^{\infty}$$

$$M_x(t) = \frac{\lambda e^{\lambda\alpha}}{t-\lambda} (0 - e^{-\lambda\alpha(t-\lambda)})$$

$$= \frac{\lambda e^{t\alpha}}{\lambda-t}$$

ii) $M'_x(t) = \lambda (e^{t\alpha}) (\lambda-t)^{-2} (-1)(-1) + \lambda (\lambda-t)^{-1} (e^{t\alpha}) (\alpha)$

$$= \frac{\lambda e^{t\alpha}}{(\lambda-t)^2} + \frac{\lambda e^{t\alpha} \alpha}{\lambda-t}$$

$$E(x) = M'_x(0) = \frac{\lambda(1)}{\lambda^2} \frac{d\lambda}{d\lambda} = \frac{1+\lambda\alpha}{\lambda} = \frac{1+\lambda\alpha}{\lambda}$$

$$M''_x(t) = \lambda e^{t\alpha} (-2)(\lambda-t)^{-3} (-1) + \lambda (\lambda-t)^{-2} e^{t\alpha} (\alpha)$$

$$+ \lambda \alpha e^{t\alpha} (\lambda-t)^{-2} (-1)(-1) + \lambda \alpha (\lambda-t)^{-1} (e^{t\alpha}) (\alpha)$$

$$= \frac{2\lambda e^{t\alpha}}{(\lambda-t)^3} + \frac{\lambda \alpha e^{t\alpha}}{(\lambda-t)^2} + \frac{\lambda \alpha e^{t\alpha}}{\lambda-t} + \frac{\alpha^2 \lambda e^{t\alpha}}{\lambda-t}$$

$$E(X^2) = M''(0) = \frac{2\lambda(1)}{\lambda^2} + \frac{2\lambda\alpha}{\lambda^2} + \frac{\lambda\alpha^2}{\lambda}$$

$$= \left(\frac{2}{\lambda} + \frac{2\alpha}{\lambda} + \alpha^2 \right)$$

$$V(X) = E(X^2) - (E(X))^2$$

$$= \frac{\alpha^2 + 2\alpha}{\lambda} + \frac{2}{\lambda^2} - \left(1 + \frac{\lambda^2\alpha^2 + 2\alpha\lambda}{\lambda^2} \right)$$

$$= \frac{\alpha^2 + 2\alpha}{\lambda} + \frac{2}{\lambda^2} - \frac{1}{\lambda^2} - \frac{\alpha^2}{\lambda} - \frac{2\alpha}{\lambda}$$

$$\boxed{V(X) = \frac{1}{\lambda^2}}$$

Q4) $G_N(t) = \left(\frac{p}{1-qt} \right)^K$, $p+q=1$

$\therefore$ Distribution is -ve binomial

$$P(V=4) = \frac{\sqrt{K-4}}{\sqrt{5JK}} p^4 q^4$$

Q5) $f(x, y) = ax^2 + axy$, $0 < x < 5$
 $10 < y < 20$

$$1 = \int_{10}^{20} \int_0^5 (ax^2 + axy) dx dy$$

$$1 = \int_{10}^{20} \left(\frac{125a}{3} + \frac{25ay}{2} \right) dy$$

Date ___/___/___

$$1 = \frac{125a}{3} \left(\frac{y}{10}\right)^{20} + \frac{25a}{2} \left(\frac{y^2}{x}\right)^{20}$$

$$a = \frac{3}{6875}$$

$$\begin{aligned} f_X(x) &= a \int_0^{20} x^2 + xy \, dy \\ &= \frac{3}{6875} \int_0^{20} x^2 + xy \, dy \end{aligned}$$

$$= \frac{3}{6875} (10x^2 + 150x)$$

$$E(Y/X) = y f(Y/X) = y \int_0^{20} \frac{f(x, y)}{f_X(x)} \, dy$$

$$= \int_0^{20} \frac{y \left(\frac{3}{6875}\right) (x^2 + xy) \, dy}{\frac{3}{6875} (10x^2 + 150x)}$$

$$\frac{3}{6875} (10x^2 + 150x)$$

$$= \left(\frac{1}{10(x+15)} \right) \left(\left[\frac{xy^2 + y^3}{2} \right]_0^{20} \right)$$

$$= \frac{1}{10(x+15)} \left(\frac{150x + 7000}{3} \right)$$

$$E(Y/X) = \frac{45x + 7000}{3(x+15)}$$

$$P\left(Y > \frac{1}{3}x + 10\right)$$

$$f_y(y) = \int_{\frac{1}{3}y-10}^{20} \frac{3}{6875} (x^2 + xy) dx$$

$$= \frac{1}{6875} \left(\frac{250 + 75y}{2} \right)$$

$$P\left(Y > \frac{1}{3}x + 10\right) = \int_{\frac{1}{3}x+10}^{20} \frac{1}{6875} (250 + 75y) dy$$

