F.M

Q1 i) $d^{(4)} = 8\%$

$d = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^4$

$= 1 - \left(1 - \frac{0.08}{4}\right)^4$

$= 0.077632$

$d = 7.7632\%$

$\delta = -\ln(1-d)$

$= -\ln(1 - 0.077632)$

$= 0.0808$

$= 8.08\%$

ii $i = ?$

$i = \frac{d}{1-d}$

$= \frac{0.077632}{1 - 0.07632}$

$i = 0.084046$

$i = 8.4046\%$

iii $d^{(12)} = 12\left[1-(1-d)^{1/12}\right]$

$= 12\left[1-(1-0.077632)^{1/12}\right]$

$= 0.080539$

$d^{(12)} = 8.0539\%$

Q2 $\delta(t) = 0.004t + 0.0002t^2$

i) $PV = 1000\left[e^{-\int_0^{10}(0.004t + 0.0002t^2)dt}\right]$

$= 1000\left[e^{-\left(0.002\times100 + \frac{0.0002\times1000}{3}\right)}\right]$

$= 1000\left[e^{-(0.2+0.0667)}\right]$

$= 1000\times e^{-(0.2667)}$

$= 765.903$

ii) $i = e^{\int_0^{10}(0.004t + 0.0002t^2)dt}$

$= e^{\left(0.002\times100 + \frac{0.002\times1000}{3}\right)}$

$= e^{(0.2+0.667)} - 1$

$i = 0.30565$

Q3) Lets assume that a person lends 1 at an effective rate of discount 'd'. As the original principle states '1-d' and d is the amount of discount. From basic defination of 'i' as the ratio of the amount of discount to the principle.

$$\therefore i = \frac{d}{1-d}$$

Above eq^n expreses i as function of d

$$i = \frac{d}{1-d}$$

$$i - id = d$$

$$d(1+i) = i$$

$$d = \frac{i}{1+i}$$

$$\therefore \text{discounting factor} = v = \frac{1}{1+i}$$

$$d = i \times \frac{1}{1+i}$$

$$\boxed{d = iv}$$

ii) a) $\frac{d^{(4)}}{4} = 2.25\%$

$d = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^4$

$= 1 - \left(1 - \frac{0.0225}{4}\right)^4$

$d = 0.022311$

$d^{(2)} = 2\left(1 - (1 - 0.022311)^{1/2}\right)$

$d^{(2)} = 2.244\%$

b) $d^{(0.5)} = 5\%$

$d = 1 - \left(1 - \frac{0.05}{0.5}\right)^{0.5}$

$d = 0.051317$

$d^{(1/2)} = 2\left(1 - 00.51317^{1/2}\right)$

$d^{(1/2)} = 5.19\%$

iii)

Lets assume that a person lends 1 at an affective rate of discout 'd'. As the original principle states '1-d and d is the amount of discount. From basic defination of 'i' as the ratio of the amount of discount to the principle.

$$\therefore i = \frac{d}{1-d}$$

Above eq^n expreses i as function of d

$$i = \frac{d}{1-d}$$

$$i - id = d$$

$$d(1+i) = i$$

$$d = \frac{i}{1+i}$$

$$\therefore \text{discounting factor} = v = \frac{1}{1+i}$$

$$d = i \times \frac{1}{1+i}$$

$$\boxed{d = iv}$$

Q4

ii) $i = 0.08$

a) $d = \frac{0.08}{1+0.08}$

$d = 0.0740741$

$d^{(12)} = 7.6715$

b) $i^{(365)} = 365\left((1+0.08)^{1/365} - 1\right)$

$i^{365} = 7.6962\%$

c) $\delta = \ln(1+i)$

$= \ln(1+1.08)$

$\delta = 7.696\%$

d) $i^{(0.5)} = 0.5\left[(1+0.08)^{1/0.5} - 1\right)$

$i^{(0.5)} = 8.32\%$

Q5 i) a) $i^{(2)} =$

$$d = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$= 1 - \left(1 - \frac{0.06}{4}\right)^4$$

