Assignment 1

24 April 2022 21:26

Q1 A stochastic model recognises the random nature of input components.

They have the following advantages

(i) They imitate the random nature of the variables involved.

(ii) It can provide information about the distribution of results

(iii) They allow the usage of Monte Carlo simulation.

b) The time spent in state H before the next visit to S has mean σ^{-1}

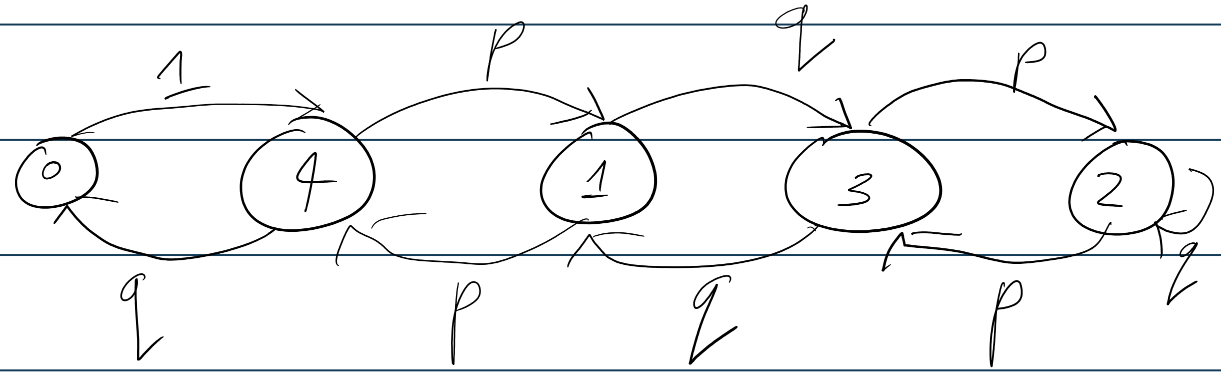
Estimate for $\sigma = \dfrac{1}{\text{Mean length of each visit}}$

$= \dfrac{\text{Number of transitions from H to S}}{\quad}$

Total time spent in state H up until the last transition from H to S.

Similarly for p.

Q2(i) $S = \{ i : i = 0, 1, 2, 3, 4 \}$



(ii)

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & q & p \\ 0 & 0 & q & p & 0 \\ 0 & q & p & 0 & 0 \\ q & p & 0 & 0 & 0 \end{pmatrix}$$

(iii) $\pi(0) = \pi(4)q$

$\pi(1) = \pi(3)q + \pi(4)p$

$\pi(2) = \pi(2)q + \pi(3)p$

$\pi(3) = \pi(1)q + \pi(2)p$

$\pi(4) = \pi(0) + \pi(1)p$

$\pi_1 = \pi_4$

$\pi_3 = \pi_4$

$\pi_2 = \pi_3$

$\pi_1 = \pi_4$

$$\pi_0 = \frac{q}{4+q}$$

(iv) P(Wet) = P(being in state 0 and raining)

raining)

$\Rightarrow P(Wet) = \left(\frac{q}{4+q}\right) p.$

(v) $P(Wet) = 0.6 \times \frac{0.4}{4+0.4}$

$= 5.45\,\%$

$\pi(N) = \frac{q}{N+q}$

$\therefore P(WET) = p\,\pi(0) = \frac{p+q}{N+q}$

For $P(WET) < \frac{1}{100}$

$\Rightarrow \frac{p+q}{N+q} < \frac{1}{100}$

$\Rightarrow N > 100 \times p \times q - q$

$\Rightarrow N > 24 - 0.4$

$\Rightarrow N > 23.6$

Q3

	State space	Time Set
Counting process	Discrete	D/C
General Random Walk	D/C	Discrete
Poisson process	Discrete	Continous
Markov Jump Chain	Discrete	Discrete
Markov Jump Process	Discrete	Continuous

a) Poisson process / Counting process
b) Markov Jump Chain / Counting process
c) Poisson process

d) Markov Jump process.

