Probability & statistics.

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Assignment - 2.

1) Let x_i be the claim size of ~~home~~ insurance
Let y_i be the claim size of car insurance.

Let $X \sim N(800, 100^2)$
$Y \sim N(1200, 300^2)$.

$$P(Y_1 + Y_2 + Y_3 > X_1 + X_2 + X_3 + X_4 + 800)$$
$$= P(Y_1 + Y_2 + Y_3 - X_1 + X_2 + X_3 + X_4) > 800).$$

$$Y_1 + Y_2 + Y_3 - (X_1 + X_2 + X_3 + X_4)$$
$$\sim N(3\times1200 - 4\times800, 300^2 + (1$$
$$\sim N(400, 310000).$$

$$P\left(Z > \frac{800-400}{\sqrt{310000}}\right).$$

$$P(Z > 0.71842) \longrightarrow 1 - P(Z < 0.71842)$$
$$= 1 - P(Z < 0.72)$$
$$= 1 - 0.76424$$
$$= 0.23576.$$

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2) $N \sim$ Negative binomial (k, p).

$P(N=n) \rightarrow {}^{n+2}C_n (0.9)^3 (0.1)^n$
${}^{(n+3)-1}C_{n-1} (0.9)^3 (0.1)^n$.

So, $k = n$ $\quad p = 0.1$
$k = 3$ $\quad p = 0.9$.

$M_y t = M_n(\log M_x t)$

$x \sim$ Gamma $(6, 2)$.
$y \rightarrow$ Total claim amount

$E(e^{yt} / N=n) = E(e^{x_1 t} + x_2 t \dots x_n t)$
$= E(e^{tx_1}) \cdot E(e^{tx_2}) \dots E(e^{tx_n})$
$= \left(1 - \frac{t}{2}\right)^{-6n}$.

So, $E(E(e^{yt} / N=n)) = E\left[\left(1 - \frac{t}{2}\right)^{-6n}\right]$.

$= E\left(e^{N \log(1-t/2)^{-6}}\right)$.
$= M_n\left[\log\left(1-\frac{t}{2}\right)^{-6}\right]$.
k.

$M_N t = \frac{pe^t}{1-qe^t} = \left[\frac{pe^{\log(1-\frac{t}{2})^{-6}}}{1-qe^{\log(1-t/2)^{-6}}}\right]$

$k = 3 \quad p = 0.9$.

$$= \left[\frac{0.9\left(1-t/2\right)^{-6}}{1-0.1\left(1-t/2\right)^{-6}}\right]^{3}$$

ii) $V(Y) = E(N)\,V(x) + \left[(E(x))^2 \cdot V(N)\right]$.

$$= \left[\frac{3}{0.9} \times \frac{6}{4}\right] + \left[\left(\frac{6}{2}\right)^2 \times \frac{3 \times 0.1}{(0.9)^2}\right]$$

$$= {}^{1}/_{0.2} + \left[6 \times {}^{0.3}/_{0.81}\right]$$

$$= 7.\overline{22}$$

$$SD = \sqrt{7.\overline{22}}$$

$$= 2.687419.$$

3) $y(x) = \lambda e^{-\lambda(x-\alpha)}$

$$M_x F = E(e^{tx}) = \int_{\alpha}^{\infty} e^{tx}\,\lambda e^{\lambda(x-\alpha)}\,dx$$

$$= \lambda \int_{\alpha}^{\infty} e^{tx} \cdot e^{-\lambda x}\, e^{\lambda\alpha}\, dx.$$

$$= e^{\lambda\alpha}\,\lambda \int_{\alpha}^{\infty} e^{-(\lambda-t)x}\, dx.$$

$$= \frac{e^{\lambda\alpha}\cdot\lambda}{t-x}\left[e^{-(\lambda-t)x}\right]_{\alpha}^{\infty}.$$

$$= e^{t\alpha}\left(\frac{\lambda}{\lambda-t}\right).$$

ii) $M_x(t) = \dfrac{\lambda e^{t\alpha}}{\lambda - t} = \lambda e^{t\alpha}(\lambda - t)^{-1}$

$M'_x t = \lambda\left[e^{t\alpha}(\lambda-t)^{-2}(1) + (\lambda-t)^{-1} e^{t\alpha}\alpha\right]$

$= \lambda e^{t\alpha}(\lambda - t)^{-2}(\alpha(\lambda - t)+1)$.

$t=0 \quad = \lambda(\lambda)^{-2}(\alpha(\lambda+1))$.

$= \dfrac{\lambda\alpha + 1}{\lambda}$.

$M''t = \lambda\left[\alpha e^{t\alpha}(\lambda-t)^{-2} + \alpha^2(\lambda-t)^{-1} e^{t\alpha} + e^{t\alpha}(2)(\lambda-1)^{-3} + (\lambda-t)^{-2}(e^{t\alpha})\right]$.