$d = 5.866\%$

$$i = \frac{0.5866}{1 - 0.5866}$$

$i = 6.232\%$

$i^{(2)} = 2\left[(1 + 0.06232)^{1/2} - 1\right]$

$i^{(2)} = 6.138\%$

b) $d^{(12)} = 12\left(1 - (1 - 0.05866)^{1/12}\right)$

$d^{(12)} = 6.030\%$

ii) Loan repaid in 93 days

$93\text{day} = 20000 + 20000\left(\frac{93}{365}\right)$

Amit had to pay Rs 205095.89

Q6 Let's take cash payment of 100

disc. value of 100 at time $0 = 70$

At six months $= 100 \times 25\%$
$= 75$

$PV_0 = AV(1-d)^n$

$70 = 75(1-d)^{1/2}$

$\left(\frac{70}{75}\right)^2 = 1-d$

$d = 0.1288889$

$d = 12.88889\%$

$d^{(12)} = 12\left(1-(1-0.1288889)^{1/12}\right)$

$= 13.7193\%$

$i^{(2)} = ?$

$i = \frac{0.1288889}{1-0.1288889}$

$= 0.1479593$

$i^{(2)} = 2\left[(1+0.479593)^{1/2} - 1\right]$

$= 14.2857\%$

$\delta = -\ln(1-d)$

$\delta = 13.7986$

Q7i) $PV_0 = 20,000$
$AV_{17} = 26,000$

$$20000 = 26000\left(1 + \frac{i^{(4)}}{4}\right)^{\frac{4 \times 17}{12}}$$

$$\left(\frac{20000}{26000}\right)^{5\frac{2}{3}} = 1 + \frac{i^{(4)}}{4}$$

$i^{(4)} = 18.96\%$

i) $i = \left(1 + \frac{0.1896}{4}\right)^4 - 1$

$= 2.034341$

$$d = \frac{0.203451}{1 + 0.2034541}$$

$= 16.905588$

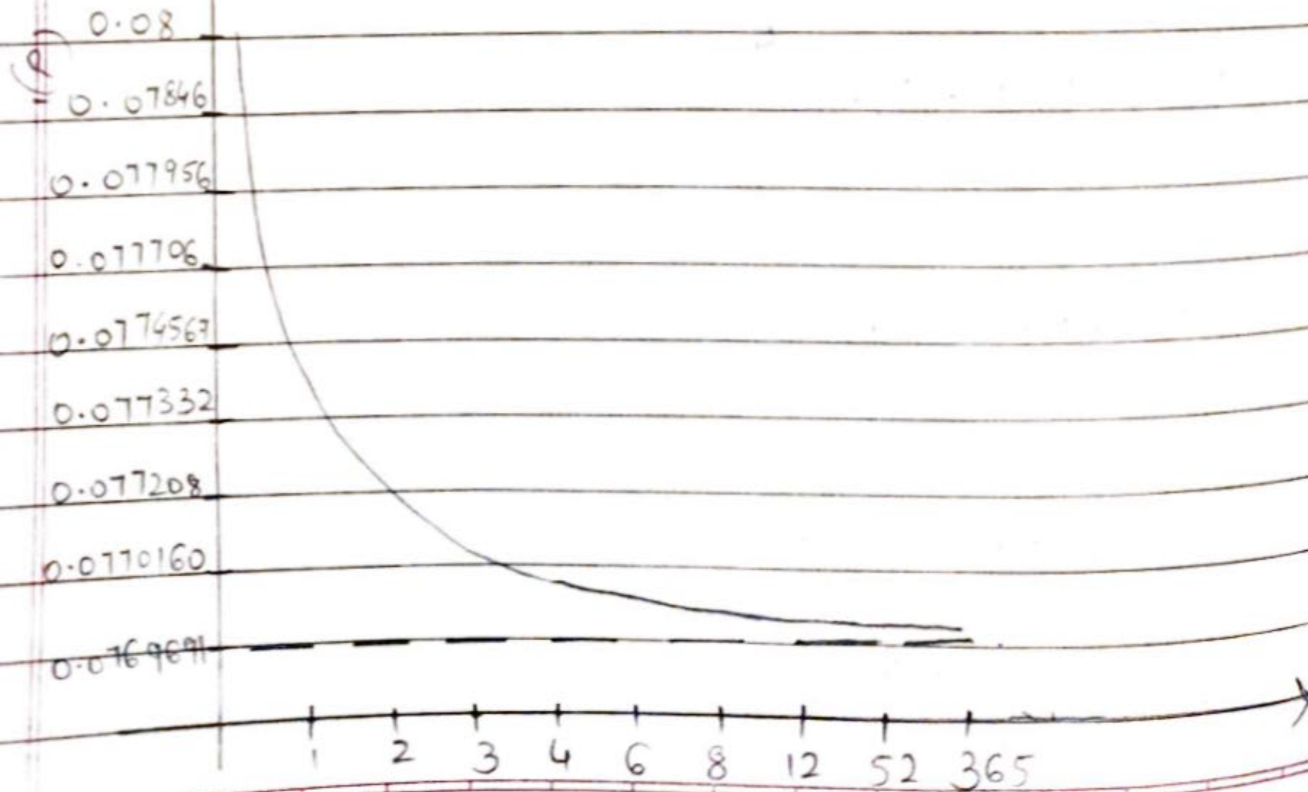
$d^{(2)} = 2(1-(1-0.1690558)$

$= 3.0628508\%$

iii) $26000 = 20000(1+i)^{17/12}$

$\left(\frac{26000}{20000}\right)^{12/17} = 1+i$

$\therefore i = 21.8\%$


(P)
0.08
0.07846
0.077956
0.077706
0.0774567
0.077332
0.077208
0.0770160
0.0769691
1
2
3
4
6
8
12
52
365

Q4 i) Force of interest refers to nominal interest rate compounded infinite number of times over a time period.

ii) $i^{(2)} = 0.0775$

$$i = \left(1 + \frac{i^{n}}{m}\right) - 1$$

$$i = \left(1 + \frac{0.0775}{2}\right)^{2} - 1$$

$i = 7.9002\%$

$\delta = \ln(1 + 0.079002)$

$= \ln(1.079002)$

$\delta = 0.07604$

$d^{(12)}$

$$d = \frac{0.079002}{1 - 0.079002}$$

$= 0.07322$

$d^{(12)} = 7.599\%$

$$i^{(12)} = \frac{1}{2}\left[(1+i)^{1/12} - 1\right]$$

$$i^{(12)} = 8.2123\%$$

