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Homework 1 - Lagrange's interpolation

1. $y = f(x)$ (7, 3) (8, 1) (9, 1) (10, 9)

y at $x = 9.5$ $f(9.5)$

x	7	8	9	10
y	3	1	1	9

$$f(9.5) = \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} \times 3$$

$$+ \frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)} \times 1$$

$$+ \frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)} \times 1$$

$$+ \frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)} \times 9$$

$$= 0.0000093626 - 0.9375$$

$$= 0.1875 - 0.3125 + 0.9375 + 2.8125$$

$$= \boxed{4.255}$$

2.

x	3	7	9	10
y	168	120	72	63

$$f(6) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} \times 168 +$$

$$\frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} \times 120 + \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)} \times 72$$

$$+ \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} \times 63$$

$$= 12 + 360 - 72 - 27$$

$$= \boxed{273}$$

3.

x	1	2	7	8
y	4	5	5	4

$$f(6) = \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)} \times 4 + \frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)} \times 5$$

$$+ \frac{(6-7)(6-2)(6-8)}{(7-1)(7-2)(7-8)} \times 5 + \frac{(6-1)(6-2)(6-7)}{(8-1)(8-2)(8-7)} \times 4$$

$$= -0.38095 - 1.66666 - 6.666 + 1.90476$$

$$= \boxed{-6.80945}$$ ——— feels wrong.

4.

x	1	2	3	4	7
y	2	4	8	16	128

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2$$

$$+ \frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4$$

$$+ \frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)} \times 8$$

$$+ \frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)} \times 16$$

$$+ \frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128$$

$$= -0.666667 + 6.4 - 24 + 42.66667 + 8.5333$$

$$= \boxed{32.9333}$$

5.

x	10	20	14	16
y	0.25	0.2	0.15	0.10

$$f(15) = \frac{(15-20)(15-14)(15-16)}{(10-20)(10-14)(10-16)} \times 0.25$$

$$+ \frac{(15-10)(15-14)(15-16)}{(20-10)(20-14)(20-16)} \times 0.2 + \frac{(15-10)(15-20)(15-16)}{(14-10)(14-20)(14-16)} \times 0.15$$

$$+ \frac{(15-10)(15-20)(15-14)}{(16-10)(16-20)(16-14)} \times 0.10$$

$$= -0.0052083 - 0.0041667 + 0.078125 + 0.0520833$$

$$\Rightarrow \boxed{0.120853}$$

Homework 2: Regula falsi method.

1. $x^3 + x - 1 = 0 = f(x)$ $\quad 0.5 < x < 1$

	x	y	
	0.5	-0.375	
x_0 ←	0.6	-0.184	→ y_0
x_1 ←	0.7	0.043	→ y_1

$$x_2 = \frac{(0.6)(0.043) - (0.7)(-0.184)}{0.043 - (-0.184)}$$

$$= 0.681057$$

$y_2 = \boxed{-0.00304180}$ — replaces y_0

$$x_3 = \frac{(0.681057)(0.043) - (0.7)(-0.00304180)}{0.043 - (-0.00304180)}$$

$$= 0.682304848$$

$y_3 = \boxed{-0.000055\,0366}$

∴ root is $\boxed{x = 0.68230484}$

2. $f(x) = x^3 - 2x + 0.5 = 0$ $\quad 0 < x < 1$

	x	y	
	0.1	0.801	
$x_0 \leftarrow$	0.2	0.108	$\longrightarrow y_0$
$x_1 \leftarrow$	0.3	−0.073	$\longrightarrow y_1$

$$x_2 = \frac{(0.2)(0.073) - (0.3 \times 0.108)}{(-0.073) - (0.108)}$$

$$= 0.259\,668\,5083$$

$y_2 =$ $\boxed{-0.001\,828\,1574}$ replaces y_1

$$x_3 = \frac{(0.2)(-0.001\,828\,1574) - (0.259\,668\,5083)(0.108)}{-0.001\,828\,1574 - 0.108}$$

$$\Rightarrow 0.258\,674\,8734$$

$y_3 =$ $\boxed{-0.000\,041\,1152}$

$\therefore$ root is $\boxed{x = 0.258\,674\,8734}$

3 $x^3 + 2x - 20 = 0 = f(x)$ $\quad 2 \leq x \leq 2.5$

	x	y
	2.3	-3.233
$x_0 \leftarrow$	2.4	$-1.376 \longrightarrow y_0$
$x_1 \leftarrow$	2.5	$0.625 \longrightarrow y_1$

$$x_2 = \frac{(2.4 \times 0.625) - (2.5)(-1.376)}{0.625 - (-1.376)}$$

$$= 2.468765817 2$$

$y_2 = \boxed{-0.015827014 9}$ — replaces y_0

$$x_3 = \frac{(2.4687 \times 0.625) - (2.5)(-0.0158270)}{0.625 - (-0.0158270)}$$

$$= 2.469537038$$

$y_3 = \boxed{-0.000174790 5}$

$\therefore$ root is $\boxed{x = 2.469537038}$

4. $f(x) = 3x - \cos x - 1 = 0$ $\quad 0 < x < 1$

	x	y	
	0.5	+0.4999619231	
$x_0 \leftarrow$	0.6	−0.1999457694	$\longrightarrow y_0$
$x_1 \leftarrow$	0.7	0.1000746303	$\longrightarrow y_1$

$$x_2 = \frac{(0.6 \times 0.1000746) - (0.7)(-0.19994576)}{0.1000746 - (-0.19994576)}$$

$$= 0.6666439914$$

$y_2 = \boxed{-0.0000003384}$

$\therefore$ root is $\boxed{x = 0.6666439914}$ Ans.

