Q1]

	0	1	2	3
2001	1245	999	610	356
2002	1455	1088	723	
2003	1567	1079		
2004	1491			

index
2001
2002
2003
2004

$(1.021) \times (1.012) \times (0.992) = 1.0249$

$1.012 \times 0.992 = 1.003904$

0.992

1

2001
2002
2003
2004

Incremental claims

	0	1	2	3
2001	1276.11	1002.9	6057.15	356
2002	1460.68	1079.3	723	
2003	1554.46	1079		
2004	1491			

Cumulative claims

	0	1	2	3
2001	1276.11	2279.01	2884.13	3240.13
2002	1460.68	2539.98	3262.98	
2003	1554.46	2653.46		
2004	1491			

	0	1	2	3
Total claims	5782.25	7452.45	6147.11	3240.13
Last claim	1491	(2633.46)	(3268.48)	(0)
	4291.25	4818.99	2884.13	3240.13
Df	1.73666	1.2756		1.12343

	0	1	2	3
2001	1276.11	2279.01	2884.13	3240.13
2002	1460.68	2539.98	3262.98	3665.74
2003	1554.46	2633.46	3359.24	3773.89
2004	1491	2589.36	3302.99	3710.69

	1	2	3
Inflation	1098.36 $\times$ 1.025	712.64 $\times$ 1.050625	407.4 $\times$ 1.076891
Inflation adj	1125.819	748.72	439.05

Total Claims = 2313.5875 (in 000s)

(iii) Ultimate amt for 2004 policies = 5250000 $\times$ 0.73

= ~~3937500~~

Cumulative DFs	–	2.48891	1.43304	1.12343
Reverse	1	1.2343	1.43304	2.48871
1/f	1	0.81017	0.6978	0.4018
1 - 1/f	0	1.8983	0.3022	0.5982

1   1165.51 $\times$ 1.025 = 1194.65

2   757.24 $\times$ 1.050625 = 795.59

3   432.61 $\times$ 1.076891 = 465.87

Total emerging liability = 2456.09 (in 000's)

Q4) (i)

AY	DY 0	1	2
2005	82	121	156
2006	128	163	
2007	98		

(ii)

AY	DY 0	1	2
2005	43512	87210	129563
2006	62889	116723	
2007	73876		

ACPC method:

AY	DY 0	1	2	Ultimate
05	530.634	720.74	830.532	830.532
06	491.320	716.092		825.1714
07	753.836			1221.4595

Ultimate claim nos.

	0	1	2	Ultimate
05	~~8~~88	121	156	156
06	~~88~~128	163		210.19
07	~~128~~98			172.7237

Projected ult. loss:

05 830.532 × 156 = 129562.99

06 1221.4595 × 172.7237 = 210,975.0042

07 825.1714 × 210.199 = 173,408.9445

Total = 513946.9407

∴ O/s claims = 513946.9407 − [73876 + 116723 + 129563]

= $\boxed{193,784.9384}$

Q6 (i) This is a heterogeneous group since parameters vary by patient, not the overall level.

(ii)

a) $E(\lambda_i) = 0.5 \times 0.1 + \frac{1}{3} \times 0.3 + \frac{1}{6} \times 0.9 = 0.3$

$E(\lambda_i^2) = 0.5 \times 0.1^2 + \frac{1}{3} \times 0.3^2 + \frac{1}{6} \times 0.9^2 = 0.17$

$Var(\lambda_i) = 0.17 - 0.3^2 = 0.08$

(b) $E(S_i|\lambda_i] = [\lambda_i m_1) \cdot [Var(S_i|\lambda_i]] - [\lambda_i m_2]$

(c) $E(S_i) = E[E(S_i|\lambda_i)] = E[\lambda_i m_1] = 0.3 m_1 = 0.3 \times 250$
$= 75$ //

(d) $\because$ ~~S~~ Variables are independent & identically distributed, mean & variance are just 100×75 & 100×23810

mean = ~~7~~,500

Variance = 2381000

(iii) Now homogeneous.

$Var(\lambda_i) = 0.5(0.2^2 + 0.4^2) - 0.3^2 = 0.01$

$E[\sum_{i=1}^{n} S_i] = n E(S_i) = 0.3 \times 100 \times 250 = 7500$

$Var[\sum S_i] = E\left[Var(\sum S_i|\lambda)\right] + Var\left[E[\sum_{i=1}^{n} S_i|\lambda]\right]$

$= E[n \lambda m_2] + Var[n \lambda m_1]$

$= 0.3 \times 100 \times 200 + 0.01 \times 100^2 \times 250^2$

$= 6256000$

(iv) Mean is same but variance is very high. This is because parameter is at overall level rather than individual.

$$l'' = \frac{22}{\lambda^2} < 0$$

It is a maxima.

