Assignment 1

(1) i) $(Ia)_{\overline{n|}} = v + 2v^2 + 3v^3 + \dots + nv^n$ — (a)

$(1+i)(Ia)_{\overline{n|}} = 1 + 2v + 3v^2 + \dots + nv^{n-1}$ — (b)

(b) − (a)

$= i(Ia)_{\overline{n|}} = 1 + v + v^2 + v^3 + \dots + v^{n-1} - nv^n$

$i(Ia)_{\overline{n|}} = \ddot{a}_{\overline{n|}} - nv^n$

$$(Ia)_{\overline{n|}} = \frac{\ddot{a}_{\overline{n|}} - nv^n}{i}$$

ii) Cost of education = 30,00,000

inflation = 6% p.a.

Contro = 2,00,000 p.a.

increase = x

PV of contribution: $2,00,000\,\ddot{a}_{\overline{10|}} + x(Ia)_{\overline{9|}}$ @ 10%

$= 2,00,000 \times 6.759026 + x \times 25.18046$

$= 1351805.20 + 25.18046x$

PV of higher edu. expense:

$30,00,000 \times (1.06)^{10} \times v^{10}$ @ 10%

$= 2071350$

$x = 28.575$

$1+i = e^{\delta}$

DATE

2. $x = 40$ $S = 100,000$ immediately on death or diagnosis of illness.

$$\begin{tikzcd} H \arrow[rr, "\sigma_x"] \arrow[dr, "\mu_x"'] & & S \arrow[dl, "\nu_x"] \\ & D & \end{tikzcd}$$

$i = 5\%$ $\mu_x = 0.004$

$\nu_x = 0.005$

$\sigma_x = 0.002$

$\delta = \ln(1+i)$

$\delta = 0.04879$

EPV of benefits:

EPV of Critically ill $= \int_0^{20} 100,000 \times e^{-\delta t} \times {}_tp_{40}^{\overline{HH}} \times \sigma_{40+t}\, dt$

$= \int_0^{20} 100,000 \times e^{-0.04879t} \times e^{-0.006t} \times 0.002\, dt$

${}_tp_{40}^{\overline{HH}} = e^{-(0.004+0.002)t}$

$= e^{-(0.006)t}$

$= \int_0^{20} 100,000 \times e^{-0.05479t} \times 0.002\, dt$

$= \$\ 200 \int_0^{20} e^{-0.05479t}$

$= 200 \left[\frac{-e^{-0.05479t}}{0.05479} \right]_0^{20}$

$= 200 \left[\frac{-e^{-0.05479 \times 20}}{0.05479} + 1 \right]$

EPV of DB $= 100,000 \int_0^{20} e^{-\delta t} \times {}_tp_{40}^{HH} \times \mu_{40+t}\, dt$

$= 100,000 \int_0^{20} e^{-0.05479t} \times 0.004\, dt$

$= 400 \left[\frac{-e^{-0.05479 \times 20}}{0.05479} + 1 \right]$

EPV of benefits $= 200 + 400 \times \left(\frac{1 - e^{-0.05479 \times 20}}{0.05479} \right)$

$= 600 \times 12.150537$

$= 7290.322$ //

3. Premium:

$P\ddot{a}^{(12)}_{40:\overline{20|}}$

$= P\left[\ddot{a}_{40:\overline{20|}} - \frac{11}{24}\left(1 - \frac{D_{60}}{D_{40}}\right)\right]$

$= P\left[13.927 - \frac{11}{24}\left(1 - \frac{882.85}{2052.96}\right)\right]$

$= 13.66576724P$

Claim Cost:

$75,00,000\, A^{1}_{40:\overline{20|}} + 50,000\, A_{[40]:\overline{20|}}$

$75,00,000\left[A^{*}_{40:20} - \frac{D_{60}}{D_{40}}\right] + 50,000\left[A_{[40]:\overline{20|}} - \frac{D_{60}}{D_{40}}\right]$

