Tisha Kothari
A -> 41.

Numerical Methods & Algebra Assignment

(1) Actual length = r_1

= 12.5 cm

Actual area = πr_1^2

= $\pi(12.5)^2$

= 490.87365 cm^2

r_2 = 12.65 cm

Approximate area = $\pi(12.65)^2$

= 502.72551 cm^2

Absolute area = Approximate area − Actual area

= 11.84886

Relative error = $\frac{\text{Absolute area}}{\text{Actual area}}$

= 0.02413

Percentage error = Relative error × 100

= 0.02413 × 100

= 2.413 %

(2) (i) $x_1 = 4$ $y_1 = 0.60206$

$x_2 = 6$ $y_2 = 0.7781513$

using linear interpolation,

$x_2 - x_1 = y_2 - y_1$

(6 − 4) = 0.7781513 − 0.60206

2 = 0.1760913

change = $\frac{0.1760913}{2}$

= 0.8804565

thus, x = 5

y = 0.60206 + 0.8804565

y = 0.69010565

(ii) $x_1 = 4.5$ $\quad y_1 = 0.6532125$

$x_2 = 5.5$ $\quad y_2 = 0.7403627$

$(5.5 - 4.5) = 0.7403627 - 0.6532125$

$1 = 0.0871502$

$\text{change} = \frac{0.0871502}{1}$

$\text{change} = 0.0871502$

Thus, $x = 5$

$y = 0.6532125 + 0.0871502$

$y = 0.6967876$

(3) $x^4 - x - 10 = 0$

$a = 1.5$

$b = 2.$

x	y
$a = x_0 = 1.5$	-6.4375
$b = x_1 = 2$	4

1) $x_2 = \frac{x_0 + x_1}{2} = 1.75$

$y_2 = -2.37109$

2) $x_3 = \frac{x_1 + x_2}{2}$

$= 1.875$

$y_3 = 0.48462$

3) $x_4 = \frac{x_2 + x_3}{2}$

$= 1.8125$

$y_4 = -1.0202$

4) $x_5 = \frac{x_4 + x_3}{2}$

$= 1.84375$

$y_5 = -0.28773$

5) $x_6 = \frac{x_5 + x_4}{2}$

$= 1.859375$

$y_6 = 0.0933$

4) $x^3 - x - 1 = 0$

$f'(x) = 3x^2 - 1$

x	y_1
0	-1
1	-1
2	5

$x_0 = 1$

$f(x_0) = -1$

$f'(x_0) = 2$

$x_1 = x_0 - F(x_0)$

$= 1 - \left(-\frac{1}{2}\right)$

$= 1.5$

$y_1 = 0.875$

$x_2 = x_1 - \frac{F(x_1)}{F'(x_1)}$

$$= 1.5 - \left(\frac{0.875}{0.75}\right)$$

$$= 1.34783$$

$F(x_2) = 0.100697$

$F'(x_2) = 4.44998$

$$x_3 = x_2 - \frac{F(x_2)}{F'(x_2)}$$

$= 1.3252$

$y_3 = 0.00209$

$F'(x_3) = 4.26846$

$$x_4 - x_3 = \frac{F(x_3)}{F'(x_3)}$$

$= 1.32472$

$y_4 = 0.00008712$

(5) $A^2 = A$

$$7A - (I + A)^3 = 7A - (I^3 + A^3 + 3A^2 I + 3A I^2)$$

$$= 7A - (I + A^2(A) + 3A + 3A)$$

$$= 7A - (I + 7A)$$

$$= -I$$

(6) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$|A| = 6 - 1(6-8) + (-1)4$

