Financial Mathematics

Assignment - 2

1]

Amount of Loan = 20,000

Monthly payments = ₹ 427.90

$$\therefore 20{,}000 = 12 \times 427.90 \cdot a^{(12)}_{\overline{5|}}$$

$$\therefore a^{(12)}_{\overline{5|}} = \frac{20{,}000}{12 \times 427.90} = 3.8949910$$

$$\therefore \frac{1 - v^5}{12\left[(1+i)^{1/12} - 1\right]} = 3.8949910$$

$$\therefore 1 - v^5 = 3.8949910 \left\{ 12\left[(1+i)^{1/12} - 1\right] \right\}$$

$$\therefore 1 - v^5 - 3.8949910 \times \left(12\left[(1+i)^{1/12} - 1\right] \right) = 0$$

$$\therefore 1 - (1+i)^{-5} - 46.7398\left[(1+i)^{1/12} - 1\right] = 0$$

$\therefore$ By linear interpolation

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$i = 10.8\%.$$

ii) APR = 10.8%. $i^{(12)} = 10.2996\%$.

~~Loan outstanding after~~ ~~1~~ repaid in 1st year,

Loan to be repaid after 1st year

$$= 12 \times 427.90 \, a^{(12)}_{\overline{4|}} \text{ @ } 10.8\%$$

$$= 12 \times 427.90 \times \frac{1 - v^4}{i^{(12)}}$$

$$= ₹\, 16775.98.$$

New monthly payments are of 274.49 which are made in arrears
Let the new term of the loan be 'n'

$\therefore 16775.98 = 12 \times 274.49 \cdot a^{(12)}_{\overline{t|}}$

$\therefore a^{(12)}_{\overline{t|}} = 5.093051$

$\therefore \frac{1-(1.108)^{-t}}{0.102996} = 5.093051$

$(1.108)^{-t} = 1-(5.093051 \times 0.102996)$

$\therefore (1.108)^{-t} = 0.475436$

$-t \cdot \ln(1.108) = \ln(0.475436)$

$\therefore -t = \frac{\ln(0.475436)}{\ln(1.108)}$

$\therefore t = 7.2498 \approx 7.25$ years.

The orignal loan had 4 years of payment remaining.

$\therefore$ Total amount paid in 4 years $= 12 \times 427.90 \times 4$

$= 20539.2$

Loan outstanding after 4 years $= 16775.98$

$\therefore$ Total interest paid would have been $= 20539.2 - 16775.98$

$= 3763.22$

~~Ne~~

New loan's term is 7.25 years.

$\therefore$ Total interest paid after restructuring $= (7.25 \times 12 \times 274.99) - 16775.98$

$= 7104.65$.

$\therefore$ Mr. & Mrs. Jones will have to pay ₹ 3341 more interest.

Q.2]

i) The discounted payback period

2) The payback period is the time required to recover the initial cost of an investment.

ii)

iii)

Cashflows of Project A.

Outgo.	Time 0	-	$-170{,}000$
	End of Year 1	-	$20{,}000$
	-11- Year 2	-	$10{,}000$

Inflow.	End of Year 1	-	$20{,}000$
	-11- 2	-	$20{,}000$
	End of Year 3	-	$200{,}000$.

$\therefore$ IRR

$$-170{,}000 - 20{,}000(1+i)^{-1} - 10{,}000(1+i)^{-2} + 20{,}000\, a_{\overline{2}|} + 200{,}000(1+i)^{-3} = 0$$

Solving in (000's)

$$\therefore -170 - 20(1+i)^{-1} - 10(1+i)^{-2} + 20\left[\frac{1-(1+i)^{-2}}{i}\right] + 200(1+i)^{-3} = 0$$

$\therefore$ By linear interpolation

$IRR_A =$

Date

Written in (000's)

$-170-20(1+i)^{-1}-10(1+i)^{-2}+20(1+i)^{-1}+20(1+i)^{-2}+200(1+i)^{-3}$

$\therefore$ By linear interpolation,

$IRR_A = 7.4\%$

Project B.

Outgo		Inflow		
Time 0	-200,000	End of Year	1	
			2	
			3	14,000
			4	
			5	
			6	
		End of Year 6		200,000

$\therefore IRR_B$ (In 000's)

$$\therefore -200+14a_{\overline{6}|}+200(1+i)^{-6}=0$$

$$\therefore -200+14\left[\frac{1-(1+i)^{-6}}{i}\right]+200(1+i)^{-6}=0$$

$\therefore$ By linear interpolation.