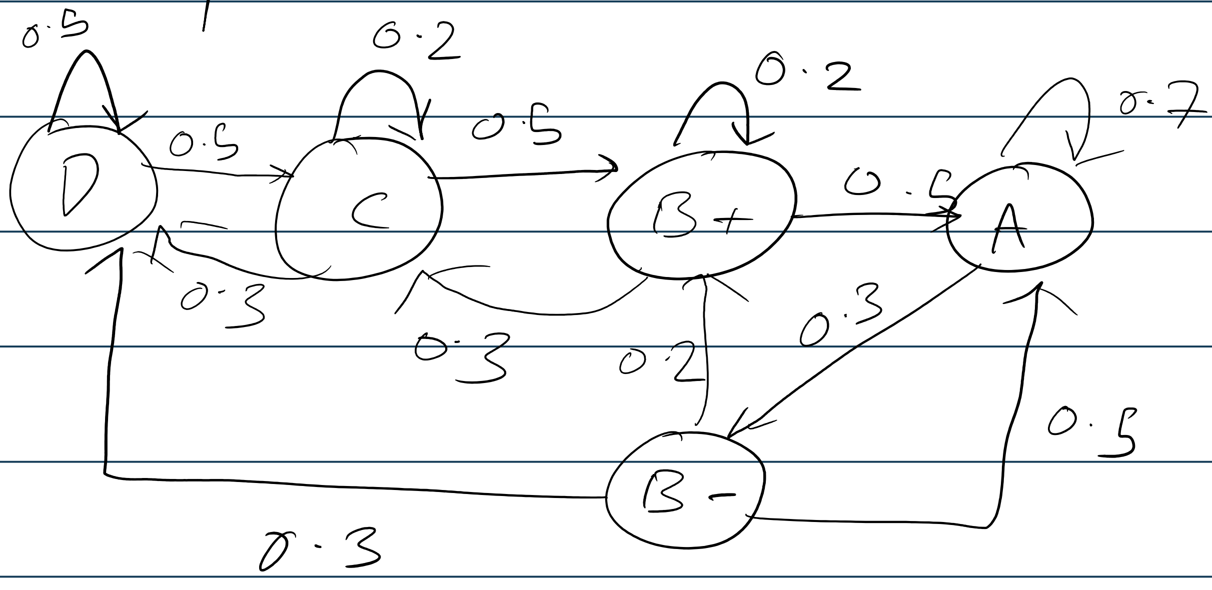
Q4 (i) Past history is needed to decide where to go in the chain.

(ii) B level is split into 2.

B+ → no reduction in performance parameter

B− → reduction in performance parameter

(iii)



(iv)

	D	C	B+	B−	A
D	0·5	0·5	0	0	0
C	0·3	0·2	0·5	0	0
B+	0	0·3	0·2	0	0·5

B+	0	0.3	0.2	0	0.5
B−	0.3	0	0.2	0	0.5
A	0	0	0	0.3	0.7

(v) $0.5\pi_D + 0.3\pi_C + 0.3\pi_{B-} = \pi_D$ (a)

$0.5\pi_D + 0.2\pi_C + 0.3\pi_{B+} = \pi_C$ (b)

$0.5\pi_C + 0.2\pi_B + 0.2\pi_{B-} = \pi_{B+}$ (c)

$0.3\pi_A = \pi_{B-}$ (d)

$0.5\pi_{B+} + 0.5\pi_{B-} + 0.7\pi_A = \pi_A$ (e)

$\pi_{B+} = 0.3\pi_A$

$\pi_C = 0.36\pi_A$

$\pi_D = 0.396\pi_A$

$\pi_A = 0.42945$

$\pi_{B-} = 0.12733$

$\pi_{D} = 0.12733$

$\pi_C = 0.1528$

$\pi_D = 0.16808$

(vi) Long run average contract value -

$100\% \, \pi_D + 120\% \, \pi_C + 150\% \, (\pi_{B+} + \pi_{B-}) + 175\% \, \pi_A$

$= 1.4762$ million USD

(vii) $m_D = 1 + 0.5 m_D + 0.5 m_C$ (i)

$m_C = 1 + 0.2 m_C + 0.5 m_B + 0.3 m_D$

$m_{B+} = 1 + 0.2 m_B + 0.5 m_A + 0.3 m_C$

$m_{B-} = 1 + 0.3 m_D + 0.5 m_A + 0.2 m_B$

$m_A = 0$

$\therefore m_D = 2 + m_C = 9.12$ years

Q5(i) There is an explicit dependence on the past behaviour of Y. Therefore Markov property doesn't hold

(ii) It does not form a Markov Chain.