$= 257193.968202 - 170521.965829$

$= 427714.965\,829$

Claim investigation expense:

$= 0.03 \times 427714.965824$

$= 12831.448975$

Commission Expense:

$0.3P\ddot{a}^{(12)}_{40:\overline{1|}} + 0.05P\ddot{a}^{(12)}_{40:\overline{20|}}$

$= 0.3P\left[\ddot{a}_{40:1} - \frac{11}{24}\left(1 - \frac{D_{41}}{D_{40}}\right)\right] + 0.05P\left[\ddot{a}_{40:20} - \frac{11}{24}\left(1 - \frac{D_{60}}{D_{40}}\right)\right]$

$= 0.3P \times 0.942596 + 0.68328836\,2P$

$= 0.966067P$

Management Expense

$= 0.1P\ddot{a}^{(12)}_{40:\overline{20|}} \times 1.04^{0.5}$

$= 0.1P \times 13.66576724 \times 1.04^{0.5}$

$= 1.39364P$

Underwriting Expense

$= 500$

Monthly premium

$13.66576724P = 427714.9658 + 12831.448975 + 0.966667P + 1.39364P + 500$

$P = 39009.7$ Monthly $= 39009.7/12$

$= 3250.8$

(ii) Accident claim

$= 7500,000\ A_{45:\overline{15}|} - 39009.7\ \ddot{a}^{(12)}_{45:\overline{15}|}$

$= 7500,000 \times 0.0359 - 39009.7 \times 11.1448$

$= 269250 - 434750.3046$

$= -165363.4377$

illness claim

$= 50,00,000\ A_{45:\overline{15}|} - 39009.7\ \ddot{a}^{(12)}_{45:15}$

$= 50,00,000 \times 0.035918293 - \cancel{39009.7}\ 434757.4699$

$= -263766.0049$

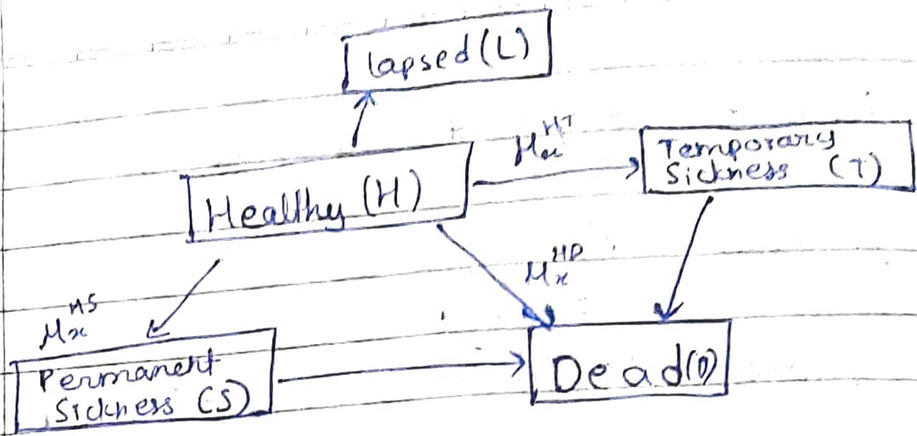
No claim mad

$= 12500000 \times 0.035918293 - 434750.6352$

$= 14,228.0273$

4.

