ii) delta = $\frac{dc}{ds}$.

iii) According to Black Scholes model

$\Delta_c = \Phi(d_1)$

$= 0.85993$

iv) ~~delta~~ $\Delta_p = -\Phi(-d_1)$

$= -[1 - \Phi(d_1)]$

$= -1 + \Phi(d_1)$

$= -1 + 0.85993$

$= -0.14007$

# Assignment - 2
## Financial engineering

Q.1. $S = 65$, $\sigma = 25\%$ p.a., $r = 2\%$
$K = 55$ (European Call option).
$T = \frac{1}{2}$

i] Call :- $C_t = S_0 \Phi(d_1) - X e^{-rt} \Phi(d_2)$

$$d_1 = \frac{\log\left(\frac{65}{55}\right) + \left(0.02 + \frac{1}{2}(0.25)^2\right) \times \frac{1}{2}}{0.25 \times \sqrt{\frac{1}{2}}}$$

$d_1 = 1.089957$
$d_2 = d_1 - \sigma\sqrt{T} = 1.089957 - 0.25\sqrt{\frac{1}{2}}$
$= 0.913180$

$\Phi(d_1) = 0.85993$, $\Phi(d_2) = 0.81859$

$C_t = 65 \times 0.85993 - 55 \times e^{-0.02 \times \frac{1}{2}} \times$
$0.81859$

$\boxed{C_t = 11.32098}$

Q.2 . i] delta ($\Delta$) = change in value of option w.r.t. change in price of underlying asset.

vega (V) :- change in option w.r.t. change in the volatility

$$V = \frac{dc}{d\sigma}$$

iii] $S = 55$, $K = 50$, $\sigma = 25\%$
$r = 5\%$, $T = 1$

using Black schole

European ~~Bt~~ call option :-

$$C_t = S_0 \cdot \Phi(d_1) - Xe^{-rt}\Phi(d_2).$$

$$d_1 = \frac{\log\left(\frac{55}{50}\right) + \left(0.05 + \frac{1}{2}(0.25)^2\right) \times 1}{0.25}$$

$d_1 = 0.7062407|$

$d_2 = 0.4562407|$

$\Phi(d_1) = 0.75804$, $\Phi(d_2) = 0.67364$

$$C_t = S\,\Phi(d_1) - Ke^{-rt}\,\Phi(d_2)$$

$$= 55 \times 0.75804 - 50 \cdot e^{-0.05 \times 1} \times 0.67364$$

$$\boxed{C_t = 9.652890}$$

using put call parity.

$$S_0 + P = C_t + Xe^{-rt}.$$

$$P = 9.652890 - 55 + 50 \times e^{-0.05}$$

$$\boxed{P = 2.214361}$$

ii) To show the vega of european call option and europeen put option to be identical,

Using put-call parity equation.

$$S + P = C + Xe^{-rt}$$

differentiating w.r.t. $\sigma$

$$\boxed{\frac{dP}{d\sigma} = \frac{dC}{d\sigma}}$$

i.e. vegas are identical.

C

iv] • delta hedged :-

portfolio for which overall delta i.e. weighted sum of delta of all individual assets equates to zero, is called as delta hedged or delta neutral.

• Vega hedged :-

portfolio for which overall vega i.e. weighted sum of vega of all individual assets equates to zero is called as vega hedged or vega neutral.

Q.3 i] price of derivative in terms of Expectation under Risk Neutral measure

$$C_t = E\left[ C_T \cdot e^{-r(T-t)} \Big/ F_t \right]$$

$C_T$ = ~~price of~~ payoff of the derivative

$C_t$ = current value of derivate.

Sundaram

$S = 50$, $K = 49$, $r = 5\%$.
$\sigma = 25\%$, $T = \frac{1}{2}$.

ii] european call option:-

$C_t = S_0 \Phi(d_1) - Xe^{-rt} \Phi(d_2)$

$d_1 = 0.3441$, $d_2 = 0.1673$
$\Phi(d_1) = 0.6346$, $\Phi(d_2) = 0.5664$

$C_t = 50 \times 0.6346 - 49 \times e^{-0.05 \times \frac{1}{2}} \times 0.5664$

$C_t = 4.66$

iii] price of american call will be same as european call option i.e. 4.66.

As it is not beneficial to excercise American call before maturity. bcz investor will loose future profit potential.

iv] $S_0 + p = c + Xe^{-rt}$.
$p = 4.66 - 50 + 49 \times e^{-0.05 \times \frac{1}{2}}$
$p = 2.45018$

v). • dividend paying stocks will cause the stock price to fall.

- Buyer of the underlying asset will receive the dividend NOT the holder of option.
- Europeen call option price will decreese ~~by~~ bcz by buying the option instead of underlying they forgoes income. So european call option buyer pay's less.
- European put option price will increase as. put buyer will get benefitted from the fall in price
- American call option will be higher than european ~~due~~ due to opportunity of early excercise.