(iii)
$$\begin{pmatrix} p & 1-p & 0 & - & - & 0 \\ 0 & pe^{-\lambda} & 1-pe^{-\lambda} & 0 & - & - \\ - & 0 & pe^{-2\lambda} & 1-pe^{-2\lambda} & 0 & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & - & - & - & - & - \end{pmatrix}$$

(iv) a) Homogenous
b) irreducible
c) No stationary distribution

(V) Probability of further error -

$$(p e^{-j\lambda})^{n} = p^{n} e^{-nj\lambda}$$

6. 3 assumptions underlying the 2 state model

a) The probabilities that a life at any given age will be found in either state at any subsequent age depends only on the ages involved and the state currently occupied.

b) $_{dt}q_{x+t} = \mu_{x+t}\, dt + o(dt)$

$(t \geq 0)$

c) For each integer x, μ_{x+t} takes a constant value μ for $0 \leq t \leq 1$.

(ii) a) Exponential distribution in each case with σ in H & ρ in S.

b) The time spent in state H before the next visit to S has mean σ^{-1}

Estimate for σ = $\dfrac{1}{\text{Mean length of each visit}}$

$= \dfrac{\text{Number of transitions from H to S}}{\text{Total time spent in state H up until the last transition from H to S.}}$

Similarly for ρ.

(iii) a) For a time inhomogeneous model the transition rates σ & ρ are functions of t.

The model can be improved by using a time inhomogeneous model.

g) State No. of successive defeats

State	No. of successive defeats
1	0
2	1
3	2
4	3
5	4

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0.7 & 0.3 & 0 & 0 & 0 \\
2 & 0.7 & 0 & 0.3 & 0 & 0 \\
3 & 0.7 & 0 & 0 & 0.3 & 0 \\
4 & 0.7 & 0 & 0 & 0 & 0.3 \\
5 & 0 & 0 & 0 & 0 & 1
\end{array}
$$

h) Not irreducible

b) Not irreducible

c) (i)

P(captain for exactly 4 matches)

$= 0.3^4 = 0.0081$

(ii) P(captain for exactly 5 matches)

$= 0.7 \times 0.3^4 = 0.00567$

(iii) P(captain for exactly 7 matches)

$= 0.7 \times 0.3^4 \times 1 \times 1 = 0.00567$

(iv) P(captain for exactly 9 matches)

$= (1 - 0.3^4) \times 0.3^4 \times 0.7 = 0.0056241$

d)

$N_1 = 1 + 0.7 N_1 + 0.3 N_2$

$N_2 = 1 + 0.7 N_1 + 0.3 N_3$

$N_3 = 1 + 0.7 N_1 + 0.3 N_4$

$N_4 = 1 + 0.7 N_1$

$\therefore N_4 = 123.46$

$N_3 = 160.49$

$N_2 = 171.6$

$N_1 = 174.94$

$\therefore E(N) = 174.94$

Q8

State	Description
T	Tie
L_F	Federer leads
L_N	Nadal leads
GF	Federer Wins
GN	Nadal Wins

$$\begin{bmatrix} 0 & 0.55 & 0.45 & 0 & 0 \\ 0.45 & 0 & 0 & 0.55 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.45 & 0 & 0 & 0.55 & 0 \\ 0.55 & 0 & 0 & 0 & 0.45 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(iii) The chain is reducible & not aperiodic

(iv) P(returning to 2 points) =

P(Federer winning 1st point) $\times$ P(Nadal winning 2nd point)

+

P(Nadal winning 1st point) $\times$ P(Federer winning 2nd point)

