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FM. Assignment 1

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① $d^{(4)} = 8\%$ p.a

$$\left(1 - \frac{d^{(p)}}{p}\right)^p = 1-d$$

$$\left(1 - \frac{d^{(4)}}{4}\right)^4 = 1-d$$

$$\left(1 - \frac{0.08}{4}\right)^4 = 1-d$$

$$d = 1 - 0.922368160$$

$$\therefore d = 0.07763184$$

(i) $\therefore \delta = -\ln(1-d)$

$$= 0.080810829 = \boxed{8.0811\%}$$

(ii) $1 + i = \frac{1}{1-d}$

$$i = \frac{1}{1-0.07763184} - 1$$

$$\therefore i = 0.084165785 = \boxed{8.417\%}$$

(iii) $d^{(12)} = ?$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = 1 - 0.07763184$$

$$1 - \frac{d^{(12)}}{12} = \sqrt[12]{0.922368160}$$

$$\therefore d^{(12)} = 0.080539339$$

$$= \boxed{8.0539339\%}$$

② $\delta(t) = 0.004t + 0.0002t^2$

(i) $C_{10} =$ Rs. 1000

$$\therefore P.V_0 = C_{10} \times \frac{1}{A(0,10)}$$

$$= 1000 \times \frac{1}{e^{\int_0^{10} \delta(t)\,dt}}$$

$$= 1000 \times \frac{1}{e^{\int_0^{10} (0.004t + 0.0002t^2)\,dt}}$$

$$= 1000 \times \frac{1}{e^{\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3}\right]_0^{10}}}$$

$$= 1000 \times \frac{1}{e^{\left(\frac{1}{5} + \frac{1}{15}\right)}}$$

$$= 1000 \times \frac{1}{e^{4/15}} = \boxed{\text{Rs. } 765.9283384}$$

(ii) $A.V. = C(1+i)^n$

$$1000 = 765.9283384(1+i)^{10}$$

$$(1+i)^{10} = 1.305605172$$

Raising by 1/10

$$(1+i) = 1.027025404$$

$$i = 0.027025404$$

$$\boxed{i = 2.7025404\%}$$

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③(i) $1+i = \frac{1}{1-d}$

$1-d = \frac{1}{1+i}$

$d = 1 - \frac{1}{1+i}$

$d = \frac{\cancel{1}+i-\cancel{1}}{1+i}$

$d = \frac{i}{1+i}$

$\therefore d = i.v$

Explanation

'd' is the amount of money invested at the beginning will earn during the period when interest is paid at the start of the period 'i'. So 'd' must be equal to P.V. at the beginning of the year payment and interest paid at the end. $\therefore d = i.v$

(ii) a) $\frac{d^{(4)}}{4} = 2.25\% = 0.0225$

$d^{(2)} = ?$

$$\left(1 - \frac{d^{(4)}}{4}\right)^4 = \left(1 - \frac{d^{(2)}}{2}\right)^2$$

Raising both sides by ½

$$(1-0.0225)^2 = 1 - \frac{d^{(2)}}{2}$$

$$\therefore \frac{d^{(2)}}{2} = 0.04449375$$

$$\therefore d^{(2)} = 0.0889875$$

$$\boxed{\therefore d^{(2)} = 8.89875\%}$$

b) $d^{(1/2)} = 5\% = 0.05$

$d^{(2)} = ?$

$$\left(1 - \frac{d^{(1/2)}}{1/2}\right)^{1/2} = \left(1 - \frac{d^{(2)}}{2}\right)^2$$

$$(1-0.05\times2)^{1/2} = \left(1 - \frac{d^{(2)}}{2}\right)^2$$

Raising both sides by ½

$$(1-0.1)^{1/4} = 1 - \frac{d^{(2)}}{2}$$

$$\frac{d^{(2)}}{2} = 0.0259962537$$

$$d^{(2)} = 0.05199250715$$

$$\boxed{\therefore d^{(2)} = 5.199250715\%}$$

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(iii) $i = 5\% = 0.05$

$n = 91$ days

~~SD = ?~~

Let Bill amount be Rs. 100 (c)

$$A.V. = C(1+i)^n$$

$$= 100(1.05)^{91/365}$$

$$= 101.2238407$$

$\therefore P.V_0 = 100 \qquad C = 101.2238407$

$\therefore P.V_0 = C(1-nd)$

$$100 = 101.2238407\left(1 - \frac{91}{365}d\right)$$

$$\boxed{\therefore d = 4.8494619\%}$$

④ (i) Same as ③(i).