(iii) a) $$\hat{\lambda} = \frac{22}{13500 + 17000/1.1 + 4\times 1600 + 6\times 1{,}600/1.1} = 0.000499$$

b) $$\int_0^M x\lambda e^{-\lambda x}\,dx + M\int_M^\infty \lambda e^{-\lambda x}\,dx = \frac{1}{\lambda}(1-e^{-\lambda M})$$

$$= \frac{1}{0.000499}(1-e^{-0.000499\times 1600})$$

$$= 1102.107$$

Claims for 2008 $\sim \exp(\mu)$

$$\mu = \frac{\lambda}{1.05\times 1.1}$$

$$1102.107 = \int_0^R x\mu e^{-\mu x}\,dx + R\int_R^\infty \mu e^{-\mu x}\,dx$$

$$= \frac{1}{\mu}(1-e^{-\mu R})$$

$$= \frac{1.05\times 1.1}{0.00499}(1-e^{-0.000499/1.05\times 1.1})$$

$$\therefore 0.476148392 = 1 - e^{-0.000432034631R}$$

~~$R = 1496.5$~~ $R = 1496.52$

[?] Claim $\sim Poi(50)$
Claim amount $\approx 100X$
$f(x) = \frac{3}{32}(6x - x^2 - 5)$

(i) $E(100X) = 100E(X)$
$= 100 \times \frac{3}{32} \int_1^5 x(6x - x^2 - 5)\, dx$
$= 100 \times \frac{3}{32} \times \int_1^5 6x^2 - x^3 - 5x \, dx$
$= \frac{300}{32} \times \left[2x^3 - \frac{x^4}{4} - \frac{5}{2}x^2\right]_1^5$
$= \frac{300}{32} \times [31.25 + 0.75]$
$= 300$

$Var(100X) = E(100X)^2 - (E(100X))^2$
$E(100X)^2 = 100^2 E(X^2)$
$100^2 \times \frac{3}{32} \cdot \int_1^5 x^2 \cdot (6x - x^2 - 5)\, dx$
$\frac{20000}{32}\left[\frac{6x^4}{4} - \frac{x^5}{5} - \frac{5x^3}{3}\right]_1^5$
$= 9800$
$Var(100X) = 98000 - (300)^2$
$= 8000$

$E(Y) = 0.3 \times 200 = 60$
$Var(Y) = 0.3 \times 200^2 - (60)^2$
$= 8400$

~~Z = Amount~~
$E(Z) = E(X + Y)$
$= E(X) + E(Y)$
$= 300 + 60$
$= 360$

$$Var(Z) = Var(X+Y)$$
$$= 8000 + 8400$$
$$= 16400$$

$$E(S) = E(N) \cdot E(Z)$$
$$Var(S) = E(N) \cdot Var(Z) + Var(N) \cdot E(Z)^2$$

$$E(S) = 50 \times 360$$
$$= 18000$$

$$Var(S) = 50 \times \cancel{16}4000 + 50 \times 360^2$$
$$= 7300000$$

Q3] $X \sim exp(\lambda)$

$Y = kX \sim Exp$

$$P(Y=y) = P(kX < y)$$
$$= P(X < y/k)$$
$$= \int_0^{y/k} \lambda e^{-\lambda z} dz$$
$$= \int_0^{y} \lambda/k \, e^{\lambda/k x} dx$$

$$Y \sim exp(\lambda/k)$$

(ii) $$L = Ce^{-4\lambda M} \times \prod(\lambda e^{-\lambda x_i}) \times e^{-6\lambda M} \times \prod\left(\frac{\lambda}{k} e^{-\lambda y_i/k}\right)$$

$x_i \rightarrow$ Claims 2006

$Y_i \rightarrow$ Claims 2007

$$l = C' - 4\lambda M + 22 \log \lambda - 13500\lambda - 6\lambda M/k - 17000\lambda/k$$
$$l' = -4M + \frac{22}{\lambda} - 13,500 - \frac{6M}{k} - \frac{17000}{k} = 0$$

$$\hat{\lambda} = \frac{22}{13500 + 17000/k + 4M + \frac{6M}{k}}$$

$$l'' \cancel{\ell} = \frac{22}{\lambda^2} < 0$$

It is a maxima.

(ii) a) $\hat{\lambda} = \dfrac{22}{13500 + 17000/1.1 + 4\times 1600 + 6\times 1,600/1.1} = 0.00499$

b) $\int_0^M x\lambda e^{-\lambda x}\,dx + M\int_M^\infty \lambda e^{-\lambda x}\,dx = \frac{1}{\lambda}(1-e^{-\lambda M})$

$$= \frac{1}{0.000499}\left(1-e^{-0.000499\times 1600}\right)$$

$$= 1102.107$$

Claims for 2008 $\sim \exp(\mu)$

$$\mu = \frac{\lambda}{1.05\times 1.1}$$

$$1102.107 = \int_0^R x\mu e^{-\mu x}\,dx + R\int_R^\infty \mu e^{-\mu x}\,dx$$

$$= \frac{1}{\mu}(1-e^{-\mu R})$$

$$= \frac{1.05\times 1.1}{0.00499}\left(1-e^{-0.00499/1.05\times 1.1}\right)$$

$$\therefore 0.476148392 = 1-e^{-0.000432034631R}$$

~~$R = 1496.5$~~ $R = 1496.52$

Q2)

05	3512	7210	9563
06	2889	6723	
07	3876		

Total claims paid = 20103000

Loss Ratio = 87%

Premium income

05	11041
06	11314
07	12549

	1	2	3
05	3512	7210	9563
06	2889	6723	
07	3876		

r	2.1767	1.3264	1
f	2.8872	1.3264	1
1-1/f	0.6536	0.24608	0

	07	06	05
EP	12549	11314	11041
IUL	10917.63	9843.18	9605.67
EL	7135.76	2422.4	0
RL	3870	6723	9563
UL	11005.76	9145.21	9563

Total UL = 29713.97 (in 000s)

Reserves = 29713970 - 20103000

= 9610970