Numerical Methods & Algebra

Assignment Questions

1) $r_i = 12.5cm$ ($r_i \rightarrow$ ideal radius)
$r_c = 12.65\,cm$ ($r_c \rightarrow$ calculated radius)

i) Absolute Error $= |r_i - r_c|$
$= |12.5 - 12.65|$
$= 0.15cm$

ii) Relative Error $= \dfrac{|r_i - r_c|}{r_i} = \dfrac{0.15}{12.5}$

$= 0.012$

iii) Percentage Error $=$ (Relative Error) $\times 100$
$= 0.012 \times 100$
$= 1.2\%$

2)

x	$f(x) = \log(x)$
4.0	0.60206
4.5	0.6532125
5.5	0.7403627
6.0	0.7781513

a) $x_1 = 4$ $\quad y_1 = 0.60206$

$x_2 = 6$ $\quad y_2 = 0.7781513$

$x = 5$

$$y = y_1 + (x - x_1)\frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$= 0.60206 + \frac{(5-4) \times (0.7781513 - 0.60206)}{(6-4)}$$

$$= 0.60206 + 0.08804565$$

$$= 0.690106$$

b) $x_1 = 4.5$ $\quad y_1 = 0.6532125$

$x_2 = 5.5$ $\quad y_2 = 0.7403627$

$x = 5$

$$y = 0.6532125 + \frac{(5-4.5)}{(5.5-4.5)} \times \begin{pmatrix} 0.7403627 - \\ 0.6532125 \end{pmatrix}$$

$= 0.6967876$

3) $x^4 - x - 10 = 0$, $a = 1.5$, $b = 2$

$f(a) = (1.5)^4 - 1.5 - 10$

$= -6.4375$ $\quad \Rightarrow y_0 = -6.4375$

$f(b) = 2^4 - 2 - 10$ $\quad \Rightarrow y_1 = 4$

$= 4$

$\cancel{y_1\ x_0 = -6.4375}$

$\cancel{y_2\ x_1 = 4}$

$\cancel{x_2 = \frac{6.4375 + 4}{2}}$

$\cancel{x_2 = -1.21875}$

$\cancel{y_2 = (-1.21875)^4 + 1.21875 - 10}$

$= \cancel{9.012481}$ (-ve)

2. $x_0 = 1.5$

$x_1 = 2$

$$x_2 = \frac{1.5+2}{2} = 1.75$$

$$y_2 = (1.75)^4 - 1.75 - 10$$
$$= -2.371094 \ (-ve)$$

$$x_3 = \frac{x_2 + x_1}{2} = \frac{1.75+2}{2}$$

$$= 1.875$$

$$y_3 = (1.875)^4 - 1.875 - 10$$
$$= 0.484619 \ (+ve)$$

$$x_4 = \frac{x_2 + x_3}{2} = \frac{1.75 + 1.875}{2}$$

$$= 1.8125$$

$$y_4 = (1.8125)^4 - 1.8125 - 10$$
$$= -1.020248 \ (-ve)$$

$$x_5 = \frac{x_4 + x_3}{2} = 1.84375$$

$$y_5 = (1.84375)^4 - 1.84375 - 10$$

$$= -0.287734 \; (-ve)$$

$$x_6 = \frac{x_5 + x_3}{2} = \frac{1.84375 + 1.875}{2}$$

$$= 1.859375$$

$$y_6 = (1.859375)^4 - 1.859375 - 10$$

$$= 0.093378$$

$\therefore$ Root of $x^4 - x - 10$ approx. is equal to 1.859375

4) $f(x) = x^3 - x - 1$, $f'(x) = 3x^2 - 1$

x	$f(x)$
0	-1
1	-1
2	5

$x_0 = 1$, $f(x_0) = -1$

$f'(x_0) = 2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \frac{(-1)}{2}$$