(ii) $i = 0.08$

a) $d^{(12)} = ?$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = \frac{1}{1+0.08}$$

$$1 - \frac{d^{(12)}}{12} = 0.993607102$$

$$\therefore d^{(12)} = 0.076714776$$

$$\boxed{\therefore d^{(12)} = 7.6714776\%}$$

b) $i^{(365)} = ?$

$$\left(1 + \frac{i^{(365)}}{365}\right)^{365} = 1.08$$

$$1 + \frac{i^{(365)}}{365} = 1.000210874$$

$$\therefore i^{(365)} = 0.076969155$$

$$\boxed{\therefore i^{(365)} = 7.6969155\%}$$

c) $\delta = ?$

$$\therefore \delta = \ln(1+i)$$

$$\therefore \delta = \ln(1.08)$$

$$\therefore \delta = 0.076961041$$

$$\boxed{\therefore \delta = 7.6961041\%}$$

d) $i^{(0.5)} = ?$

$$\left(1 + \frac{i^{(0.5)}}{0.5}\right)^{0.5} = 1.08$$

$$1 + \frac{i^{(0.5)}}{0.5} = 1.1664$$

$$\therefore i^{(0.5)} = 0.0832$$

$$\boxed{\therefore i^{(0.5)} = 8.32\%}$$

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⑤(i) $d^{(4)} = 6\% = 0.06$

a) $i^{(2)} = ?$

$$\left(1 - \frac{d^{(4)}}{4}\right)^4 = \frac{1}{1+i}$$

$$\left(1 - \frac{0.06}{4}\right)^4 = \frac{1}{1+i}$$

$$1+i = \frac{1}{0.941336551}$$

$$i = 0.062319315$$

$$\therefore \left(1 + \frac{i^{(2)}}{2}\right)^2 = 1+i$$

$$1 + \frac{i^{(2)}}{2} = \sqrt{1.062319315}$$

$$\therefore i^{(2)} = 0.061377515$$

$$\boxed{\therefore i^{(2)} = 6.1377515}$$

b) $d^{(12)} = ?$

$$\left(1 - \frac{d^{(4)}}{4}\right)^4 = 1-d$$

$$1-d = 0.941336551$$

$$\therefore d = 0.058663449$$

$$\therefore \left(1 - \frac{d^{(12)}}{12}\right)^{12} = 1-d$$

$$1 - \frac{d^{(12)}}{12} = 0.994974790$$

$$\therefore d^{(12)} = 0.060302525$$

$$\boxed{\therefore d^{(12)} = 6.0302525\%}$$

(ii)

200,000	220,000
15th April, 2005	15th April, 2006

200,000	?
15th April, 2005	17th July, 2005

April	May	June	July	T
15	31	30	17	93

$n = 93$ days

If loan is repaid on 15th April, 2006

$$\therefore 1+i = \frac{220000}{200000} = \frac{11}{10}$$

$$\therefore i = 0.1$$

$$i = 10\%$$

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Interest paid for 93 days $= \frac{10}{100} \times \frac{93}{365} = 0.025479452055$

Amount to be paid if the contract is terminated on 17th July, 2005 $= 200000 + (200000 \times 0.025479452055)$

$= 200000 + 5095.89043$

$= \boxed{Rs.\ 205095.8904}$

⑥(i) Let the retail price of the toy be Rs.100

a) Cash payment $= 100 - \frac{30}{100} \times 100 = Rs.\ 70$

b) Six month credit $= 100 - \frac{25}{100} \times 100 = Rs.\ 75$

$\therefore d = ?$

$70 = 75\,(1-d)^{0.5}$

$d = 0.128888889$

$\boxed{\therefore d = 12.88889\%}$

(ii) $d^{(12)} = ?$

$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = 1-d$

$\therefore d^{(12)} = 0.137195439$

$\boxed{\therefore d^{(12)} = 13.7195439\%}$

$\delta = \ln(1+i)$

$= \ln\left(\frac{1}{1-d}\right)$

$= \ln\left(\frac{1}{1-0.1288889}\right)$

$= 0.137985743$

$= \boxed{13.7985743\%}$

$i^{(2)} = ?$

$\left(1 + \frac{i^{(2)}}{2}\right)^2 = \frac{1}{1-d}$

$1 + \frac{i^{(2)}}{2} = 1.07142857143$

$\therefore i^{(2)} = 0.1428571429$

$\boxed{\therefore i^{(2)} = 14.28571429\%}$

(7) $C = Rs.\ 20000$

$A.V = Rs.\ 26000$

$n = 17\ m$

(i) $i^{(4)} = ?$

$\therefore A.V = P\left(1 + \frac{i^{(4)}}{4}\right)^{4 \times \frac{17}{12}}$

$\frac{26000}{20000} = \left(1 + \frac{i^{(4)}}{4}\right)^{17/3}$

$\therefore i^{(4)} = 0.189552546$

$\boxed{\therefore i^{(4)} = 18.9552546\%}$

(ii) $\left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(\frac{1}{1 + \frac{i^{(4)}}{4}}\right)^4$

