Assignment: Numerical methods of Algebra

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Section B Roll No. 77.

Q1. measured radius = 12.65 cm
Actual radius = 12.5 cm

Area measured $= \pi r^2 = \frac{22}{7}(12.65)^2 = 502.927\ cm^2$

Actual Area $= \pi r^2 = \frac{22}{7}(12.5)^2 = 491.0714\ cm^2$

Absolute Error $= |502.9279 - 491.0714|$
$= |11.8565\ cm^2|$

Relative Error $= \frac{11.8565}{491.0714} = |0.0241|$

Percentage Error $= |2.41\%|$

Q2

x	y(f(x))	Δy (Δf(x))
4	0.60206	0.1760913
6	0.7781513	

$h = |x_1 - x_2| = |4 - 6| = 2$

$u = \frac{x - x_0}{h} = \frac{5-4}{2} = 1/2$

$$f(x) = 1 \cdot f(x_0) + u\,\Delta f(x_0)$$

$$\log 5 = 0.60206 + (-1/2)(0.1760913)$$

$$= 0.60206 - 0.0880505$$

$$= \boxed{0.5140\underline{1}}$$

(b)

x	$y(f(x))$	$\Delta y(\Delta f(x))$
4.5	0.6532125	0.0871502
5.5	0.7403627	

$$h = |4.5 - 5.5| = 1.$$

$$u = \frac{5 - 4.5}{1} = 0.5$$

$$\log(5) = (0.6532125) + (-0.5)(0.0871502)$$

$$= 0.6532125 - 0.0435751$$

$$= \boxed{0.6096874}$$

Q3

x	$f(x)$
1.5	-6.4375
2	4

$x = x$

$f(x) = x^4 - x - 10$

$\bar{x}_1 = \frac{1.5+2}{2} = 1.75 \Rightarrow f(x_1) = -2.3721$

$\bar{x}_2 = \frac{1.75+2}{2} = 1.875 \Rightarrow f(x_2) = 0.4842$

$\bar{x}_3 = \frac{1.875+1.75}{2} = 1.8125 \Rightarrow f(x_3) = -1.0202$

$\bar{x}_4 = \frac{1.8125+1.875}{2} = 1.84375 \Rightarrow f(x_4) = -0.2877$

$\bar{x}_5 = \frac{1.84375+1.875}{2} = 1.859375 \quad f(x_5) = 0.0934$

Approx value of $x = 1.859375$

Q4

x	y
0	-1
1	-1
2	5

Equation: $f(x) = x^3 - x - 1$.

Newton Raphson method:

$f'(x) = 3x^2 - 1$.

$x_0 = 1$

$f(x_0) = -1$

$f'(x_0) = 2$.

$x_1 = 1.5$ $\left(x_1 = 1 + \frac{1}{2}\right)$

$f(x_1) = 0.875$

$f'(x_1) = 5.75$.

$x_2 = 1.3478$ $\left(x_2 = 1.5 - \frac{0.875}{5.75}\right)$

$f(x_2) = 0.1006$

$f'(x_2) = 4.4497$

$x_3 = 1.3252$ $\left(x_3 = 1.3478 - \frac{0.1006}{4.4497}\right)$

$f(x_3) = 0.0019$

$f'(x_3) = 4.2684$

$x_4 = 1.3247$ $\left(x_4 = 1.3252 - \frac{0.0019}{4.2684}\right)$

$f(x_4) = -0.000076$

$\therefore$ Approx value of $x = 1.3247$.

Q5 $7A - (I + A)^3$

$= 7A - (I^3 + 3I^2A + 3A^2I + A^3)$

$= 7A - I - 3A - 3A - 3A$

$= -I$

Q6 $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$|A| = 1(-6) + 1(-4) + 2(4)$

$= -6 - 4 + 8$

$|A| = -2$

$A^{-1} = \frac{1}{|A|} \cdot A$

$\therefore A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$.