$$= 1.5$$

$$f(x_1) = (1.5)^3 - 1.5 - 1$$

$$= 0.875$$

$$f'(x_1) = 3 \times (1.5)^2 - 1$$

$$= 5.75$$

$$x_2 = 1.5 - \frac{0.875}{5.75}$$

$$= 1.347826$$

$$f(x_2) = 0.100682$$

$$f'(x_2) = 4.449905$$

$$x_3 = 1.347826 - \frac{0.100682}{4.449905}$$

$$= 1.325200$$

$f(x_3) = 0.002059$

$f'(x_3) = 4.268469$

$x_4 = 1.3250 - \dfrac{0.002059}{4.268469}$

$= 1.324718$

$f(x_4) = -0.0000009$

$\therefore$ Root of Eq $x^3 - x - 1$ is 1.324718

5) $A^2 = A$

$7A - (I+A)^3 = 7A - [I^3 + A^3 + 3A^2 I + 3AI^2]$

$= 7A - I - A^3 - 3A^2 - 3AI^2 \quad (A^2 = A)$

$= 7A - I - A - 3A - 3A$

$= 7A - 7A - I$

$= -I$

6) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$|A| = 1(0-6) + 1(-4) + 2(4)$ [Expansion using R_1]

$= -6 - 4 + 8$

$= -2$

$\text{Adj } A = \begin{bmatrix} -6 & +4 & 4 \\ +1 & -1 & -1 \\ -6 & +2 & 4 \end{bmatrix}^T$

$= \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \cdot \text{Adj } A$

$$\frac{1}{-2}\begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$$

7) $\vec{a} = 8\hat{i} - 9\hat{j}$, $\vec{b} = 7\hat{i} - 9\hat{j}$

$\vec{a}.\vec{b} = |\vec{a}|.|\vec{b}|. \cos\theta$

$$|\vec{a}| = \sqrt{8^2 + (-9)^2} = \sqrt{64+81} = \sqrt{145}$$

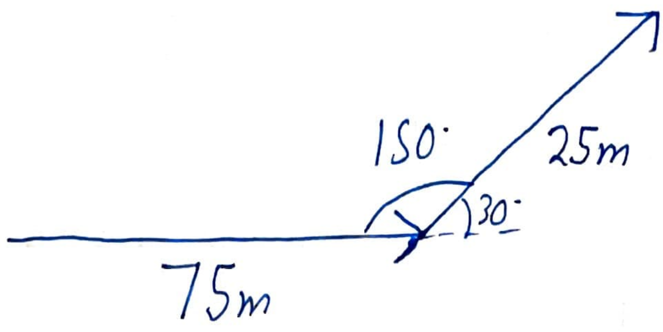
$$|\vec{b}| = \sqrt{7^2 + (-9)^2} = \sqrt{49+81} = \sqrt{130}$$

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|} = \frac{(8 \times 7) + ((-9)\times(-9))}{\sqrt{145} \times \sqrt{130}}$$

$$\theta = \cos^{-1}\left(\frac{56+81}{\sqrt{145 \times 130}}\right) = \cos^{-1}\left(\frac{137}{\sqrt{18850}}\right)$$

$$\boxed{\theta = 3.758556^\circ}$$

8)