(i)



(ii) $x = 50$ ceases at 65

$n = 15$ years

EPV (Premiums) = EPV (Benefits)

$$EPV(T) = 10{,}000 \int_0^{15} v^t \, {}_tp_{50}^{HT} \, dt$$

$$EPV(S) = 20{,}000 \int_0^{15} v^t \, {}_tp_{50}^{HS} \, dt$$

$$EPV(\text{Death}\ \cancel{\text{Benefits}}) = 100{,}000 \int_0^{15} v^t \underbrace{\left({}_tp_{50}^{HH} \mu_{50+t}^{HD} + {}_tp_{50}^{HT} \mu_{50+t}^{TD} + {}_tp_{50}^{HS} \mu_{50+t}^{SD} \right)}_{u} dt$$

$$EPV(\text{Premiums}) = P \int_0^{15} {}_tp_{50}^{HH} \, v^t \, dt$$

$$P = \frac{10{,}000 \int_0^{15} v^t \, {}_tp_{50}^{HT} \, dt + 20{,}000 \int_0^{15} v^t \, {}_tp_{50}^{HS} \, dt + 100{,}000 \int v^t \cdot u \, dt}{\int_0^{15} {}_tp_{50}^{HH} \, v^t \, dt}$$

5. (i) Gross premiums for compound revisionary policy:

EPV premiums $= P\ddot{a}_{[40]:25}$

$= 15.887P$

EPV expenses $= 1200 + 500\bar{A}_{[40]} + 2\% \times P \times (\ddot{a}_{[40]:25} - 1)$

$= 1200 + 500 (1.04)^{0.5} \times 0.23041 + 0.02 \times P \times (15.887 - 1)$

$= 1317.487 + 0.29774P$

EPV benefits $= \frac{2,00,000}{1.04^{0.5}} (q_{[40]} + (1.04)\, {}_{1|}q_{[40]} + \cdots)$

$= 196116.14$

$15.887P = 1317.487 + 0.29774P + 196116.14$

$P = 12664$

(ii) EPV (benefits) $= 2,00,000 \times b(\bar{I}\bar{A})_{[40]} + 2,00,000 (1-b)\bar{A}_{[40]}$

$(1.04)^{0.5} [2,00,000 \times b(IA)_{[40]} + 2,00,000 (1-b) A_{[40]}]$

$= 196116.14$

$b = 9\%$

(iii) Net premium

$P\ddot{a}_{40:\overline{25}|} = 2,00,000 \bar{A}_{[40]}$

$P = 2960.544$

NPR at $t=10$

${}_{10}V = 3,00,000\bar{A}_{50} - P\ddot{a}_{50:15}$

$= 3,00,000 \times 1.04^{0.5} \times 0.32907 - 2960.54 \times 11.253$

$= 67361.10$

Q. $n = 20$ $\quad x = 45$ $\quad P = ?$

DB $\rightarrow$ SA $= 100{,}000$ EOYOD $\quad i = 5\%$

Cancer $\rightarrow$ SA $= 100000$ EOY $\quad \delta = \ln(1.05)$

Survives $\rightarrow$ Premium returned $\quad = 0.04879$

$$\begin{tikzcd} & H \arrow[dl, "\sigma_x"'] \arrow[dr, "\mu_x"] & \\ C \arrow[rr, "\nu_x"'] & & D \end{tikzcd}$$

$\mu_x = 0.006$

$\sigma_x = 0.002$

$\nu_x = 0.010$

IC $= 2\%$ $\quad$ IE $= 2\% \times P$ $\quad$ Profit margin $= 10\% \times P$

EPV of benefits :

$$= 100{,}000 \int_0^{20} e^{-\delta t} \; {}_tp_{45}^{HH} \left(\mu_{45+t} + \sigma_{45+t}\right) dt$$

$$= 100{,}000 \int_0^{20} e^{-0.04879} \; {}_tp_{45}^{HH} (0.008)\, dt$$

$${}_tp_{45}^{HH} = e^{-0.008t}$$

$$= 100{,}000 \int_0^{20} e^{-0.05679t} \times 0.008\, dt$$

$$= 800 \times \left[\frac{-e^{-0.05679 \times 20}}{0.05679} + 1 \right]$$

$$= 800 \times \left(\frac{1 - e^{-0.05679 \times 20}}{0.05679} \right)$$

$$= 9562.7387$$

7. EPV of CF = EPV of prem − EPV of benefits − EPV of Expense − EPV of Commission

EPV premium $= 30,000\ \ddot{a}_{[45]:\overline{20}|}$
$= 30,000 \times 11.88$
$= 356,640$