5. $f(x) = x^3 - 3x - 5 = 0$ $\quad 2 < x < 2.5$

	x	y	
	2.1	−2.039	
$x_0 \leftarrow$	2.2	−0.952	$\longrightarrow y_0$
$x_1 \leftarrow$	2.3	0.267	$\longrightarrow y_1$

$$x_2 = \frac{(2.2 \times 0.267) - (2.3)(-0.952)}{0.267 - (-0.952)}$$

$$= 2.2780968007$$

$y_2 = \boxed{-0.0715944071}$ replaces y_0

$$x_3 = \frac{(2.278096 \times 0.267) - (2.3)(-0.011594)}{0.287 - (-0.011594)}$$

$$= 2.279011107$$

$$y_3 = \boxed{-0.000009661}$$

∴ root is $\boxed{x = 2.279011107}$ Ans.

6. $f(x) = x \log_{10} x - 1.2 = 0 \qquad 2 \le x \le 3$

	x	y	
	2.6	−0.121069293	
$x_0 \leftarrow$	2.7	−0.0353178	$\rightarrow y_0$
$x_1 \leftarrow$	2.8	0.052042 4	$\rightarrow y_1$

$$x_2 = \frac{(2.7 \times 0.0520424) - (2.8)(-0.0353178)}{0.0520424 - (-0.0353178)}$$

$$= 2.792154119$$

$y_2 = \boxed{0.0457314 9}$ → replaces y_1

2.792154119

$$x_3 = \frac{(2.7 \times 0.0457314 9) - (2.792154119)(-0.0355317 8)}{0.0457314 9 - (-0.0353178)}$$

$$= 2.740456802$$

$$y_3 = \boxed{-0.000016552}$$

∴ root is $\boxed{x = 2.740456302}$ Ans.

4. $f(x) = x^3 - 4x + 1 = 0$

	x	y	
	1.7	−0.887	
$x_0 \leftarrow$	1.8	−0.368	$\longrightarrow y_0$
$x_1 \leftarrow$	1.9	0.259	$\longrightarrow y_1$

$$x_2 = \frac{(1.8 \times 0.259) - (1.9)(-0.368)}{0.259 - (-0.368)}$$

$$= 1.858692185$$

$y_2 = \boxed{-0.013476748}$ → replace y_0

$$x_3 = \frac{(1.858692185 \times 0.259) - (1.9)(-0.013476748)}{0.259 - (-0.013476748)}$$

$$= 1.860735277$$

$y_3 = \boxed{-0.000450?}$

∴ root is $\boxed{x = 1.860735277}$

8. $f(x) = x^3 + 2x^2 - 8 = 0$

	x	y	
	1.4	−1.336	
$x_0 \leftarrow$	1.5	−0.125	$\rightarrow y_0$
$x_1 \leftarrow$	1.6	1.216	$\rightarrow y_1$

$$x_2 = \frac{(1.5 \times 1.216) - (1.6)(-0.125)}{1.126 - (-0.125)}$$

$$= 1.509321401$$

$y_2 = \boxed{-0.005586540}4$ → replaces y_0.

$$x_3 = \frac{(1.5093214 \times 1.216) - (1.6)(-0.00558654)}{1.216 - (-0.00558654)}$$

$$= 1.509736089$$

$y_3 = \boxed{-0.0002477}$

∴ root is $\boxed{x = 1.509736089}$

9. $f(x) = x^3 - 9x + 1 = 0$

x	y	
0	1	
$x_0 \leftarrow 0.1$	0.101	$\longrightarrow y_0$
$x_1 \leftarrow 0.2$	-0.792	$\longrightarrow y_1$

$$x_2 = \frac{(0.1)(-0.792) - (0.2 \times 0.101)}{-0.792 - 0.101}$$

$$= 0.1113101904$$

$$y_2 = \boxed{-0.00041258}$$

$\therefore$ root is $\boxed{x = 0.1113101904}$

10. $f(x) = x^3 - x - 4 = 0$

x	y	
1.6	-1.504	
$x_0 \leftarrow 1.7$	-0.787	$\longrightarrow y_0$
$x_1 \leftarrow 1.8$	0.032	$\longrightarrow y_1$

$$x_2 = \frac{(1.7 \times 0.032) - (1.8)(-0.787)}{0.032 - (-0.787)}$$

$$= 1.796092796$$

$y_2 = \boxed{-0.000198843999}$ nearer to y_0

$$x_3 = \frac{(1.796092796 \times 0.032) - (1.8)(-0.00198843 99)}{0.032 - (-0.00198843 99)}$$

$$= 1.796321381$$

$$y_3 = \boxed{-0.00000453}$$

$\therefore$ One root is $\boxed{x = 1.796321381}$ Ans