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Pricing & Reserving for life Insurance Products - Assignment 1.

1] i) Constant force of mortality states that the force of mortality between the age x & age $x+1$ is constant.

ii) $_1p_{55}$

$\therefore$ under CFM,

$p_x = e^{-\mu x}$

$\cancel{\mu =}$ $p_{55} = e^{-\mu(55)}$

$\mu_{55} = -\ln(p_{55})$

$\mu_{55} = -\ln(1 - q_{55})$

$\mu_{55} = -\ln(1 - 0.004469)$

$\boxed{\mu_{55} = 0.00447902}$

2] i) Select mortality is mortality rates depending on duration as well as age. as in real life these factors need to be accounted for.

ii) $_{20}p_{[25]:[30]}$

$= {_{20}p_{[25]}} \times {_{20}p_{[30]}}$

$= \dfrac{l_{45}}{l_{[25]}} \times \dfrac{l_{50}}{l_{[30]}}$

$= \dfrac{9801.3123}{9951.9913} \times \dfrac{9712.0728}{9923.7497}$

$= 0.96385$

3) UDD

$$_{1.4}q_{54.5} = \cancel{0.5}\,q \quad 1 - {}_{1.4}p_{54.5}$$

$$= 1 - {}_{0.5}p_{54.5} \times {}_{0.9}p_{55}$$

$$= 1 - \left[(1 - {}_{0.5}q_{54.5})(1 - {}_{0.9}q_{55})\right]$$

$$= 1 - \left\{\left[1 - \frac{0.5\,q_{54}}{1 - 0.5\,q_{54}}\right]\right\}(1 - 0.9\,q_{55})$$

$$= 1 - \left[\left(1 - \frac{0.5 \times 0.00714}{1 - 0.5 \times 0.00714}\right)\right](1 - 0.9 \times 0.00792)$$

$$= 0.0107307$$

4) $$P\,\ddot{a}_{35:\overline{30|}} = 5000000\,A^{1}_{35:\overline{30|}} + 30\,P\,A_{35:\overline{30|}}^{\;\;1}$$

AM92

$$\ddot{a}_{35:\overline{30|}} = 14.35$$

$$A^{1}_{35:\overline{30|}} = A_{35} - v^{30}\,{}_{30}p_{35}\,A_{65}$$

$$= \cancel{0.09468} \quad 0.03257$$

$$A_{35:\overline{30|}}^{\;\;1} = v^{30}\,{}_{30}p_{35}$$

$$= \cancel{0.13322581} \quad \cancel{0.1}$$

$$= (1.06)^{-30} \times \frac{8821.2612}{9894.4299}$$

$$= 0.1332581$$

$$P(14.35 - 30(0.1332581)) = 5000000 \times 0.03257$$

$$P = 16771.98381$$

5) i) $1\,\bar{a}_x$

ii) $PV = \bar{a}_{\overline{T_x|}}$ when $T_x > 0$

$$= \int_0^\infty v^s\,{}_sp_x\,dx$$

6) i) $P = 5000000\, A^{1}_{45:\overline{20}|}$

where

$A^{1}_{45:\overline{20}|} = A_{45:\overline{20}|} - A_{45:\overline{20}|}^{\;\;1}$

$= 0.46998 - (1.04)^{-20}\left(\dfrac{8821.2612}{9801.3123}\right)$

$= 0.059228$

$P = 5000000 \times 0.059228 = 296140.0863$

Single premium = 2961400863

(ii) $V = 5000000\, A^{1}_{46:\overline{19}|} - 296140.0863$

$= 5000000\left[0.34579 - 1.06^{-19}\left(\dfrac{8821.2612}{9716.9324}\right)\right]$

$- 296140.0863$

$= -55694.26417$

$DSAR = 5000000 - (-55694.26417)$

$= 5055694.26417$

$EDS = 10000\, q_{45} \times DSAR = 74065920.97$

$ADS = 50556942.6417$

$MP = EDS - ADS = 23508978.33$

7) $EPV = 100000\, A^{1}_{40:\overline{20}|} + 200000\, A_{60}\, v^{20}\, {}_{20}p_{40}$

$= 100000\left(A_{40} - v^{20}\, {}_{20}p_{40}\, A_{60}\right) + 200000\, v^{20}\, {}_{20}p_{40}\, A_{60}$

$v^{20}\, {}_{20}p_{40} = 0.293802$

$A_{40} = 0.12313$

$A_{60} = 0.32692$

$EPV = 21917.97945$

$EPV^2 = 100000^2\left({}^{2}A^{1}_{40:\overline{20}|}\right) + 200000^2\; {}^{2}A_{60}\, v^{20}\, {}_{20}p_{40}$