$1 - \frac{d^{(2)}}{2} = \left(\frac{1}{1.047388136}\right)^2$

$\therefore d^{(2)} = 0.1768823531$

$\boxed{\therefore d^{(2)} = 17.68823531\%}$

(iii) $26000 = 20000\,(1 + ni)$

$1.3 = 1 + \frac{17}{12} i$

$\therefore i = 0.2117647059$

$\boxed{\therefore i = 21.17647059\%}$

(iv) Interest rates keep getting lower as & when the frequency gets higher.

$d < d^{(12)} < i^{(4)} < i$

The S.I. has to be higher than the C.D. to maintain the same amount.

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(10) $\delta = \begin{cases} 0.08 & ; 0 \leq t \leq 5 \\ 0.06 & ; 5 \leq t \leq 10 \\ 0.04 & t \geq 10 \end{cases}$

$v(t) = e^{-\delta}$ or $\frac{1}{A(t)}$

$= \frac{1}{A(0,5) \times A(5,10) \times A(10,t)}$

$= e^{-\int_0^5 0.08\,dt} \times e^{-\int_5^{10} 0.06\,dt} \times e^{-\int_{10}^{t} 0.04\,dt}$

$= e^{-[0.08t]_0^5} \times e^{-[0.06t]_5^{10}} \times e^{-[0.04t]_{10}^{t}}$

$= e^{-0.4} \times e^{-0.3} \times e^{-0.04t + 0.4}$

$= \cancel{e^{-0.4}} \times e^{-0.3} \times e^{-0.04t} \times \cancel{e^{0.4}}$

$\boxed{\therefore v(t) = e^{-[0.3 + 0.04t]}}$

(11) a) $P.V. = C\,(v(t))$

$P.V. = 50000\,(1 - 0.18)^{9/12}$

$= 50000\,(0.82)$

$\boxed{\therefore P.V. = Rs.\ 43085.42623}$

b)(i) $\delta = 6\%$ $\quad d^{(12)} = ?$

$\delta = \ln\left(\frac{1}{1-d}\right)$

$0.06 = \ln\left(\frac{1}{1 - \frac{d^{(12)}}{12}}\right)^{12}$

$\therefore \frac{d^{(12)}}{12} = 0.004987620807$

$d^{(12)} = 0.05985024969$

$\boxed{\therefore d^{(12)} = 5.985024969\%}$

(ii) $d^{(4)} = 6\% = 0.06$ $\quad i^{(12)} = ?$

$1 + i = \frac{1}{1-d}$

$\left(1 + \frac{i^{(12)}}{12}\right)^{12} = \frac{1}{\left(1 - \frac{d^{(4)}}{4}\right)^4}$

$\therefore 1 + \frac{i^{(12)}}{12} = \left(\frac{1}{1 - \frac{0.06}{4}}\right)^{1/3}$

$\therefore i^{(12)} = 0.06060708865$

$\boxed{\therefore i^{(12)} = 6.060708865\%}$

(12) $A.V. = 7049 \times A(0,2) \times A(2, 4\tfrac{1}{2}) \times A(4\tfrac{1}{2}, 6)$

$$= 7049 \times \left(1+\frac{i^{(12)}}{12}\right)^{12} \times (1+10i) \times \frac{1}{\left(1-\frac{d^{(12)}}{12}\right)^{12\times\frac{3}{2}}}$$

$$= 7049 \times \left(1+\frac{0.06}{12}\right) \times (1+0.15) \times \left(\frac{1}{1-\frac{0.06}{12}}\right)^{18}$$

$$= 7049 \times (1.127159776) \times (1.15) \times (1.0904421325)$$

$$= \boxed{Rs.\ 9999.893613}$$

(13) Let initial investment be Rs. x

$A.V. = 2x$ ~~$= 2x$~~

(i) $2x = x(1+i)^n$

$2 = (1+0.01)^n$

$2 = (1.01)^n$

Taking log on both sides

$\ln 2 = n \ln(1.01)$

$\boxed{\therefore n = 7.272540902 \text{ years}}$

(ii) $d^{(12)} = 10\%$

$x = 2x(1-d)^n$

$x = 2x\left(1-\frac{d^{(12)}}{12}\right)^{12n}$

$\frac{1}{2} = \left(1-\frac{d^{(12)}}{12}\right)^{12n}$

Taking log on both sides

$\ln \frac{1}{2} = 12n \ln\left(1-\frac{0.1}{12}\right)$

$\boxed{\therefore n = 6.902550392 \text{ years}}$