EPV Benefits $= 1000000 \times A_{[45]:\overline{20}|}$
$= 1000000 \times 0.32711$
$= 327110$

EPV Expenses $= 5000 + 2.5\% \times 30,000 \times (\ddot{a}_{[45]:\overline{20}|} - 1)$
$+ 500 \times P_{[45]} \times 1.06^{-1} \times (\ddot{a}_{[45]:\overline{20}|})_{@4\%}$

Renewal Commission
$= 2\% \times 30,000 \times (\ddot{a}_{[45]:\overline{20}|} - 1)$
$= 2\% \times 30,000 \times (11.888 - 1)$
$= 6533$

PV of net C.F. $= 356640 - 327110 - 19438 - 6533 - I$
$= 3559 - I$

Margin = PV of net CF / Annual Prem
$10\% = (3559 - I)/30,000$
$I = $ Rs 559

8.

(i) Assumption of independence of decrements is used to derive single decrement table from multiple decrement table

$(a\mu)_x^j = \mu_x^j$ for all j & all x

(ii) In cases of term assurance, it is likely that people who lapse their policies have a lower than avg mortality rate.

(iii)

a) $1000 \int_0^5 e^{-\delta t} \left({}_tP_{45}^{PI} \mu_{45+t} + {}_tP_{45}^{PP} \nu_{45+t} \right) dt$

b) $250 \int_0^5 e^{-\delta t} \, {}_tP_{45}^{IP} \nu_{45+t} \, dt$

c) $750 \int_0^5 e^{-\delta t} \, {}_tP_{45}^{II} \, dt$

d) $250 \int_0^5 e^{-\delta t} \, {}_tP_{45}^{PP} \nu_{45+t} \, dt$

(iv) $(aq)_{45}^c = \frac{0.05}{0.18} (1 - e^{-0.18}) = 0.045758$

$(aq)_{45}^c = \frac{0.13}{0.18} (1 - e^{-0.18}) = 0.118972$

$(ap)_{45}^c = 1 - 0.04578 - 0.118972 = 0.835270$

$(aq)_{46}^c = \frac{0.06}{0.16} (1 - e^{-0.16}) = 0.055446$ $\quad (aq)_{45}^s = \frac{0.1}{0.16} (1 - e^{-0.16}) = 0.092410$

$(ap)_{46} = 1 - 0.055446 - 0.092410 = 0.852144$

$0.835270 \times 0.852144 = 0.711770$

9.(i) In a with profits contract, the surplus on fund is distributed to the policyholders in a pre-determined manner. As the company receives surplus it is shared with the policy holders in the form of bonus. Also, company can declare bonuses on its expectation of how well a company will do & it does not have to be fixed & hence the need of bonuses under with profit policies

(ii) Investment Surplus, Mortality Surplus, Surrender Surplus, Reinsurance surplus, Expense Surplus.

(iii) Terminal bonus could be a strategy to build lower guarantees over the term of with profit policies. The companies would have greater freedom in choosing the investment assets. Which means they can take more risk & return higher terminal bonus to policyholders.

10. EPV of premiums:

$$12P\ddot{a}^{(12)}_{50:\overline{10}|} \quad @6\%$$

PE

$$\ddot{a}^{(12)}_{50:\overline{10}|} = \ddot{a}_{50:\overline{10}|} - \frac{11}{24}\left(1 - {}_{10}p_{50}\, v^{10}\right)$$

$$= 7.694 - \frac{11}{24}\left(1 - (1.06)^{-10} \times \frac{9287.2164}{9712.0728}\right)$$

$$= 7.4804$$

$\therefore$ EPV of premiums $= 12 \times P \times 7.4804$

$$= 89.7648P$$

EPV of benefits $= 10,00,000 \times \left(q_{50} \times v + 1.019231 \cdot v^2 \cdot {}_{1|}q_{50} + \dots \right.$

