

Numerical Methods and Algebra

Assignment -1

1) Let $x = \sqrt{7 + \sqrt{7 + \sqrt{7} \dots}}$

$$x = \sqrt{7 + x}$$

$$x^2 = 7 + x$$

$$x^2 - x - 7 = 0$$

$$x = \frac{1 \pm \sqrt{1 + 28}}{2}$$

$$x = \frac{1 \pm \sqrt{29}}{2}$$

x can't be ~~neg~~ -ve

$$x \neq \frac{1 - \sqrt{29}}{2}$$

$$x = \frac{1 + \sqrt{29}}{2}$$

2) $7x^2 - 6y^2 = -11x^2 - 4y^2$

$$18x^2 = 2y^2$$

$$\frac{x^2}{y^2} = \frac{2}{18} = \frac{1}{9}$$

$$\frac{x}{y} = \pm \frac{1}{3}$$

$$3) \quad x + 3y = 4 \quad \dots \textcircled{1}$$

$$x^2 + y^2 - 4y = 12 \quad \dots \textcircled{2}$$

From $\textcircled{1}$, $x = 4 - 3y$

Subs $x = 4 - 3y$ in eq $\textcircled{2}$, we get

$$(4 - 3y)^2 + y^2 - 4y = 12$$

$$10y^2 - 28y + 16 = 12$$

$$10y^2 - 28y + 4 = 0$$

$$y = \frac{28 \pm \sqrt{784 - 160}}{20}$$

$$y = \frac{28 \pm \sqrt{624}}{20}$$

$$y = \frac{28 \pm \sqrt{39 \times 16}}{20} = \frac{28 \pm 4\sqrt{39}}{20}$$

$$y = \frac{7 \pm \sqrt{39}}{5}$$

$$x = \frac{-1 - 3\sqrt{39}}{5} \quad \text{and} \quad x = \frac{3\sqrt{39} - 1}{5}$$

4) ~~$d = 2\pi$~~ $d = 2x^2 - 16x + 34$

a) when $x = 3.5$
 $d = 2(3.5)^2 - 16(3.5) + 34$
 $d = 2.5$

b) $\frac{d}{dx}(d) = 4x - 16 \Rightarrow x = 4$, when $\frac{d}{dx} = 0$

$\frac{d^2}{dx^2}(d) = 4$

Minimum $x = 4$ per min

c) $100 = 2x^2 - 16x + 34$
 $0 = 2x^2 - 16x - 66$

$x = \frac{16 \pm \sqrt{256 + 528}}{4}$

$x = 11$ or $x = -3$

as $\downarrow$ Speed cannot be -ve

5)

a) Company 1 :- $1500 + 25x > 0$

b) Company 2 :- $2100 + 22y > 0$

c) $1500 + 25x < 2100 + 22x$

$$3x < 600$$

$$x < 200$$

b) $x^3 - 7x^2 + 14x - 6 = 0$

a) $[0, 1]$

a	b	t	f(t)
0	1	0.5	2.625
0	1.5	0.75	0.98437
0	0.75	0.375	-1.68164
0.375	0.75	0.5625	0.16186
0.5625	0.75	0.68625	0.4557

b) $[1, 3.2]$

a	b	t	f(t)
3.2	1	2.1	1.791
3.2	2.1	2.65	0.55215
3.2	2.65	2.925	0.8586
3.2	2.925	3.0625	-0.0564

c) [3.2, 4]

a	b	t	f(t)
3.2	4	3.6	0.336
3.2	3.6	3.4	-0.016
3.4	3.6	3.5	0.128
3.4	3.5	3.4	10.046125
3.4	3.45	3.425	0.01302
3.4	3.425	3.4125	-1.99×10^{-3}
3.4125	3.4125	3.4149	5.38×10^{-3}
3.4125	3.41875		

27) $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$
 $f'(x) = 920x^3 + 54x^2 + 8x - 221$

i) [-1, 0]

$x_1 = 0$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_2 = 0 - \frac{(-9)}{-221} = -0.040723$$

$$x_3 = -0.040723 - 0.0409288$$

$$x_4 = -0.04065 \Rightarrow -0.04065988$$

ii) Same as above

8)

$$a) \frac{70 + 86 + 81 + 83 + n}{5} = \frac{320 + n}{5} = u$$

$$b) 90 > \frac{320 + n}{5} \geq 80$$

$$c) 450 > 320 + n \geq 400$$

$$130 > n \geq 80$$

$$9) \begin{aligned} x_1 &= 4\% & , & \quad y_1 = 8.54 & , & @ \text{ IRR, } y=0 \\ x_2 &= 20\% & , & \quad y_2 = -11.81 \end{aligned}$$

$$\frac{x_1 - 4}{20 - 4} = \frac{0 - 8.54}{-11.81 - 8.54}$$

$$x = 10.71\%$$

$$10) \begin{aligned} R(n) &= 32n - 0.21n^2 \\ C(n) &= 195 + 12n \end{aligned}$$

$$R(n) - C(n) = \text{Profit}$$

$$f(n) = 32n - 0.21n^2 - 195 - 12n$$

$$- 0.21n^2 + 20n - 195 = 0$$

$$0.21n^2 - 20n + 195 = 0$$

$$n = \frac{20 \pm \sqrt{(-20)^2 - 4 \times (0.21) \times (195)}}{2 \times 0.21}$$

$$n = 84.2114 \quad \text{or} \quad n = 11.0266$$

$$f'(n) = 0.42n - 20 > 0$$

$$0.42n > 20$$

$$n > 47.6$$

as order should be more than 47.6

$$n = 84.2114$$