F.M. Assignment

1.) $d^{(4)} = 8\%$.

$$(1-d) = \left(1 - \frac{d^4}{4}\right)^4$$

$$\therefore d = 1 - \left(1 - \frac{d^4}{4}\right)^4$$

$$d = 1 - (1 - 0.02)^4 \Rightarrow 7.763184\%$$

(I) equivalent rate of interest –

$$\delta = -\log(1-d) \qquad \left[\delta = \log\left(\frac{1}{(1-d)}\right)\right]$$

$$\therefore \delta = -\log(1 - 0.7763184)$$

$$\boxed{\delta = 8.08108293\%}$$

(II) equivalent rate per annum :-

$$(1+i)(1-d) = 1$$

$$\therefore i = \frac{1}{1-d} - 1 = \frac{d}{1-d}$$

$$\boxed{i = 8.416578\%}$$

(III) equivalent nominal rate of discount per ~~annum~~ annum –

$$d^{12} = 12 \times \left[1 - (1-d)^{\frac{1}{12}}\right]$$

$$d^{12} = 12 \times \left[1 - (1 - 7.763184\%)^{\frac{1}{12}}\right]$$

$$\boxed{d^{12} = 8.05393\%}$$

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2) $\delta(t) = 0.004t + 0.0002t^2$

(1) Let Amt. due $= C$.
$\therefore C = £1000$

$$PV_0 = C \cdot V(t) \Rightarrow \frac{C}{e^{\int_0^{10} \delta(t)dt}}$$

$$\int_0^{10} 0.004t + 0.0002t^2 \, dt$$

$$= \left(0.002t^2 - \frac{0.002t^3}{3}\right)_0^{10}$$

$$= (0.26666667)$$

$$\therefore PV_0 = 1000 \times e^{-0.2666667}$$

$$\boxed{PV_0 = £765.93.}$$

(ii) $(1+i)^{10} = e^{(0.2666667)}$
$\therefore \boxed{i = 2.702540} \%$

3)
(i) Interest earned on an investment, paid at beginning of a period is 'd'. Interest earned on an investment, paid at the end of the period is 'i'.
$\therefore$ We can discount 'i' from end of period with discounting factor as 'v', we get 'd'.

$$V = \frac{1}{1+i} \quad -(1) \quad \& \quad (1+i)(1-d) = 1$$

$$\therefore 1-d = \frac{1}{1+i} \quad -(2)$$

$$d = \frac{i}{1+i}$$

from ① - $v = \frac{1}{1+i}$

$\therefore$ $\boxed{d = vi}$

(II) [a] $\frac{d^4}{4} = 2.25\%$

$$\left(1 - \frac{d^2}{2}\right)^2 = \left(1 - \frac{d^4}{4}\right)^4$$

$$\therefore \quad d^2 = 2\left[1 - \left(1 - \frac{d^4}{4}\right)^2\right]$$

$$\boxed{d^2 = 8.89875\%}$$

~~(III)~~ [b]

$$d^{1/2} = 5\%$$

$$\left[1 - \frac{d^{1/2}}{1/2}\right]^{1/2} = \left(1 - \frac{d^2}{2}\right)^2$$

$$\therefore \quad d^2 = 2\left[1 - (1 - 2d^{1/2})^{1/4}\right]$$

$$\boxed{d^2 = 5.19925072\%}$$

(III) $v(0, 91/365)$

$$\therefore \quad \frac{1}{(1.05)^{91/365}} = \left(1 - \frac{91}{365}d\right)$$

$$\therefore \quad d = \frac{365}{91}\left[1 - \frac{1}{(1.05)^{91/365}}\right]$$