Putting $t=0$.

$= \lambda\left[\dfrac{2\alpha}{\lambda^2} + \dfrac{\alpha^2}{\lambda} + \dfrac{2}{\lambda^3}\right]$.

$E(x^2) = \dfrac{2\alpha}{\lambda} + \dfrac{\alpha^2}{\cancel{\lambda}} + \dfrac{2}{\lambda^2}$

$V(x) = E(x^2) - [E(x)]^2$

$= \dfrac{2\alpha}{\lambda} + \alpha^2 + \dfrac{2}{\lambda^2} - \left[\alpha + \dfrac{1}{\lambda}\right]^2$.

$= \dfrac{1}{\lambda^2}$

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4) $G_V(t) = \left(\dfrac{p}{1-qt}\right)^k \quad \because p+q=1$

$G_V t = \Sigma t^v \cdot P(V=v)$

$P(V=4) = \dfrac{\sqrt{k+4}}{\sqrt{5}\,\sqrt{k}} \; p^k q^k \quad \because G_V(t) = p^k(1-q$

$\cdot = \dfrac{\sqrt{k+2}}{4!\,\sqrt{k}} \; p^k q^k.$

$= \dfrac{(k+3)!}{(k-1)!\,4!} \; p^k (1-p)^k.$

5) $a = ?$

$a \int_{10}^{20} \int_{0}^{5} (x^2 + xy)\, dx\, dy = 1.$

$= \int_{10}^{20} \left[\dfrac{x^3}{3} + \dfrac{x^2}{2} y\right]_0^5 dy = \dfrac{1}{a}.$

$= \left[\dfrac{125y}{3} \cdot + \dfrac{25}{2} \dfrac{y^2}{2}\right]_{10}^{20} = \dfrac{1}{a}.$

$\dfrac{1250}{3} + \dfrac{25}{2} \times \dfrac{100}{2}. = \dfrac{1}{a}.$

$a = \dfrac{3}{3125} = 0.00096$

iii) $E(y/x)$

$$f_{y/x} = \frac{f(x,y)}{f(x)}$$

$$f(x) = \frac{3}{6875}\int_{10}^{20} x^2 + xy \, dy$$

$$= \frac{3}{6875}\left(10x^2 + 150x\right).$$

$$= \frac{30x}{6875}(x+15).$$

$$f_{y/x} = \frac{3}{6875} \frac{x(x+y)}{30(x)} \times \frac{6875}{x+15} = \frac{x+y}{10(x+15)}$$

$$\therefore E(Y/X) = \int_{10}^{20} \frac{(x+y)}{10(x+15)} \, dy$$

$$= \frac{1}{10(x+15)}\left[\frac{xy^2}{2} + \frac{y^3}{3}\right]_{10}^{20}$$

$$= \frac{1}{x+15}\left(15x + \frac{700}{3}\right).$$

$P(Y > 1/3 x + 10).$

$$f(y) = \frac{3}{6875}\int_0^5 x^2 + xy \, dx$$

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$$= \frac{3}{6875}\left[\frac{x^3}{3} + \frac{x^2}{2} y\right]_0^5$$

$$= \frac{3}{6875}\left(\frac{125}{3} + \frac{25}{2} y\right)$$

So, $\frac{3}{6875}\left(\int_{3x+10}^{20} \frac{125}{3} + \frac{25}{2} y \, dy\right)$.

$$= \frac{3}{6875}\left[\frac{125}{3} y + \frac{25}{2} y^2\right]_{\frac{1}{3}x+10}^{20}$$

$$= \frac{3}{6875}\left(\frac{1250}{3} + 1875 - \frac{25x^2}{36} - \frac{125x}{7} - \frac{25x}{6}\right.$$

6) $X \sim$ Poisson (λ). $\quad$ $Y \sim$ Poisson (μ)

$M_x(t) = e^{\lambda(e^t - 1)}$ $\quad$ $M_y(t) = e^{\mu(e^t - 1)}$

So, $Z = X - Y$.

$$M_z(t) = E(e^{tz}) = E(e^{t(x-y)}) = E(e^{tx} \cdot e^{-ty})$$

$$= \frac{E(e^{tx})}{E(e^{ty})}.$$

$$= \frac{e^{\lambda(e^t - 1)}}{e^{\mu(e^t - 1)}} = \left(e^{(\lambda - \mu)(e^t - 1)}\right).$$

So by uniquness of MGF of $z = e^{(\lambda - \mu)(e^t - 1)}$

$\sim$ Poisson $(\lambda - \mu)$

7)

			X		
		1	2	3	4
y	1	0.2	0	0.05	0.15
	2	0	0.3	0.1	0.2.

