

Calculus - Assignment 2.

① $\int_{\frac{1}{50}}^2 \frac{e^{2/w}}{w^2} dw$

Let $t = e^{2/w}$

diff wrt w .

$$\frac{dt}{dw} = -2e^{2/w} \frac{1}{w^2}$$

$$= \int_{\frac{1}{50}}^2 \frac{tw^2}{-2w^2e^{2/w}}$$

$$= -\frac{1}{2} \int_{\frac{1}{50}}^2 dt$$

$$= -\frac{1}{2} [t]_{\frac{1}{50}}^2$$

$$= -\frac{1}{2} (2 - 0.02)$$

$$= -0.99$$

② $\int \frac{2t^3 + 1}{(t^4 + 2t)^3} dt$

Let $x = (t^4 + 2t)^3$

diff wrt t .

$$\frac{dx}{dt} = 3(t^4 + 2t)^2(4t^3 + 2)$$

$$= \int \frac{2t^3 + 1}{x(3)(t^4 + 2t)^2(4t^3 + 2)}$$

Multiply & divide by x .

$$= \int \frac{4t^3 + 2}{(x)(3)(4t^3 + 2)(x)^{2/3}}$$

$$= \frac{1}{6} \int x^{-5/3} dx$$

$$= \frac{1}{6} \left(\frac{x^{-5/3+1}}{-5/3+1} \right) + C$$

$$= -\frac{1}{4} x^{-2/3} + C$$

$$= -\frac{1}{4} (t^4 + 2t)^{3(-2/3)} + C$$

$$= -\frac{1}{4} (t^4 + 2t)^{-2} + C$$

③ $f'(x) = x^4 + 3x - 9$
 $f(x) = \int f'(x) dx$

$$f(x) = \int (x^4 + 3x - 9) dx$$

$$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$$

④ $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$

$$\int_{-2}^3 f(x) dx = \int_{-2}^1 f(x) dx + \int_1^3 f(x) dx$$

$$= \int_{-2}^1 3x^2 dx + \int_1^3 6 dx$$

$$= [x^3]_{-2}^1 + [6x]_1^3$$

$$= (1 + 8) + (18 - 6)$$

$$= 4$$

⑤ $\int 3(8y - 1) e^{4y^2 - y} dy$

let $u = e^{4y^2 - y}$

diff w.r.t y

$$\frac{du}{dy} = (e^{4y^2 - y}) (8y - 1)$$

$$= \int \frac{3(ey-1)(e^{4y^2-y})}{(e^{4y^2-y})(8y-1)} dy$$

$$= 3x + C$$

$$= 3[e^{4y^2-y}] + C$$

$$\textcircled{6} \quad f''(x) = 15\sqrt{x} + 5x^3 + 6$$

$$f'(x) = \int 15\sqrt{x} + 5x^3 + 6 dx$$

$$= \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + K dx$$

$$f(x) = \int \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + K dx$$

$$= 4x^{5/2} + \frac{x^4}{4} + 3x^2 + Kx + C$$

$$f(1) = -\frac{5}{4}$$

$$4(1)^{5/2} + \frac{(1)^4}{4} + 3(1)^2 + K(1) + C = -\frac{5}{4}$$

$$K + C = -8.5 \rightarrow \textcircled{1}$$

$$f(4) = 404$$

$$4(4)^{5/2} + \frac{(4)^4}{4} + 3(4)^2 + K(4) + C = 404$$

$$4K + C = 164 \rightarrow \textcircled{2}$$

subtracting $\textcircled{1}$ & $\textcircled{2}$

$$4K + C = 164$$

$$(-) \quad K + C = -8.5$$

$$\hline 3K = 172.5$$

$$K = 57.5$$