IRR

Risk discount rate ie $i = 6\%$

$$\therefore NPV_A = -170{,}000 - \overset{000}{20}(1.06)^{-1} - \overset{000}{10}(1.06)^{-2} + \overset{000}{20}(1.06)^{-1} + \overset{000}{20}(1.06)^{-2} + 200(1+i)^{-3}$$

$$= Rs\,6823.82$$

$$NPV_B = -200{,}000 + 14\left[\frac{1-(1.06)^{-6}}{0.06}\right] + 200(1.06)^{-6}$$

$$= -200{,}000 + 68842.54 + 140992.10$$
$$= Rs\,9834.64$$

Q.3] Value of all annuities on 1/11/1985 is as follows:-

$$\text{Annuity } 1 = 200\, \ddot{a}_{\overline{22|}}\, v^{\left(\frac{1}{4}\right)} \quad @\ 8\%$$

$$= 200 \times 0.980944 \times 11.01676$$
$$= 2161.36$$

$$\text{Annuity } 2 = 320 \times (1+i)^{1/12} \times a^{(4)}_{\overline{16.25|}}$$

$$= 320 \times 1.0064 \times 9.1842$$
$$= 2957.872$$

$$\text{Annuity } 3 = 180 \times a^{(12)}_{\overline{18.75|}}$$

$$= 1780.665$$

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Total = 6899.90

X - be the revised annuity payment.

$\therefore$ Revised Amount $= X\ (1+i)^{1/4}\ a^{(2)}_{\overline{21.5}|}$

$$\therefore X = \frac{6899.90}{1.019427 \times 10.30886}$$

$= 656.56$

$\therefore$ Revised amount of annuity is ₹656.56.

Q.4] $(I\ddot{a})_{\overline{n}|}$ - Increasing annuity due for n year

$\downarrow$

$PV_0 = 1 + 2v + 3v^2 + 4v^3 + 5v^4 \ldots\ldots nv^{(n-1)}$ — ①

Multiplying v on both sides.

$\therefore v(I\ddot{a})_{\overline{n}|} = v + 2v^2 + 3v^2 \ldots\ldots nv^n$ — ②

$\therefore$ By subtracting equation ② from ①

$(1-v)(I\ddot{a})_{\overline{n}|} = 1 + v + v^2 + v^3 + v^4 \ldots + v^{n-1} + nv^n$

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$\therefore (1-v)(I\ddot{a})_{\overline{n|}} = \left[\dfrac{1-v^n}{(1-v)}\right] - nv^n$

$d(I\ddot{a})_{\overline{n|}} = \ddot{a}_{\overline{n|}} - nv^n \quad \because 1-v = d$

$$\boxed{\therefore I\ddot{a}_{\overline{n|}} = \frac{\ddot{a}_{\overline{n|}} - nv^n}{d}}$$

Q.5] i) TWRR for the year 2014 is as follows

for property fund

$\therefore (1+i) = \dfrac{16.4}{12.4}$ — [Start of Year 2015 i.e end of 2014) — [Start of Year 2014]

$\therefore i = \dfrac{16.4}{12.4} - 1$

$= 0.3226$

$= 32.26\%$

for equity fund

$(1+i) = \dfrac{15.5}{12.1}$

$\therefore i = \dfrac{15.5}{12.1} - 1 = ~~0.32810~~ ~~0.28~~ 0.28099$

$= ~~28.10\%~~ \; 28.099\%$

ii) Let us assume the ~~amount of~~ ~~1000~~ ~~invest~~ units are bought.

$\therefore 1240(1+i) + 1310(1+i)^{3/4} + 1480(1+i)^{1/2} + 1580(1+i)^{1/4}$

$= 1640 \times 4$

$\therefore$ By linear interpolation $i = 29.32\%$

iii) Assume that ₹ 100 is invested each time at start ~~of ea~~

$\therefore$ 4 quarters = 400 p.a.

$$\therefore 400\,\ddot{s}^{(4)}_{\overline{1|}} = 100 \times 16.4\left(\frac{1}{12.4} + \frac{1}{13.1} + \frac{1}{14.8} + \frac{1}{15.8}\right)$$

$$\therefore \ddot{s}^{(4)}_{\overline{1|}} = 1.1801$$

$$\therefore i = 0.2979$$
$$= 29.79\%.$$

iii) a)

$$1210(1+i) + 920(1+i)^{3/4} + 1030(1+i)^{1/2} + 1310(1+i)^{1/4} = 4 \times 1550$$

$\therefore$ By linear interpolation

$$i = 67.31\%.$$

b) $$400 \times \ddot{s}^{(4)}_{\overline{1|}} = 100 \times 15.5\left(\frac{1}{12.1} + \frac{1}{9.2} + \frac{1}{10.3} + \frac{1}{13.1}\right)$$

$$\therefore \ddot{s}^{(4)}_{\overline{1|}} = 1.4135$$

$$\therefore i = 0.7088$$
$$= 70.88\%.$$

Q.6] Loan amount = 160,000

term = 10 years.

i = 8%

Level annuity.