$= 0.55 \times 0.45 + 0.45 \times 0.55$

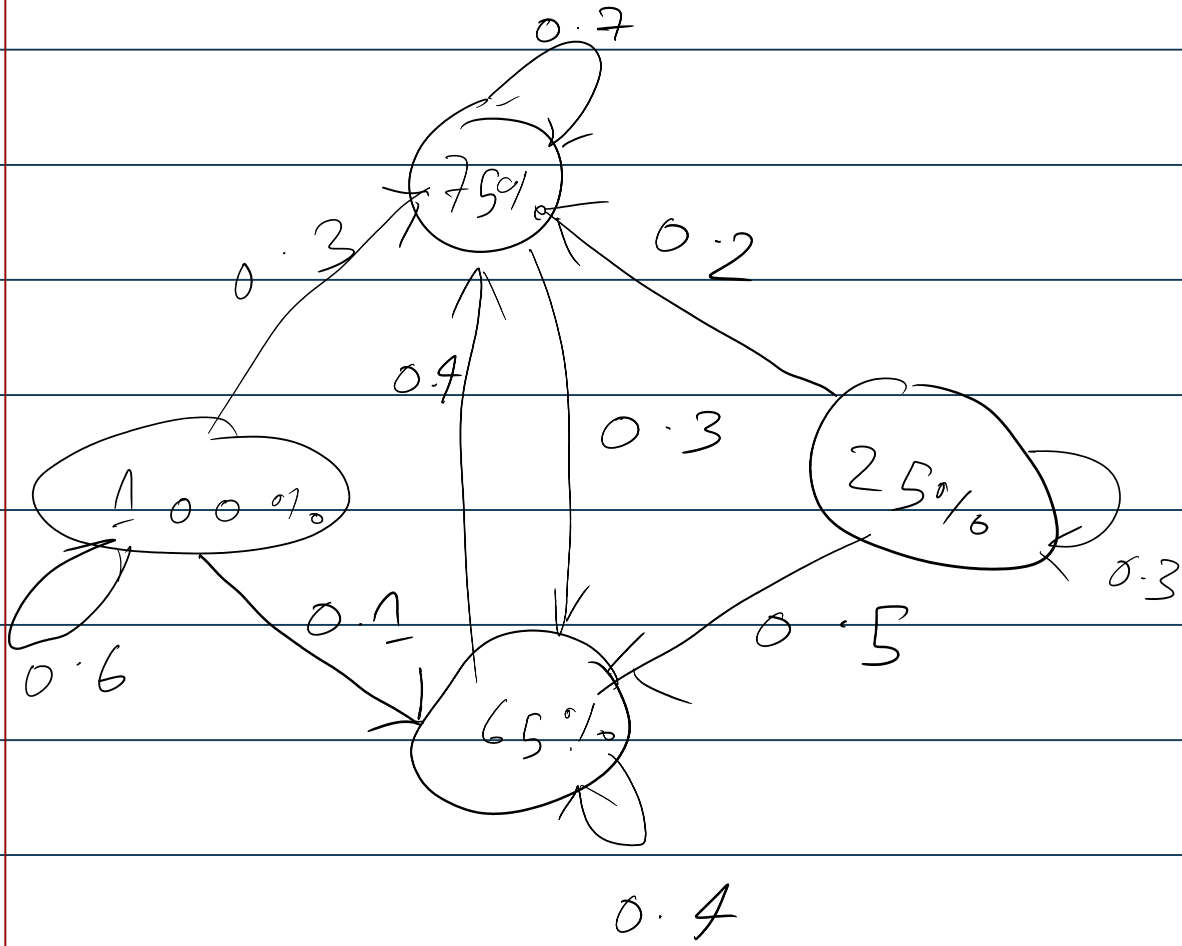
$= 0.495$

$0.495^N = 1 - 0.95$

$$N = \frac{\ln 0.05}{\ln 0.495}$$

$$\Rightarrow \boxed{N = 4.26}$$

Q9



$$S_1 = S_0 P$$

$$(0.05, 0.3, 0.45, 0.2)\begin{pmatrix} 0.6 & 0.3 & 0.1 & - \\ - & 0.7 & 0.3 & - \\ - & 0.4 & 0.4 & 0.2 \end{pmatrix}$$

$$\begin{vmatrix} - & 0.4 & 0.4 & 0.2 \\ - & 0.2 & 0.5 & 0.3 \end{vmatrix}$$

$$= (0.03, 0.445, 0.375, 0.15)$$

$\therefore$ % of corporate buyers having a target of 65% is 36.15%

$$x_1 = 0.6x_1$$

$$x_2 = 0.3x_1 + 0.7x_2 + 0.4x_3 + 0.2x_4$$

$$x_3 = 0.1x_1 + 0.3x_2 + 0.4x_3 + 0.5x_4$$

$$x_4 = 0.2x_3 + 0.3x_4$$

$$x_1 + x_2 + x_3 + x_4 = 1$$

$$x_1 = 0$$

$$x_2 = 0.5423$$

$$x_3 = 0.3559$$

$x_q = 0.1017$

c) 37.5% in 1 year

36.2% in 2 years

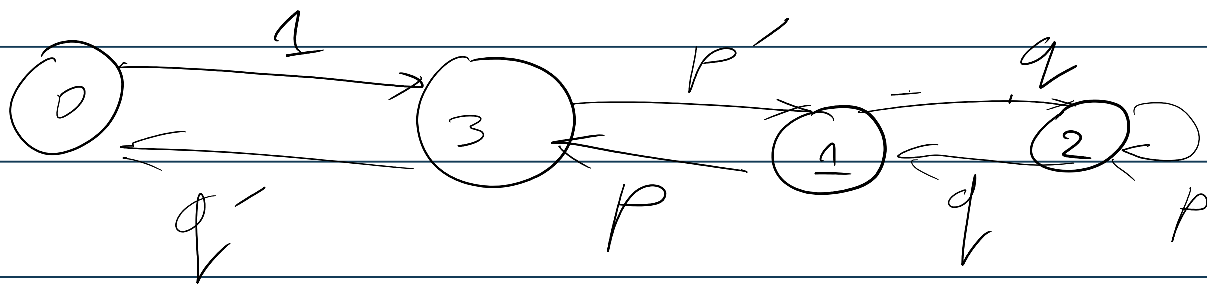
It should reach 35.59% in a few years time