$$K + C = -8.5$$

$$c = -66$$

$$\therefore f(x) = 4x^{5/2} + \frac{x^4}{4} + 3x^2 + 57.5x - 66$$

$$(9) \frac{dv}{dt} = 9.8 - 0.196v$$

$$\int \frac{dv}{9.8 - 0.196v} = \int dt$$

$$\log(9.8 - 0.196v) = -0.196t + c$$

$$(12) \text{ Taylor series for } f(x) = x^3 - 10x^2 + 6 \quad \boxed{x=3}$$

$$f(x) = x^3 - 10x^2 + 6$$

$$f'(x) = 3x^2 - 20x$$

$$f''(x) = 6x - 20$$

$$f'''(x) = 6$$

$$f(3) = -57$$

$$f'(3) = -33$$

$$f''(3) = -2$$

$$f'''(3) = 6$$

$$f(x) = f(3) + (x-3)f'(3) + \frac{(x-3)^2 f''(3)}{2!} + \frac{(x-3)^3 f'''(3)}{3!}$$

$$= -57 + (x-3)(-33) + \frac{(x-3)^2(-2)}{2} + \frac{(x-3)^3(6)}{6}$$

$$= -57 + (x-3)(-33) + (x-3)^2(-1) + (x-3)^3$$

$$(13) \text{ Maclaurian series for } (1+x)^n$$

$$f'(x) = n(1+x)^{n-1}$$

$$f''(x) = n(n-1)(1+x)^{n-2}$$

$$f'''(x) = n(n-1)(n-2)(1+x)^{n-3}$$

$$f(x) = f(0) + \frac{x^1 f'(0)}{1!} + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!}$$

$$(1-x)^n = 1 + n + (n-1)(n) + n(n-1)(n-2)$$

$$(14) \quad f(x) = \sqrt{1+x}$$

$$f'(x) = \frac{1}{2} (1+x)^{-1/2}$$

$$f'(0) = \frac{1}{2}$$

$$f''(x) = \frac{-1}{4} (1+x)^{-3/2}$$

$$f''(0) = -\frac{1}{4}$$

$$f'''(x) = \frac{-3}{8} (1+x)^{-5/2}$$

$$f'''(0) = -\frac{3}{8}$$

$$\therefore f(x) = \sqrt{1+x} + \frac{x!}{1!} \left(\frac{1}{2} \right) + \frac{x^2}{2!} \left(-\frac{1}{4} \right) + \frac{x^3}{3!} \left(-\frac{3}{8} \right)$$

$$(15) \quad f(x) = e^{-6x}$$

$$f(x) = e^{-6x}$$

$$f'(x) = e^{-6x} (-6)$$

$$f''(x) = e^{-6x} (-6)(-6)$$

$$f'''(x) = e^{-6x} (-6)^3$$

$$\boxed{x=4}$$

$$f(-4) = e^{24}$$

$$f'(-4) = e^{24} (-6)$$

$$f''(-4) = e^{24} (-6)^2$$

$$f'''(-4) = e^{24} (-6)^3$$

$$e^{-6x} = e^{24} + \frac{(x-4)}{1!} (e^{24} (-6)) + \frac{(x-4)^2}{2!} (e^{24} (-6)^2) + \frac{(x-4)^3}{3!} (e^{24} (-6)^3)$$

$$(16) \quad f(x) = \ln(3+4x)$$

$$f'(x) = \frac{4}{3+4x}$$

$$\boxed{x=0}$$

$$f'(0) = \frac{4}{3}$$

$$f''(x) = (-4)(4x+3)^{-2}(4)$$

$$f''(0) = (-1)(4)^2(3)^2$$

$$f'''(x) = (4)(4x+3)^{-3}(4)^2$$

$$f'''(0) = (-1)(-2)(-3)^2(4)^3$$

$$\ln(3+4x) = \ln(3) + \frac{x}{1!} \left(\frac{4}{3} \right) + \frac{x^2}{2!} (-1)(4)^2(3)^2 + \frac{x^3}{3!} (-1)(-2)(-3)^2(4)^3$$