Q.5 i] delta under Black scholes model

$\Delta = \Phi(d_1)$

ii] $S_0 = 40$, $r = 2\%$, $K = 45.91$
$t = 5$, $\Delta_c = 0.6179$
$\sigma = ?$

Assuming $\sigma = 20\%$

$C_t = S_0 \Phi(d_1) - Xe^{-rt} \Phi(d_2)$

$$d_1 = \frac{\ln\left(\frac{40}{45.91}\right) + \left(0.02 + \frac{1}{2} \times (0.20)^2\right) \times 5}{0.2 \times \sqrt{5}}$$

$d_1 = 0.1390755$
$d_2 = -0.308138$

$\Phi(d_1) = 0.55172$,
$\Phi(-d_2) = 1 - \Phi(0.308138)$
$\Phi(d_2)$ $1 - 0.61791 = 0.38209$

$\boxed{C_t = 6.196366}$

$\boxed{\Delta_c = 0.55172}$ For, $\sigma = 0.20\%$

~~So, For $\Delta_c = 0.6179$~~

~~$\Delta_c = \Phi(d_1)$~~

~~So, $d_1 = 0.30$, $d_2 =$~~

~~$\Phi(d_1) = 0.6179$~~

~~$\Phi(d_2) =$~~

Assuming $\sigma =$ ~~20%~~ 35%

$d_1 =$ ~~0.211883~~ 0.34300

$d_2 =$ ~~-0.347133~~ -0.439615

~~$\Phi(d_1) = 0.57926$~~

~~$\Phi(d_2) = 1 - \Phi(0.347133)$~~

~~$= 1 - 0.63307$~~

~~$= 0.36693$~~

~~$\Delta_c = 0.579$~~

$\Phi(d_1) = 0.63307$

$\Phi(d_2) = 1 - \Phi(0.439615)$

$= 1 - 0.66640$

$\Phi(d_2) = 0.3336$

$\boxed{\Delta_c = 0.63307}$ for, $\sigma = 35\%$

$\Delta_c = 0.55172$ for $\sigma = 20\%$

$\Delta_c = 0.63307$ for $\sigma = 35\%$

$\Delta_c = 0.6179$ $\sigma = ?$

By Interpolation

$\boxed{\sigma = 32.2\%}$

iii) As Both Stocks are independent of each other.
So, By Risk Neutral measure.

$$\mathbb{Q}\left[\frac{S_1}{S_0} < K_S\right] \cdot \mathbb{Q}\left[\frac{R_1}{R_0} < K_R\right] \cdot Ce^{-rT}.$$

iv) So if the stocks are perfectly correlated.

$$\mathbb{Q}\left[S_1/S_0 < K_S\right] Ce^{-rt} \quad \text{--- (1)}$$

$$\mathbb{Q}\left[R_1/R_0 < K_R\right] \cdot Ce^{-rt} \quad \text{--- (2)}$$

Q.6 $S = 6.4$, , $X = 241830$.

$X$ = units of underlying

$y$ = units of derivative

$T = 1$

$y = 100{,}000 \Rightarrow$ ~~un~~ european put (written).

$X = 6.3$

$r = 3\%$ p.a.

~~Portfolio A = −100,000 × 6.4~~ 6.3

~~Portfolio B = Cash = 24830 × 6.4~~

i) a) delta of the put option.

$$\Delta p = -1 + \Phi(d_1)$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$\Delta p = \Phi(d_1) - 1$$

b) In this case,

$$100000 \times \Delta p = -24830$$

$$\Delta p = \frac{-24830}{100{,}000} = -0.25$$

11) To. volatility is $7.1\%$

$$\Delta p = -0.25 \quad (-0.24830)$$

$$\Delta p = \Phi(d_1) - 1$$

$$-0.24830 + 1 = \Phi(d_1)$$

$$\Phi(d_1) = 0.7517$$

$$d_1 = 0.68$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right).T}{\sigma\sqrt{T}}$$

$$0.68.\sigma = \ln\left(\frac{6.4}{6.3}\right) + \left(0.03 + \frac{1}{2}\sigma^2\right)$$

$$0.68\sigma = 0.015748 + 0.03 + \frac{1}{2}\sigma^2$$

$$0.68\sigma = 0.045748 + \frac{1}{2}\sigma^2$$

$$0.5\sigma^2 - 0.68\sigma + 0.045748 = 0$$

$$\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{0.68 \pm \sqrt{}(0.60901)}{2 \times 0.5} = \frac{0.68 - 0.60901}{1}$$

$$= \cancel{7\%}\ \cancel{7.99\%}\ 7.099\%$$

$$= 7.01\%$$

iii] a] ~~price of put~~ european put

$$P_t = K.e^{-rt}\,\Phi(-d_2) - S_0\,\Phi(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{6.4}{6.3}\right) + \left[0.03 + \frac{1}{2}(0.071)^2\right]}{0.071}$$

$d_1 = 0.67984$ , $d_2 = 0.608843$

~~$\Phi(d_1)$~~ $\Phi(-d_1) = 1 - \Phi(d_1)$
$= 1 - \Phi(0.67984)$
$= 1 - 0.74857$
$= 0.25143$

$\Phi(-d_2) = 1 - \Phi(0.608843)$
$= 1 - 0.72575$
$= 0.27425$

$P_t = 6.3 \times e^{-0.03} \times 0.27425 - 6.4 \times 0.25143$

$= 0.0675598$
$= 6.7559\,p.$

Option price $= 6.7559\,p.$

Q.7 i] delta :- change in option price w.r.t. change in price of underlying.

gamma :- change in deta w.r.t. change in underlying asset.

vega :- change in price of option w.r.t change in volatility.

ii] $\Delta = \Phi(d_1)$.

$$d_1 = \frac{\ln\left(\frac{60}{50}\right) + \left[0.03 + \frac{1}{2}(0.25)^2\right] \times 3}{0.25 \times \sqrt{3}}$$

$d_1 = 0.845406$

$d_2 = 0.41239$.

$\Delta = \Phi(d_1) = 0.79955$

iii] delta hedging.

delta = 0.801

As selling european put option ⇒ so change ~~..~~ occur in price of option.

$17.91 - 0.801 \times 60 = -30.15$

-30.15 short in ~~..~~ cash.

Q.10 portfolio which is long one call plus cash of $Ke^{-rT}$ and short one put.

The portfolio has a payoff at time of expiry of $S_T$.

$$C + Ke^{-r(T-t)} - P_t = S_t$$