i) $V(X/Y \cancel{=} 2) = E(X^2/Y=2) - [E(X/Y=2)]^2$

$E(X^2/Y=2) = \Sigma x^2 \dfrac{P(X=x, y=2)}{P(Y=2)}$

when $x=1$ + when $x=2$ + when $x=3$ + when $x=4$

$\dfrac{1^2(P(x=1, y=2)}{y=2} + \dfrac{4(0.3)}{0.6} + \dfrac{9(0.1)}{0.6} + \dfrac{16(0.2)}{6.}$

$= 8.83333$

$E(X/Y=2) = \sum_x \dfrac{P(X=x \& y=2)}{P(y=2)}.$

$\dfrac{1(0)}{0.6} + \dfrac{2(0.3)}{0.6} + \dfrac{3(0.1)}{0.6} + \dfrac{4(0.2)}{0.6}$

$= 2.8333$

$V(X/Y=2) = \underline{8.333 - (2.8333)^2}$

$= 0.80757.$

ii) $f(u,v) = \frac{48}{67}(2uv - u^2)$. $\quad 0 < u < 1$

$\frac{u}{2} < v < 2$

$E(u/v = v) = ?$

$\frac{f(u,v)}{f(v)} \rightarrow f\ v/v = v$.

$\therefore f(v) = \frac{48}{67}\int_0^1 (2uv - u^2)\,du$

$= \frac{48}{67}\left[u^2 v - \frac{u^3}{3}\right]_0^1$

$= \frac{48}{67}\left(v - \frac{1}{3}\right)$

$= \frac{16}{67}(3v - 1)$.

$\frac{f(u,v)}{f(v)} = \frac{\cancel{48}^{3}}{\cancel{67}} \frac{(2uv^2 - u^2)}{(3v-1)} \times \frac{\cancel{67}}{\cancel{16}}$

$= \frac{3}{3v-1}(2uv^2 - u^2)$.

$E(u/v = v) = \frac{3}{3v-1}\int_0^1 u(2uv^2 - u^2)\,du$

$= \frac{3}{3v-1}\left[\frac{2u^3v^2}{3} - \frac{u^4}{4}\right]_0^1$

$= \frac{8v^2 - 3}{4(3v-1)}$

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8) $X \sim \text{Gamma}(3, 2)$. ; $E(X) = 3/2$; $V(X) = 3/$

$E(Y/X = x) = 3x+1$ $\quad$ $Var(Y/X = x) = 2x^2+5$

$E(Y) = E(E/Y = x) \rightarrow E(3x+1$

$= 3(E(x)) + 1$

$= 3\lambda \cdot 3/2 + 1$

$= \frac{11}{2}$.

$V(Y) = E[V(X/Y)] + V[E(X/Y)]$.

$= E(2x^2+5) + V(3x+1)$

$= 2E(x^2) + 5 + 9V(x)$

$V(x) = E(x^2) - [E(x)]^2$.

$E(x^2) = \frac{3}{4} + \frac{9}{4} = 3$.

$\therefore V(Y) = 2(3) + 5 + 9(3/4)$.

$= \frac{71}{4}$

9) $E(x) = 3$ $\quad$ $E(Y) = 4$

$V(x) = 4$ $\quad$ $V(Y) = 1$

Correlation coefficient $= -0.3\sqrt{4} \rightarrow cov(x,y)$

$= -0.6$.

$z = x + y$.

i) $cov(x+y) = cov(x,y) + cov(x,y)$.

$= V(x) + cov(x,y)$

$= 4 + (-0.6)$.

$= 3.4$

ii) $var(z) = cov(x+x, x+y)$.

$= cov(x;x) + 2cov(x,x) + cov(y,y)$.

$= V(x) + 2cov(x,y) + v(y)$

$= 4 + 2(-0.6) + 1$

$= 5 - 1.2$.

$= 3.8$.

10) $f(x,y) = kx^{-\alpha} e^{-y/\beta}$

$$= k \int_1^\infty \int_1^\infty x^{-\alpha} e^{-y/\beta} \, dx \, dy = 1$$

$$\therefore \; k \int_1^\infty e^{-y/\beta} \left[\frac{x^{-\alpha+1}}{1-\alpha} \right]_1^\infty = 1$$

$$\therefore \; \frac{k}{1-\alpha} \int_1^\infty e^{-y/\beta} = 1$$

$$\frac{-k}{\alpha-1} \left[\frac{e^{-y/\beta}}{-1/\beta} \right]_1^\infty = 1.$$

$$\frac{-k}{\alpha-1} \beta \left(e^{-1/\beta} \right) = 1$$

$$k = \frac{\alpha-1}{\beta\left(e^{-1/\beta}\right)}.$$