Q10 $P_{1,3} = p$

$P_{1,2} = q$

$P_{3,1} = 0.25(1+3p) = p'$

$P_{3,0} = 0.75(1-p) = q'$



$$\begin{array}{c|cccc}
 & q' & & p & q \\
 & 0 & 1 & 2 & 3 \\
\hline
0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & q & p \\
2 & 0 & q & p & 0 \\
3 & q' & p' & 0 & 0
\end{array}$$

$\pi_0 = q' \pi_3$

$\pi_3 = \pi_0 + p \pi_1$

$\pi_1 = q \pi_2 + p' \pi_3$

$\pi_2 = q \pi_1 + p \pi_2$

$\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1$

$\pi_1 = \pi_2$

$$p.\pi_0 = \frac{0.75(1-p)p}{2 \times 0.25(1+3p) + p + 0.75pq}$$

(iii) $p = 0.7$

Required probability $= 4.58\%$

$$\pi_N = \frac{p}{(N-1)\times 0.25(1+3p) + p + 0.75pq}$$

Q11

$$P = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1/4 & 1/4 & 0 & 1/4 & 1/4 & 0 \\ 0 & 0 & 1/2 & 0 & 0 & 1/2 \\ 0 & 0 & 1/2 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 1/2 & 1/2 & 0 \end{pmatrix}$$

(ii) irreducible

not aperiodic

(iii) $\pi P = P$

(iv) $\dfrac{1}{\pi(1)} = 12$

(v) $\psi(i) = E(\text{no. of steps to reach stage } 5 \mid X_0 = i)$

$\psi(5) = 0$

$\psi(6) = 1 + \frac{1}{2}\psi(5) + \frac{1}{2}\psi_4$

$\psi_4 = 1 + \frac{1}{2}\psi_6 + \frac{1}{2}\psi_3$

$\psi_3 = 1 + \frac{1}{4}\psi_1 + \frac{1}{4}\psi_2 + \frac{1}{4}\psi_4$
$+ \frac{1}{4}\psi_5$

$\psi_1 = 1 + \psi_3$

$\psi_2 = 1 + \psi_3$

$\therefore \boxed{\psi_1 = 7}$

Q.12 $m_{i4} = \begin{cases} 0 & ; i = 4 \\ 1 + \sum p_{ij} m_{j4}, & i \neq 4 \end{cases}$

$m_{24} = 1 + (m_{34})$

$m_{44} = 0$

$m_{34} = 1 + \frac{1}{3} m_{44} + \frac{2}{3} m_{24}$

$m_{34} = 1 + 0 + \frac{2}{3}(1 + m_{34})$

$\Rightarrow m_{34} = 5$

$\therefore m_{24} = 6$ steps

(ii) $P(1, 2, 3, 2, 3, 4)$

$= P(X_0 = 1) \times p_{12} \times p_{23} \times p_{32} \times p_{23} \times p_{34}$

$= \frac{3}{4} \times \frac{3}{5} \times 1 \times \frac{2}{3} \cancel{\times} 1 \times \frac{1}{3}$

$= \frac{1}{10}$

Q13 a) $P_{RR} = \frac{6}{14} = \frac{3}{7}$

$P_{RS} = \frac{4}{7}$

$P_{SR} = \frac{8}{15}$

$P_{SS} = \frac{7}{15}$

b)

1/7	2/7	3/7	
S	S	R	
8/14	7/15	8/15	0.142222
S	R	R	
8/14	8/15	6/14	0.136112

8/14 R	8/15 S	6/14 R	0.130612
6/14 R	8/14 R	8/15 R	0.130612
6/14	6/14	6/14	0.07872

$\therefore$ Total = 0.482164

(ii) a) $\mu = 3$

$P(X > 6) = 1 - p(x_6)$

$= 1 - 0.049787$

$= 0.95021$

b) $P(2 \leq X < 5) = P(x_2) + P(x_3) + P(x_4)$

$= 0.61611$

c) Avg. no. of policies sold per

$$day = \frac{3}{5} = 0.6$$

$$\therefore P(X) = 0.32928$$