$\qquad + 1.019231^{15} \times v^{15} \times {}_{14|}q_{50} + \dots$

$\qquad \left. + 1.019231^{15}\, v^{15}\, {}_{15}p_{50}\right)$

$$= 10,00,000 \times \frac{1}{1.019231} A_{50:\overline{15}|} \quad @4\%$$

$A_{50:\overline{15}|} = 0.56719$

$\therefore$ EPV of benefits $= \dfrac{10,00,000 \times 0.56719}{1.019231}$

$$= 566004.1$$

EPV of expense & commission:

$$\frac{(20+35)}{100} \times 12P + \frac{(2+1.5)}{100} \times {}_1p_{50} \times 1.06^{-1} \times 12P\ddot{a}^{(12)}_{51:\overline{9}|}$$

$$+ 2500 + 400 \times {}_1p_{50} \times 1.06^{-1} \times \ddot{a}_{51:\overline{9}|} \quad @6\%$$

$= 9.33P + 6115.06$

$EPV(\text{Premium}) = EPV(\text{Benefits}) + EPV(\text{Expenses})$

$89.7648P = 5,66,004.1 + 9.33P + 6115.06$

$P = 7113.12$

(ii) Sum assured at the end of 10^{th} year + Accrued bonuses

$X = 10,00,000 \times 1.02^{10}$

$X = 1218994$

PV of future benefits at the end of 10^{th} Year

$= \frac{X}{1.05} q_{60} + \frac{X \times 1.04}{1.05^2} {}_{1|}q_{60} + \cdots \frac{X}{1.05^5} \times 1.04^4 \times {}_{4|}q_{60}$

$+ \frac{X}{1.05^5} \times 1.04^5 \times {}_5p_{60}$

$= 11,60,774$

PV of expenses $= 400 \times \ddot{a}_{60:\overline{5}|}$ @5%

$= 400 \times (1 + 1.05^{-1} \times {}_1p_{60} + 1.05^{-2} \times {}_2p_{60} + \cdots 1.05^{-4} \times {}_4p_{60})$

$= 1787.39$

Reserves required = PV of Benefits + PV of Expenses

$= 11,62,561$

11. $n = 20$ years with profit endowment assurance
$x = 30.$ $S = Rs\,400000$
regular bonus → rate of 40 per 1000 SA which would vest at end of each year.
Net premium prospective reserve at time 7 = ?
EOYOD → DB AM92 Ultimate $i = 6\%$ p.a.

⇒ $R = 400000$

$$P\ddot{a}_{30:\overline{20|}} = 400,000 \times A_{30:\overline{20|}}$$

$$\ddot{a}_{30:\overline{20|}} = \ddot{a}_{30} - {}_np_x v^n \ddot{a}_{50} = 12.08705$$

$$A_{30:\overline{20|}} = A_{30:\overline{20|}}^{1} + A_{30:\overline{20|}}^{\ \ 1} = 0.315817$$

$$A_{30:\overline{20|}}^{\ \ 1} = v^{20} \cdot {}_{20}p_{30}$$

$$A_{30:\overline{20|}}^{1} = A_{30} - v^{20}\,{}_{20}p_{30}\,A_{50}$$

$$P = \frac{400,000 \times 0.315817}{12.08705}$$
$$= 10451.42$$

Total bonus till time 7
$7 \times 4\% \times 400,000 = 112000$

∴ Guaranteed SA $= 400,000 + 11200 = 512000.$

∴ Prospective net premium reserves at time 7 is
EPV (Benefit) − EPV (Premium)
$512000\, A_{37:\overline{13|}} - 10451.42\, \ddot{a}_{37:\overline{13|}}$
$= 512000 \times 0.471638 - 10451.42 \times 9.333835$
$= 143926.89$