$= 100000^2\,(0.01415) + 200000^2$ ______

$EPV^2 = 1798358788$

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$SD = \sqrt{EPV^2 - (EPV)^2}$

$= 36303.73211$

8) $A_{65} = 0.5412$ $\quad A_{60} = 0.5591$ $\quad q_{[65]} = 0.75 \times q_{65}$

A — doubt

9) i) $P = 120000\, a_{70}$

$= 120000(\ddot{a}_{70} - 1)$

$= 1267440$

ii) $P(L_0 > 0) = 1$

~~P(L0 < 0)~~

10) i) $PV = \bar{a}_{\overline{T_x}|}$

$EPV = \int_0^\infty a_{\overline{t}|}\ {}_tp_x\, \mu_{x+t}\, dt = \bar{a}_x$

(ii) $\int_0^\infty e^{-(\mu+\delta)} dt = 16$

$\frac{1}{-(\mu + 0.04)} \left[e^{-(\mu+\delta)t} \right]_0^\infty = 16$

$\mu = 0.0225$

12) $S_{55:\overline{10}|} = a_{55:\overline{10}|} (1+i)^{10}\ {}_{10}p_{55}$

$= 12.57810568$

$\ddot{a}^{(4)}_{[40]:\overline{20}|} = \ddot{a}^{(4)}_{[40]} - v^{20}\ {}_{20}p_{[40]}\ \ddot{a}^{(4)}_{60}$

$= \ddot{a}_{[40]} - \frac{3}{8} - v^{20}\ {}_{20}p_{[40]} \left(\ddot{a}_{60} - \frac{3}{8}\right)$

$= (15.494 - 3/8) - (1.06)^{-20} (11.891 - 3/8) \left(\frac{9287.2164}{9854.3036} \right)$

$= 14.6735$

13) $P\ddot{a}_{45:\overline{20}|} = 100000\, A_{45:\overline{20}|} + 100000\, A_{45:\overline{20}|}^{1}$

$\ddot{a}_{45:\overline{15}|} = 11.886$

$A_{45:\overline{20}|} = 0.46991$

$A_{45:\overline{20}|}^{1} = 0.410952$

$P = 7734.603748$

ii $_{13}V = 100000\, A_{58:\overline{7}|} + 100000\, A_{58:\overline{7}|}^{1} - P\ddot{a}_{58:\overline{2}|}$

$_{13}V = 132603.42\cancel{89}$

$DSAR = 100000 - {}_{13}V$

$= -32603.429$

$EDS = -36657.6653$

$APS = -130413.7158$

$MP = 93756.01046$

~~14) (i) 0 ; $T_{63} \leq 2$; $5000\, \bar{a}_{\overline{T_{63}-2|}}$~~

14) i) $0 \quad ; \quad T_{63} \leq 2$

$5000\, \bar{a}_{\overline{T_{63}-2}|}\, v^2 \quad ; \quad T_{63} > 2$

ii) $5000 \times 100 \times {}_2p_{63} \times v^2\, \bar{a}_{65}$

$500000\,(14.871 - 0.5)\,(1.04)^{-2}\left(\dfrac{9703.108}{9775.888}\right)$

6594347.317

iii) Variance $= 5000^2 \times 100$

$$= \frac{1}{\delta^2}\left(v^{2n}{}_n q_x + \delta v^{2n}{}_n p_x \, {}^2\bar{A}_{x:n} \right) - \left(v^n{}_n q_x + v^n{}_n p_x (\bar{A}_{x+n})^2 \right)$$

$$= 5000^2 \times 100 \times \frac{1}{\delta^2} \left(v^4{}_2 q_{63} + v^4{}_2 p_{63} \, {}^2\bar{A}_{65} - \left(v^2{}_2 q_{63} + v^2{}_2 p_{63} (\bar{A}_{65})^2 \right)^2 \right)$$

$\bar{A}_{65} = 1 - \delta \bar{a}_{65}$

$\bar{A}_{65} = 0.436354$

${}^2A_{65} = (1.04)^{0.5} (0.20847) = 0.212955$

$\text{Var} = 5000^2 \times 100 \times 13.53$

$= (183937.6604)^2$

15) a) $({}_tV + P)(1+i) \times (1+i)^{11/12} - (1+i)^{10/12} - S\, q_{x+t} = {}_{t+1}V\, p_{x+t}$

b) $({}_tV + P)(1+i) - S\, q_{x+t} - R\, a_r\, p_{x+t}$

$= {}_{t+1}V \times p_{x+t}$.