Q10 $$v(t) = e^{-\int_0^5 \delta(s)\,dt}$$

$$= e^{-\int_0^5 0.08\,dt} \cdot e^{-\int_5^{10} 0.06\,dt - \int_{10}^{t} 0.04\,dt}$$

$$= e^{-0.4} \cdot e^{-0.3} e^{-(0.04t - 0.4)}$$

$$= e^{-0.4-0.3-0.04t+0.4}$$

$$v(t) = e^{-0.3-0.04t}$$

Q11 a) $$PV_{a/12} = 50,000$$

$$d = 18\%$$

$$PV = 50,000(1-0.18)^{1/12}$$

$$PV = 43085.43$$

b) $\delta = 6\%$

$d = 1 - e$

$d = 5.82355\%$

$d^{(12)} = 12\left[1 - (1 - 0.0582355)^{1/12}\right]$

$d^{(12)} = 5.99\%$

ii) $d^{(4)} = 6\%$

$i^{(2)} = ?$

$d = 1 - \left(1 - \frac{0.06}{4}\right)^4$

$= 5.866345\%$

$i = \frac{0.05866345}{1 - 0.05866345}$

$= 6.23\%$

$i^{(2)} = 12\left[(1 + 0.0623)^{1/12} - 1\right]$

$= 6.0589\%$

$i^{(12)} = 6.059\%$

Camlin KOKUYO

a) $d < \delta < i^{(4)} < i$

Lets assume that a person lends 1 at an effective rate of discount 'd'. As the original principle states '1-d and d is the amount of discount. From basic defination of 'i' as the ratio of the amount of discount to the principle.

$$\therefore i = \frac{d}{1-d}$$

Above eq^n expreses i as function of d

$$i = \frac{d}{1-d}$$

$$i - id = d$$

$$d(1+i) = i$$

$$d = \frac{i}{1+i}$$

$$\therefore \text{discounting factor} = v = \frac{1}{1+i}$$

$$d = i \times \frac{1}{1+i}$$

$$\boxed{d = iv}$$

Camlin KOKUYO

Q12 $PV_0 = 7049$

$i^{(2)} = 6\%$

$i^{(4)} = 1.5\%$

$d^{(12)} = \cancel{1.8}\ 6\%$

$$d = 1 - \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

$$= 0.0584$$

$$AV_6 = 7049\left[\left(1 + \frac{i^{(12)}}{12}\right)^{12\times 2} \cdot \left(1 + \frac{i^{(4)}}{4}\right)^{4} \cdot \left(1 - d\right)^{18/...}\right]$$

$$= 7049\left[(1.127150) \times (1 - 0.0384) \times (0.9136...)\right]$$

$$AV_6 = 7536.4154$$

Q3) $(1+i)^n = 2$

$(1+i)^n = 2$

$n = 7.3$ years

ii) $d^{(12)} = 10\%$

$d = 1 - \left(1 - \frac{d^{(12)}}{12}\right)^{12}$

$d = 9.55\%$

$x = 2x(1-d)^n$

$\frac{1}{2} = (1-d)^n$

$\frac{\ln(0.5)}{\ln(0.9045)} = n$

$n = 6.9$ years

$\longleftarrow \; \rangle \; \longrightarrow$