Numeral Methods and Algebra
Assignment - 1

Q1) Actual length $= r_1 = 12.5 cm$

Actual Area $= \pi r^2$

$= \pi (12.5)^2 = 490.87385 cm^2$

Approximate radius $= 12.65$

Approximate area $= \pi r^2 = \pi (12.65)^2$

$= 502.72551 cm^2$

1) Absolute error = Approx area - Actual Area

$= 502.72551 - 490.87385$

$= 11.58166 cm^2$

2) Relative error $= \frac{\text{Absolute error}}{\text{Actual Area}}$

$= \frac{11.58166}{490.87385} = 0.024144$

3) Percentage error = Relative error × 100

$= 0.024144 \times 100$

$= 2.4144\%$

Q.2) 1) Using linear interpolation

$x_1 = 4$ $\quad y_1 = 0.60206$

$x_2 = 6$ $\quad y_2 = 0.7781513$

$(6.4) = (0.7781513 - 0.60206)$

$2 = 0.1760913$

1 cause a change of $\frac{0.1760913 \times 1}{2}$

$\therefore 1 = 0.08804563$

$\therefore x = 5$

$y = 0.60206 + 0.08804565$

$= 0.69010565$

iii) $x_1 = 4.5$ $\qquad y_1 = 0.6532125$

$x_2 = 5.5$ $\qquad y_2 = 0.7403627$

$(5.5 - 4.5) = 0.7403627 - 0.06532125$

$1 = 0.0871502$

$\therefore$ Change caused by $0.5 = 0.0435751$

$\therefore x = 5$

$y = 0.6532125 + 0.0435751$

$= 0.6967876$

Q3) $x^4 - x - 10 = 0$

$x_0 = 1.5$ $\qquad y_1 = -6.4375$

$x_1 = 2$ $\qquad y_2 = 4$

$\rightarrow x_2 = \frac{x_0 + x_1}{2} = 1.75$

$y_2 = -2.37109$

$\rightarrow x_3 = \frac{x_1 + x_2}{2} = 1.875$

$y_3 = 0.48462$

$\rightarrow x_4 = \frac{x_2 + x_3}{2} = 1.8125$

$y_4 = -1.02025$

$\rightarrow x_5 = \frac{x_4 + x_3}{2} = 1.84375$

$y_5 = -0.28773$

$\rightarrow x_6 = \frac{x_3 + x_5}{2} = 1.859375$

$y_6 = 0.93378$

Q4) $x^3 - x - 1 = 0$ $\quad f'(x) = 3x^2 - 1$

x	y
0	-1
1	-1
2	5

$x_0 = 1$

$f(x_0) = 1$

$f'(x_0) = 2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 + \frac{(-1)}{2} = 1.5$$

$y_1 = 0.875 \quad f'(x_1) = 5.75$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.5 - \frac{(0.875)}{5.75}$$

$= 1.34783$

$f(x_2) = 0.100699$

$f'(x_2) = 4.44994$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.3252$$

$y_3 = 0.00205$

$f'(x_3) = 4.26846$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.32472$$

$y_4 = 0.000000872$

Q5) $A^2 = A$

$7A - (I+A)^3 = 7A - (I^3 + A^3 + 3A^2 I + 3AI^2)$

$= 7A - [I + A^2(A) + 3A^2 I + 3A]$

$= 7A - (I + A^2 + 6A)$

$= 7A - I - 7A = -I$

Q6) $R = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$$|A| = 1\begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix} - (-1)\begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix} + 2\begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= 1(2-6) + 1(-4-0) + 2(4-0)$$

$$= -6 + -4 + 8 = -2$$

$$\text{adj}(A) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

$$= \frac{-1}{2}\begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$$

Q7) $\vec{a} = 8\hat{i} - 9\hat{j}$ $\vec{b} = 7\hat{i} - 9\hat{j}$

$$\vec{a}\cdot\vec{b} = ab\cos\theta$$

$$\vec{a}\cdot\vec{b} = (8\hat{i} - 9\hat{j})(7\hat{i} - 9\hat{j})$$

$$= 56 + 81 = 137$$

$$a = \sqrt{64+81} = \sqrt{145}, \quad b = \sqrt{49+81} = \sqrt{130}$$

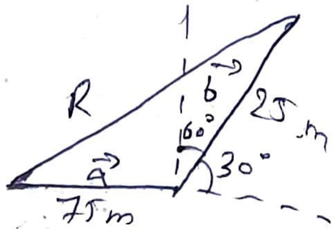
$$\vec{a}\cdot\vec{b} = ab\cos\theta$$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{ab}$$