$$R^2 = A^2 + B^2 + 2AB\cos 30°$$

$$R = \sqrt{75^2 + 25^2 + 2 \times 75 \times 25 \times \frac{\sqrt{3}}{2}}$$

$$R = 97.46\,m$$

9) 101, 104, 107, ... 998

$a = 101$, $d = 3$, $a_n = 998$

$998 = 101 + (n-1)3$

$\frac{897}{3} = n-1$

$299 = n - 1$

$\boxed{n = 300}$

$$S_n = \frac{n}{2}\left[a + l\right]$$

$$= \frac{300}{2}\left[101 + 998\right] = 150 \times 1099$$

$$\Rightarrow 164850$$

10) $a_6 = 32, \quad a_8 = 128$

$$ar^5 = 32, \quad ar^7 = 128$$

$$ar^5 . r^2 = 128$$

$$r^2 = \frac{128}{32}$$

$$r = 4^{1/2} = 2$$

11) $(4+3x)^5 = 1 \times 4^5 \times (3x)^0 + 5 \times 4^4 \times (3x)^1$

$$+ 10 \times 4^3 \times (3x)^2 + 10 \times 4^2 \times (3x)^3$$

$$+ 5 \times 4^1 \times (3x)^4 + 1 \times 4^0 \times (3x)^5$$

$$\Rightarrow 1024 + 3840x + 5760x^2 + 4320x^3$$

$$1620x^4$$

$$+ \cancel{4050x} + 243x^5$$

12) $\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$A - \lambda I$

$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

$|A - \lambda I| = 0$

$(2-\lambda)[(3-\lambda)(-5-\lambda) - 0] + 3[2(3-\lambda) - 0] = 0$

$= (2-\lambda)[-15 + \lambda^2 + 2\lambda] + 3[6 - 2\lambda] = 0$

$= -30 + 2\lambda^2 + 4\lambda + 15\lambda - \lambda^3 - 2\lambda^2 + 18 - 6\lambda = 0$

$\Rightarrow -\lambda^3 + 0\lambda^2 + 13\lambda - 12 = 0$

$\Rightarrow \lambda^3 - 13\lambda + 12 = 0$

$\lambda = 1$ satisfied

1	1	0	-13	12
	1	1	1	-12
	1	1	-12	0

$$\lambda^2 + \lambda - 12 = 0$$

$$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$$

$$\lambda = -4, 3$$

Eigen Values $\rightarrow$ 1, -4, 3

13) $f(x) = x^3 - 7x^2 + 8x - 3, \quad x_0 = 5$

$f(x_0) = 5^3 - 7 \times 5^2 + 8 \times 5 - 3$

$= -13$

$f'(x) = 3x^2 - 14x + 8$

$f'(x_0) = 3 \times 5^2 - 14 \times 5 + 8$

$= 13$

$x_1 = x_0 - \dfrac{(-13)}{13}$

$= 5 + 1 = 6$

$f(x_1) = 6^3 - 7 \times 6^2 + 8 \times 6 - 3$

$= 9$

$f'(x_1) = 3 \times 6^2 - 14 \times 6 + 8$

$= 32$

$x_2 = 6 - \dfrac{9}{32}$

$x_2 = 5.71875$

14) $(1-x+x^2)^4$, Let $a=(1-x)$, $\cancel{b=x^2}$

$= \cancel{(x-1)^2}\ (a+x^2)^4$

$= 1.a^4(x^2)^0 + 4\times a^3.(x^2)^1 + 6\times a^2\times(x^2)^2$
$+ 4\times a\times(x^2)^3 + 1.a^0.(x^2)^4$

$= a^4 + 4a^3x^2 + 6a^2x^4 + 4ax^6 + x^8$
$= (1-x)^4 + 4(1-x)^3.x^2 + 6(1-x)^2.x^4$
$+ 4(1-x).x^6 + x^8$

15) $e^{-x} = 3\log(x)$

$f(x) = 3\log(x) - e^{-x}$

x	$f(x)$
1	-0.367879
2	1.944106

$x_2 = \frac{x_1+x_0}{2} = \frac{1+2}{2} = 1.5$

$f(x_2) = 3\log(1.5) - e^{-1.5} = +0.993265$

$$x_3 = \frac{x_0 + x_2}{2} = \frac{1+1.5}{2} = 1.25$$

$$f(x_3) = 3\log(1.25) - e^{-1.25}$$
$$= 0.382926$$

$$x_4 = \frac{x_0 + x_3}{2} = \frac{1+1.25}{2} = 1.125$$

$$f(x_4) = 3.\log(1.125) - e^{-1.125}$$
$$= 0.0286966$$

$$x_5 = \frac{x_0 + x_4}{2} = \frac{1+1.125}{2} = 1.0625$$

$$f(x_5) = 3\log(1.0625) - e^{-1.0625}$$
$$= -0.163717$$

$$x_6 = \frac{x_4 + x_5}{2} = \frac{1.125 + 1.0625}{2}$$

$$= 1.09375$$

$$f(x_6) = 3\log(1.09375) - e^{-1.09375}$$
$$= -0.066122$$

Root of $3\log(x) = e^{-x}$ is 1.09315