$$\boxed{d = 4.849462\%}$$

4.) (ii) $i = 8\%$

[a] $d = \dfrac{i}{1+i}$

$d = 7.407407407\%$

$\therefore d^{12} = 12 \times \left[1 - (1-d)^{1/12}\right]$

$\boxed{d^{12} = 7.671478\%}$

[b] $i^{365} = \cancel{365}\, n \left[(1+i)^{\frac{1}{n}} - 1\right]$

$i^{365} = 365\left[(1.08)^{1/365} - 1\right]$

$\boxed{i^{365} = 7.696914\%}$

[c] $\delta = \log(1+i)$

$\delta = \log(1.08)$

$\therefore \boxed{\delta = 7.696104\%}$

[d] $i^{1/2} = \dfrac{1}{2}\left[(1.08)^2 - 1\right]$

$\boxed{i^{1/2} = 8.32\%}$

5.) (i) $d^4 = 6\%$

$d = 1 - \left(1 - \dfrac{d^4}{4}\right)^4$

$d = 1 - (1 - 0.015)^4$

$\boxed{d = 5.8663449\%}$

[a] $i = \frac{d}{1-d}$

$\therefore i = 6.2319315.38\%$

$i^{2} = 2\left[(1+i)^{1/2} - 1\right]$

$\boxed{i^{2} = 6.137752\%}$

[b] $d^{12} = 12\left[1 - (1-d)^{1/12}\right]$

$\boxed{d^{12} = 6.030253\%}$

(11) [a] $i = \frac{20000}{200000} \times 100$

$i = 10\%$

no. of days = 93.

$AV_{\frac{93}{365}} = 20000 \times (1+i)^{93/365}$

$= 200000 \times (1.1)^{91/365}$

$\boxed{AV_{93/365} = ₹204916.3564}$

6.) let retail price $= x$

$\Rightarrow$ if retailler pays cash $\rightarrow \frac{70}{100} \times x$

$\therefore PV = 0.7x$

$\Rightarrow$ If retailler uses 6 mnth credit then -

$AV(1-d)^{n} = 0.75x\,(1-d)^{1/2}$

but since it is equal -

$$0.7x = 0.75x(1-d)^{1/2}$$

$$1-d = \cancel{\left(\frac{0.7}{0.75}\right)} \left(\frac{7}{7.5}\right)^2$$

$$\boxed{d = 12.88889\%}$$

(III) [a] $d^{12} = 12 \times \left[1 - (1-d)^{1/12}\right]$

$$\boxed{d^{12} = 13.719544\%}$$

[b] $i = \dfrac{d}{1-d}$

$\therefore \cancel{\boxed{i = 14.7959184\%}}$

$\therefore i = 14.7959184\%$

$$i^2 = 2\left[(1+i)^{1/2} - 1\right]$$

$$\boxed{i^2 = 14.285714\%}$$

[c] $\delta = \log(1+i)$

$\therefore \boxed{\delta = 13.7985742\%}$

7.) Amt. invested = 20000

$AV_{17\text{ mnths}} = AV_{1.41666667} = 26000$

$AV_n = C \times (1+i)^n$

$\therefore 26000 = 20000(1+i)^{1.41666667}$

$\therefore \boxed{i = 20.34570672\%}$

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(I) $i^4 = 4 \times \left[(1+i)^{1/4} - 1 \right]$

$\boxed{i^4 = 18.955254\%}$

(II) $\left(1 - \frac{d^p}{p}\right)^p = (1-d)$

& $(1-d) = \frac{1}{1+i}$

$\therefore \frac{1}{1+i} = \left(1 - \frac{d^p}{p}\right)^p$

$\therefore \cancel{d^2} = \frac{1}{1+i} = \left(1 - \frac{d^2}{2}\right)^2$

$\therefore d^2 = 2\left[1 - (1.20345 7067)^{-1/2}\right]$

$\boxed{d^2 = 17.6882353\%}$

(III) $\cancel{i_5} = 20000\,(1 + 1.41666667 i_s)$

$\therefore \boxed{i_s = 21.1764706\%}$

(IV) Numerically, 'i' has the highest value & 'd' has the lowest value, yet both will give the same value.

Simple interest is greater than 'i' since there is no accumulation in S.I., and hence be g higher numerical value to compensate for no compounding

9.) (I) Force of interest -
The measure of interest at any given point of time or the interest at any moment is called force of interest.