$$= \frac{137}{\sqrt{145}\sqrt{130}} = 0.99784946$$

$$\theta = 3.75856^\circ$$

8) Displacement vector = $\vec{R}$



$\vec{R} = \vec{a} + \vec{b}$

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$\theta = 30°$

$R^2 = 75^2 + 25^2 + 2(75)(25)\cos 60°$

$R^2 = 9497.595264$

$R = 97.4556$ metres

9) Common difference $\Rightarrow d = 3$

$AP = 101, 104, 107, \ldots, 998$

$a = 101$

$a_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$347 = (n-1)3$

$n = 300$

$S_n = \frac{n}{2}(a + a_n)$

$= \frac{300}{2}(101 + 998)$

$= 164850$

10) $a_6 = 32$ & $a_8 = 128$

$ar^5 = 32$ $\quad ar^7 = 128$

$\frac{ar^7}{ar^5} = \frac{128}{32}$

$r^2 = 4$

$r = 2$ or -2 (-2 rejected)

$\therefore r = 2$

Q11) $(3x+4)^5$

Using Pascal's Triangle →

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

$(a+b)$

$(a+b)^2$

$(a+b)^3$

$(a+b)^4$

$(a+b)^5$

$(3x+4)^5 = 3x^5 + (5 \times (3x)^4 \times 4) + (10 \times (3x)^3 \times 4^2)$
$+ (10 \times (3x)^2 \times 4^3) + (5 \times 3x \times 4^4) + 4^5$
$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3480x + 1024$

$(3x+4)^3 = 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3480x + 1024$

Q12) $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

$|A - \lambda I|$

Expanding using 3rd row

$|A - \lambda I| = 0 - 0 + (3-\lambda)[(2-\lambda)(-5-\lambda)+6]$

$= (3-\lambda)[-10-2\lambda+5\lambda+\lambda^2+6]$

$= (3-\lambda)[\lambda^2+3\lambda-4]$

$= 3\lambda^2 + 9\lambda - 12 - \lambda^3 - 3\lambda^2 + 4\lambda$

$= -\lambda^3 + 13\lambda - 12$

$\lambda^3 - 13\lambda + 12 = 0$

Comparing coefficient

$a + b + c + d = 1 - 13 + 12$

$= 0$

$\therefore$ only one root $= 1$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -13 & 12 \\ & & 1 & 1 & -12 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

$\lambda^2 + \lambda - 12 = 0$

$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$

$\lambda(\lambda + 4) - 3(\lambda + 4) = 0$

$(\lambda - 3)(\lambda + 4) = 0$

$\lambda = 3$ or -4

$\therefore$ Eigen values $= -4, 3 \,\&\, 1$

Q(4) $(x^2 - x + 1)^4 = ((x-1)^2 + x)^4$

Using Pascal's triangle

1	1
1 1	$a+b$
1 2 1	$(a+b)^2$
1 3 3 1	$(a+b)^3$
1 4 6 4 1	$(a+b)^4$

$((x-1)^2 + x)^4 = (x-1)^8 + 4(x-1)^6 x + 6(x-1)^4 x^2 + 4(x-1)^2 x^3 +$

$(x^2 - x + 1)^4 = (x-1)^8 + 4(x-1)^6 x + 6x^2 (x-1)^4 + 4x^3 (x-1)^2$

$+ x$

Q13) $f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

$f(5) = -13$

$f'(5) = 13$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 5 - \frac{(-13)}{13} = 6$$

$f(x_1) = 9$

$f'(x_1) = 32$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 6 - \frac{9}{32}$$

$= 5.71875$, $f(x_2) = 0.84782$

Q15) $e^{-x} = 3\log(x)$

$3\log x + e^{-x} = 0$

x	y
0.1	-2.09516
0.2	-1.27818
0.3	-0.82782
0.4	-0.5235
0.5	-0.29656
0.6	-0.11673
0.7	0.03188

$x_0 = 0.6$ $\quad y_0 = -0.11673$

$x_1 = 0.7$ $\quad y_1 = 0.03188$

$\rightarrow x_2 = \frac{x_0 + x_1}{2} = 0.65$ $\quad y_2 = -0.03921$

$\rightarrow x_3 = \frac{x_2 + x_1}{2} = 0.675$ $\quad y_3 = -0.00293$

$\rightarrow x_4 = \frac{x_3 + x_1}{2} = 0.6875$ $\quad y_4 = 0.01465$

$\rightarrow x_5 = \frac{x_4 + x_3}{2} = 0.68125$ $\quad y_5 = 0.0059$