

CALCULUS

$$1) \lim_{h \rightarrow 0} \frac{2(h-3)^2 - 18}{h}$$

$$\lim_{h \rightarrow 0} \frac{2[(h-3)^2 - 9]}{h}$$

$$\lim_{h \rightarrow 0} \frac{2[(h-3)^2 - (3)^2]}{h}$$

$$\lim_{h \rightarrow 0} \frac{2[(h-3-3)(h-3+3)]}{h}$$

$$\lim_{h \rightarrow 0} 2(h-6)$$

put limit $h \rightarrow 0$

$$2(0-6)$$

$$2(-6) = -12$$

30.)

$$\lim_{x \rightarrow 9} \frac{x-9}{3-\sqrt{x}}$$

$$\lim_{x \rightarrow 9} \frac{x-9}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}}$$

$$\lim_{x \rightarrow 9} \frac{x-9}{(9-x)} \times \frac{3+\sqrt{x}}{3+\sqrt{x}}$$

$$\lim_{x \rightarrow 9} \frac{(x-9)}{(x-9)} \times \frac{3+\sqrt{x}}{3+\sqrt{x}}$$

put limit

$$\begin{aligned} & - (\sqrt{9} + 3) \\ & - (3 + 3) \\ & - 6 \end{aligned}$$

Q4)

a

(a) a) $x = -1$ b) $x = 0$ c) $x = 3$

$$f(x) = \frac{4x+5}{9x-3x}$$

at $x = 1$

$$f(-1) = \frac{-4+5}{9+3} = \frac{1}{12}$$

at $x = 0$

$$f(x) = \frac{5}{9}$$

at $x = 3$

$$f(x) = \frac{17}{0}$$

It's discontinuous

Q6)

$$g(y) = \begin{cases} y^2 + 5 & \text{if } y < -2 \\ 1 - 3y & \text{if } y \geq -2 \end{cases}$$

a) $\lim_{y \rightarrow 6} g(y) = \lim_{y \rightarrow 6} 1 - 3y$

put limit

$$1 - 3(6) = 1 - 18 = -17$$

$$\lim_{y \rightarrow -2} g(y)$$

$$\begin{aligned} \text{RHL} \\ \lim_{y \rightarrow -2^+} &= \lim_{y \rightarrow -2^+} 1 - 3y \\ &= 1 - 3(-2) \\ &= 1 + 6 = 7 \end{aligned}$$

$$\begin{aligned} \text{LHL} \\ \lim_{y \rightarrow -2^-} &= y^2 + 5 \\ &= \frac{4 + 5}{9} \end{aligned}$$

8.)

$$R(w) = \frac{3w + w^4}{2w^2 + 1} \quad \frac{u}{v}$$

$$\begin{aligned} R(w) &= \frac{(2w^2 + 1)(3 + 4w^3) - (3w + w^4)(4w)}{(2w^2 + 1)^2} \\ &= \frac{[6w^2 + 8w^5 + 3 + 4w^3] - [12w^2 + 4w - 3]}{(2w^2 + 1)^2} \\ &= \frac{4w^5 + 4w^3 - 6w^2 + 3}{(2w^2 + 1)^2} \end{aligned}$$

9a.)

$$f(x) = 2e^x - 8^x$$

$$f'(x) = 2e^x - 8^x \log 8$$

10.)

$$y = z^5 - e^z \ln(z)$$

$$\frac{dy}{dx} = 5y^4 - \left[\frac{e^z}{z} + \ln(z) e^z \right]$$

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$$a.) f(x) = (6x^2 + 7x)^4$$

$$f'(x) = 4(6x^2 + 7x)^3 (12x + 7)$$

$$b.) f(t) = 5 + e^{4t+t^2}$$

$$f'(t) = e^{4t+t^2} (4 + 2t)$$

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$$f(x) = x^3 - 3x^2 - 9x + 12$$

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0$$

$$3x^2 - 6x - 9 = 0$$

$$x^2 - 2x - 3 = 0$$

$$x^2 - 3x + x - 3 = 0$$

$$x(x-3) + 1(x-3) = 0$$

$$x(x+1)(x-3) = 0$$

$$x = -1, 3$$

~~13~~

$$f''(x) = 6x - 6$$

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$$\text{at } x = -1$$

$$= 6(-1) - 6$$

$$= -6 - 6$$

$$= -12$$

maximum

at $x = -1$

$$\text{at } x = 3$$

$$= 6(3) - 6$$

$$= 18 - 6$$

$$= 12$$

minimum

at $x = 3$

14.)

$$t(x) = 4x^3 - 18x^2 + 24x - 7$$

$$t'(x) = 12x^2 - 36x + 24$$

$$t'(x) = 0$$

$$12x^2 - 36x + 24 = 0$$

$$x^2 - 3x + 2 = 0$$

$$x^2 - 2x - x + 2 = 0$$

$$x(x-2) - 1(x-2) = 0$$

$$(x-2)(x-1) = 0$$

$$x = 2, 1$$

$$t''(x) = 24x - 36$$

at $x=2$

$$t''(2) = 48 - 36$$

$$= 12$$

min at $x=2$ at $x=1$

$$= 24 - 36$$

$$= -12$$

max at $x=1$