X – Annual repayment amount.

$\therefore 160{,}000 = X \cdot a_{\overline{10|}} @ 8\%$.

$\therefore X = \dfrac{160{,}000}{6.7101}$

$\therefore X = 23844.65$.

ii) After 4^{th} repayment Capital outstanding is

$23844.65 \cdot a_{\overline{6|}} @ 8\% = 110231.44$.

New interest $i = 10\%$.

X – new installment

$\therefore 110231.44 = X \cdot a_{\overline{6|}} @ \cancel{8\%}, 10\%$.

$\therefore X = 25{,}309.72$.

iii) Loan outstanding after 7^{th} repayment.

$25309.72 \times a_{\overline{3|}} @ 10\%$.

$= 62942.75$

$\therefore i = 9\%$.

$\therefore$ New installment $\times a_{\overline{3|}} @ 9\% = 62942.75$

$\therefore$ New installment $= 24865.78$

20.51%

Q.1] i) 1) Investing in property requires availability of huge amount of funds whereas relatively investing in government securities is lot cheaper.

2) The property needs to be maintained and can cost a lot to do it, whereas no such thing is required for government securities.

3) Government securities are considered to be a risk-free investment.

4) The return on property may be lower than the return on securities.

ii) Advantage.

Government knows the prices beforehand can do the appropriate planning for it

Disadvantage.

It won't be able to meet its financial requirement i.e it may lie Govt default.

iii)

1) Index-linked bond - Returns are linked to a certain index

2) Equity - Dividend is not payable everytime

3) Property - Rental payments are uncertain

Date

Q.8] Purchase price = 2.7 crores.

Annual operation cost = 0.2 cr.

Rig will be sold at 0.1cr.

∴ i)

$2.7 + 0.2 a_{\overline{3|}}$ → P.V of outgo.

Expected present value of income $= 0.1v^3 + \bar{a}_{\overline{10|}} (0.25 + 0.2v^2 + 0.1v^3)$

∴ By equating both.

$2.7 + 0.2 a_{\overline{3|}} = 0.1v^3 + \bar{a}_{\overline{10|}} (0.25 + 0.2v^2 + 0.1v^3)$

∴ By linear interpolation,

$i = 9.3\%$.

ii) $i = 10\%$.

Oil strike occur in 3rd year.

$\therefore NPV = 0.1v^3 + 0.85\bar{a}_{\overline{10|}} v^3 - 2.7 - 0.2 a_{\overline{3|}}$

$= 0.1(1.1)^{-3} + 0.8\left(\frac{1-(1.1)^{-3}}{0.1}\right)(1.1)^{-3} - 2.7 - 0.2\left(\frac{1-(1.1)^{-3}}{0.1}\right)$

$= 7100089$.

The investment has a positive NPV, so the investment can be done.

Q.9] 1) TWRR.

$(1+i)^3 = \frac{33}{30.5} \times \frac{41.05 - 4.5}{33+6} \times \frac{45.6}{41.05} \times \frac{47}{45.6 - 2.5}$

$= 1.228313$

$i = (1.228313)^{1/3} - 1$

$\therefore i = 7.0951\%$

MWRR.

$30.5(1+i)^3 + 6 \times (1+i)^{2.25} + 4.5(1+i)^{1.5} - 2.5(1+i)^{65}$

$= 47$

$\therefore$ By linear interpolation

~~i~~ $i = 7.20\%$.

ii) Linked rate of return.

iii) ~~Money rated~~ TWRR is more useful than MWRR because it takes account the time value of the money. The timing of outflow and inflow and the amount is not in the hands of the fund manager. ~~The~~ Due to this TWRR is a better judge of the performance

Q.10] $\therefore$ Loan amount $= 80\%$ of X $= 0.8X$.

'X' be the ~~loan amount~~ house cost.

$i = 12\%$.

$\therefore$ Amount of loan granted is as follows.

$$0.8X = 1{,}80{,}000\, a_{\overline{15|}} + 20{,}000 (Ia)_{\overline{15|}}$$

ii) Loan outstanding after 8^{th} repayment

2029
2015
(14)

Q.13 Loan amount = 988,000

Let X be the monthly repayment

i = 7%

n = 25 years

$$\therefore 988,000 = X\, a^{(12)}_{\overline{25|}} \quad @ 7\%$$

$$988000 = X\, a_{\overline{25|}} \times \frac{i}{i^{(12)}}$$

$$\therefore X = \frac{988000}{12.022914}$$

$$= ₹82176.42$$

∴ Monthly installment is of ₹ 82176.42

ii) Loan outstanding after 10/3/2029

$$82176.42 \cdot a^{(12)}_{\overline{11|}} \quad v^{3/12}$$

iii)