$$= \frac{1}{6875(2)} \left[10000 - \frac{1750x}{3} - \frac{25x^2}{3} \right]$$

b) $X \sim \text{Poi}(\lambda) \quad Y \sim \text{Poi}(\mu)$

$$M_X(t) = e^{\lambda(e^t-1)} \quad M_Y(t) = e^{\mu(e^t-1)}$$

$$M_{X-Y}(t) = E(e^{t(X-Y)})$$

$$= E(e^{tX - tY})$$

$$= \frac{E(e^{tX})}{E(e^{tY})} = \frac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}} = e^{\lambda - \mu(e^t-1)}$$

$$M_{X+Y}(t) = E(e^{t(X+Y)}) \quad \text{--- HP}$$

$$= E(e^{tX}) E(e^{tY})$$

$$= e^{\lambda(e^t-1)} e^{\mu(e^t-1)}$$

$$= e^{\lambda + \mu(e^t-1)} \quad \text{--- HP}$$

Date ___/___/___

$$\begin{aligned}
 \text{d7) i) } \text{Var}(X/Y=2) &= E(X^2/Y=2) - (E(X/Y=2))^2 \\
 &= \sum_x x^2 \frac{P(X=x, Y=2)}{P(Y=2)} - \left(\sum_x x \frac{P(X=x, Y=2)}{P(Y=2)} \right)^2 \\
 &= \frac{1^2(0) + 2^2(0.3) + 3^2(0.1) + 4^2(0.1)}{0.6} - \left[\frac{1(0) + 2(0.3) + 3(0.1) + 4(0.1)}{0.6} \right]^2 \\
 &= \frac{53}{6} - \frac{289}{36} = \frac{29}{36}
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } f(u, v) &= \frac{48}{67} (2uv - v^2) \quad 0 < u < 1, 0 < v < 2 \\
 E(U|v=v) &= \int_0^1 u f(u, v=v) du \\
 &= \int_0^1 \frac{u f(u, v=v)}{f(v=v)} dv \quad \text{--- (1)}
 \end{aligned}$$

$$\begin{aligned}
 f_v(v=v) &= \int_0^1 \frac{48}{67} (2uv - v^2) du \\
 &= \frac{48}{67} \left[\frac{2v^2u}{2} - \frac{v^2}{3} \right]_0^1 \\
 &= \frac{48}{67} \left(\frac{v-1}{3} \right) = \frac{48}{67} \left(\frac{3v-1}{3} \right) = \frac{16}{67} (3v-1)
 \end{aligned}$$

$$\begin{aligned}
 \text{from (1)} \quad E(U|v=v) &= \frac{\int_0^1 u \left(\frac{48}{67} \right) (2uv - v^2) du}{\frac{16}{67} (3v-1)}
 \end{aligned}$$

$$= \frac{3}{3v+1} \int_0^1 -2v^2x - v^3 dx$$

$$= \frac{3}{3v+1} \left[\frac{3v}{3} x - \frac{v^3}{v} x \right]_0^1$$

$$= \frac{8v-3}{4(3v+1)}$$

$$18) \quad \lambda = 3 \quad \lambda = 2$$

$X \sim \text{binomial}(3, 2)$

$$E(Y | X=x) = 3x + 1$$

$$\text{Var}(Y | X=x) = 2x^2 + 5$$

$$E(Y) = E(E(Y | X=x))$$

$$= E(3x + 1)$$

$$= 3E(X) + 1$$

$$= 3 \left(\frac{3}{2} \right) + 1 = \frac{11}{2}$$

$$E(Y) = \frac{11}{2}$$

$$V(Y) = E(V(Y | X=x)) - V(E(Y | X=x))$$

$$= E(2x^2 + 5) + V(3x + 1)$$

$$= 2(3) + 5 + 9 \left(\frac{3}{4} \right) = 6 + 5 + 6.75$$

$$= 17.75$$

Date ____ / ____ / ____

Q9) $Z = X + Y$

$$E(X) = 3$$

$$E(Y) = 4$$

$$\sigma_X = \sqrt{V(X)} = 2$$

$$\sigma_Y = \sqrt{V(Y)} = 1$$

$$\begin{aligned} \text{Cov}(X, Z) &= \text{Cov}(X, X+Y) \\ &= \text{Cov}(X, X) + \text{Cov}(X, Y) \\ &= V(X) + (-0.6) \end{aligned}$$

(5.4) (3.4)

$$\text{Var}(Z) = \text{Cov}(X, \text{Var}(X+Y))$$

$$= V(X) + V(Y) + 2\text{Cov}(X, Y)$$

$$= 4 + 1 + 2(-0.6)$$

(3.8)

Q10) $f(x, y) = K x^\alpha e^{-y/\beta}$

$$1 < \alpha < 2$$

$$1 < y < 8$$

$$\alpha > 1, \beta > 0$$

$$1 = K \int_1^\infty \int_1^\infty x^\alpha e^{-y/\beta} dy dx$$

$$1 = K \left(\frac{1}{\alpha+1} \right) \left(-\beta e^{-y/\beta} \right)$$

$$K = \frac{-1 + \alpha}{\beta e^{-1/\beta}}$$