(II) $i^{(2)} = 7.75\%$

[a] $i = \left(1 + \frac{i^{(2)}}{2}\right)^2 - 1$

$\therefore$ $\boxed{i = 7.900156\%}$

[b] $\delta = \log(1+i) = \log(1.07900156)$

$\boxed{\delta = 7.603613\%}$

[c] $d = \frac{i}{1+i} = \frac{0.07900156}{1.07900156}$

$\boxed{d = 7.32172827\%}$

$d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$

$\boxed{d^{(12)} = 7.5795759\%}$

[d] $i^{(1/2)} = \frac{1}{2}\left[(1+i)^2 - 1\right]$

$\boxed{i^{(1/2)} = 8.212219\%}$

10) $\delta(t) = \begin{cases} 0.08 & , 0 \leq t \leq 5 \\ 0.06 & , 5 \leq t < 10 \\ 0.04 & t \geq 10 \end{cases}$

→ when $0 \leq t \leq 5$,

$$V(t) = e^{-\int_0^t 0.08\, ds}$$

$$V(t) = e^{-0.08t}$$

& $$PV = e^{-0.08t}$$

→ when $5 \leq t < 10$

$$V(t) = e^{-\left(\int_0^5 0.08\, ds + \int_5^t 0.06\, ds\right)}$$

$$V(t) = e^{-(0.06t + 0.1)}$$

$$\therefore PV = e^{-(0.06t + 0.1)}$$

→ when $t \geq 10$,

$$V(t) = e^{-\left(\int_0^5 0.08dt - \int_5^{10} 0.06dt - \int_{10}^t 0.04\, ds\right)}$$

$$V(t) = e^{-(0.06 + 0.1 + [0.04]_{10}^{t})}$$

$$V(t) = e^{-(0.04t + 0.3}$$

$$\therefore PV = e^{-(0.04t + 0.3)}$$

11.) [a.] $n = \frac{9}{12} = 3/4$ yr.

amt. due $= C = ₹50000$

$d = 18\%$

$PV_0 = C(1-d)^n$

$PV_0 = 50000(1-0.18)^{3/4}$

$\boxed{PV_0 = ₹43085.43}$

[b]

(1) $\delta = 6\%$

$e^{-\delta} = \left(1 - \frac{d^{12}}{12}\right)^{12}$

$d^{12} = 12\left[1 - e^{-\delta/12}\right]$

$\boxed{d^{12} = 5.985025}$

(11) $d^4 = 6\%$

$d = 1 - \left(1 - \frac{d^4}{4}\right)^4$

$d = 5.8663449\cancel{2}4$

$i = \frac{d}{1-d} = 6.231931537\%$

$i^4 = 4\left[(1+i)^{1/4} - 1\right]$

$\boxed{i^4 = 6.0913705\%}$

$$i^{12} = 12\left[(1+i)^{1/12} - 1\right]$$

$$\boxed{i^{12} = 6.060709\%}$$

[c] ascending order –

$$d < \delta < i^{p} < i$$

12) $C = ₹7049$

$$AV_6 = C \cdot A(0,6)$$

$$AV_6 = C \cdot A(0,2) \times (A\ 2,4.5) \cdot A(4.5,6)$$

$$\Rightarrow A(0,2) = \left(1 + \frac{6}{12}\%\right)^{2\times12}$$

$$= (1.005)^{24}$$

$$\Rightarrow A(2,4.5) = \left[1 + (4.5-2)\times 4(0.015)\right]$$

$$A(2,4.5) = (1 + 0.15)$$

$$A(2,4.5) = (1.15)$$

$$\Rightarrow A(4.5,6) = \frac{1}{\left(1 - \frac{6}{12}\%\right)^{12\times1.5}} = \frac{1}{(1-0.005)^{18}}$$

$$\therefore AV_6 = (1.005)^{24} \times (1.15) \times \frac{1}{(0.995)^{18}} \times 7049$$

$$\therefore \boxed{AV_6 = ₹9999.8945}$$

13) $c = y$

$AV_n = 2y$

time ~~[illegible]~~ = n yrs

(i) $AV_n = c(1+i)^n$, $i = 10\%$

$$2y = y(1+0.1)^n$$
$$2 = (1.1)^n$$
$$n \log(1.1) = \log 2$$
$$\boxed{n = 7.27 \text{ yrs}}$$

(ii) $AV_n = \dfrac{c}{\left(1 - \frac{d^{12}}{12}\right)^{12n}}$

$$2y = \frac{y}{\left(1 - \frac{d^{12}}{12}\right)^{12n}}$$

$$\left(1 - \frac{d^{12}}{12}\right)^{12n} = \frac{1}{2}$$

$$12n \log\left(1 - \frac{0.1}{12}\right) = \log\left(\frac{1}{2}\right)$$

$$\boxed{n = 6.902 \text{ yrs}}$$