$= -2$

$\text{Adj}(A) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$$A^{-1} = \frac{Adj(A)}{|A|}$$

$$= \frac{-1}{2}\begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$$

(7) $\vec{a} = 8i - 9j$
$\vec{b} = 7i - 9j$

$\vec{a}\cdot\vec{b} = ab\cos\theta$
$= (8i - 9j)(7i - 9j)$
$= 56 + 81$
$= 137$

$a = \sqrt{64 + 72} = \sqrt{145}$
$b = \sqrt{49 + 81} = \sqrt{130}$

$\vec{a}\cdot\vec{b} = ab\cos\theta$

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{ab}$

$= \frac{137}{\sqrt{130} \times \sqrt{145}}$

$\cos\theta = 0.9720224206.$

(8) $\vec{R} = \vec{a} + \vec{b}$
$R^2 = A^2 + B^2 + 2AB\cos\theta$
$\theta = 30^\circ$
$R^2 = (75)^2 + (25)^2 + 2(75)(25)\cos 30^\circ$
$R^2 = 9497.595284$

(9) $d = 3$

$AP = 101, 104, 107, \dots, 998.$

$a = 101$

$a_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$\frac{998 - 101}{3} = n - 1$

$n = 300.$

$S_n = \frac{n}{2}(a_n + a)$

$= \frac{300}{2}(998 + 101)$

$= 150(1099)$

$= 164850$

(10) $a_6 = 32$

$ar^5 = 32$

$\frac{ar^7}{ar^5} = \frac{128}{32}$

$r^2 = 4$

$r = \pm 2.$

(11) $(3x+4)^5 = (3x)^5 + 5(3x)^4(4) + 10(3x^3)(4)^2$

$+ (10)(3x^2)(4^3) + 5(3x)(4^4) + (4)^5$

$= 243x^5 + 1620x^4 + 4320x^3 +$

$5760x^2 + 3840x + 1024$

(12) $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$|A - \lambda I| = -\lambda^3 + 13\lambda - 12$

$-\lambda^3 + 13\lambda - 12 = 0$

$\lambda^3 - 13\lambda + 12 = 0$

Compairing coefficient

$1 - 13 + 12$

$= 0.$

Root $= 1$

$\lambda^2 - \lambda - 12 = 0$

$\lambda^2 - 4\lambda + 3\lambda - 12 = 0$

$\lambda(\lambda - 4) + 3(\lambda - 4) = 0$

$(\lambda + 3)(\lambda - 4) = 0$

$\lambda = -3 \quad \lambda = 4$

Eigen values $= \{4, -3, 1\}$.

(13) $f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

$F(5) = -13$

$F'(5) = 13$

$x_1 = x_0 - \dfrac{F(x_0)}{F'(x_0)}$

$= 5 - \left(\dfrac{-13}{13}\right)$

$= 6$

$F(x_1) = 9$
$F'(x_1) = 32$

$x_2 = x_1 - \frac{F(x_1)}{F'(x_1)}$

$x_2 = 6 - \frac{9}{32}$

$x_2 = 5.7875$
$F(x_2) = 0.847787.$

(14) $(x^2 - x + 1)^4 = [(x-1)^2 + x]^4$
using pascal's triangle,

$[(x-1)^2 + x]^4 = (x-1)^8 + 4(x-1)^6 x + 6(x-1)^4 (x)^2 + 4(x-1)^2 x^3 + x^4$

$[x^2 - x + 1]^4 = (x-1)^8 + 4x(x-1)^6 + 6x^2(x-1)^4 + 4x^3(x-1)^2 + x^4.$

(15) $e^{-x} = 3\log(x)$
$3\log x + e^{-x} = 0$

x	y
0.1	-2.0951625
0.2	-1.278179
0.3	-0.8278180
0.4	-0.5235
0.5	-0.29656
0.6	-0.11673
0.7	-0.03188.

$x_0 = 0.6 \qquad y_0 = -0.11673$

$x_1 = 0.7 \qquad y_1 = 0.03188$

$$x_2 = \frac{x_0 + x_1}{2} = 0.65$$

$y_2 = -0.03921$

$$x_3 = \frac{x_1 + x_2}{2} = 0.675$$

$y_3 = -0.00293.$

$$x_4 = \frac{(x_2 + x_3)}{2} = 0.6875$$

$y_4 = 0.01465$

$$x_5 = \frac{x_3 + x_4}{2} = 0.68125$$

$y_5 = 0.0059.$