Q7 $A = 8\hat{i} - 9\hat{j} \Rightarrow |A| = \sqrt{8^2 + (-9)^2} = 12.042$

$B = 7\hat{i} - 9\hat{j} \Rightarrow |B| = \sqrt{7^2 + (-9)^2} = 11.402$

$A \cdot B = (8\hat{i} - 9\hat{j}) \cdot (7\hat{i} - 9\hat{j})$

$= (8)(7) + (-9)(-9)$

$= 56 + 81$

$A \cdot B = 137.$

$\cos\theta = \frac{A \cdot B}{|A||B|} = \frac{137}{(12.042)(11.402)}$

$\cos\theta = 0.998$

$\theta = \cos^{-1}(0.998)$

$\theta = 3.62°$

Q8 $mag. = \sqrt{75^2 + 25^2}$

$= \sqrt{5625 + 625} = \sqrt{6250} = 79.057$

Q9 101, 104 998

$a = 101, \quad d = 3, \quad a_n = 998$

$a_n = a + (n-1)d.$

$998 = 101 + (n-1)3.$

$3(n-1) = 998 - 101$

$(n-1) = \frac{897}{3}$

$n - 1 = 299 \quad \therefore n = 300.$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= 150\left[2(101) + 8(300-1)3\right]$$

$$= 150\,(202 + 897)$$

$$= 750\,(1099)$$

$$\boxed{S_{300} = 164850}$$

Q10 $T_6 = ar^5 = 32$

$T_8 = ar^7 = 128.$

$\therefore r^2 = 4. \Rightarrow \boxed{r = 2}$

Q11 $(4+3x)^5 = {}^5C_0(4)^5 + {}^5C_1(4)^4(3x) + {}^5C_2(4)^3(3x)^2$
$+ {}^5C_3(4)^2(3x)^3 + {}^5C_4(4)(3x)^4$
$+ {}^5C_5(3x)^5.$

$= 1024 + 3840x + 5760x^2 + 4320x^3$
$+ 1620x^4 + 243x^5$

$\therefore$ ~~$4+x\,(4+3x)^5 =$~~

$$\boxed{(4+3x)^5 = 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024}$$

Q12 $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\rightarrow |A - \lambda I| = 0$

$$= \left| \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0 \right|$$

$$= \begin{vmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$$

$= (2-\lambda)[(3-\lambda)(-5-\lambda) - 0] + 3[2(3-\lambda) - 0] + 0 = 0$

$= (2-\lambda)[-15 - 3\lambda + 5\lambda + \lambda^2] + 3[6 - 2\lambda] = 0$

$= (2-\lambda)[\lambda^3 - 2\lambda - 15] + 3(6 - 2\lambda) = 0.$

$\therefore 2\lambda^2 + 4\lambda - 30 - \lambda^3 - 2\lambda^2 + 15\lambda + 18 - 6\lambda = 0$

$= -\lambda^3 + 13\lambda - 12 = 0.$

$(\lambda - 1)(-\lambda^2 + 12) = 0$

$\therefore \lambda = 1$ or $\lambda^2 = +12$

i.e $\lambda = \pm\sqrt{12}$

$\therefore \lambda = 1, \sqrt{12}, -\sqrt{12}$

Thus Eigen values : $1, \sqrt{12}, -\sqrt{12}$

Q13 $f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8.$

$x_0 = 5$

$f(x_0) = -13$

$f'(x_0) = 13.$

$x_1 = 6$ $\left(x_1 = 5 - \frac{(-13)}{13} \right\} = 6. \right)$

$f(x_1) = 9$

$f'(x_1) = 32.$

$\boxed{x_2 = 5.72}$ $\left(x_2 = 6 - \frac{9}{32} = 5.72 \right)$

Q14 $(1 - x + x^2)^4 = \underbrace{(x^2 - x + 1)}_{a}^{4}$ b

$= (x^2 - x)^4 + 4(x^2 - x)^3 + 6(x^2 - x)^2 + 4(x^2 - x) + 1$

$= (x^8 - 4x^7 + 6x^6 - 4x^5 + x^4) + 4(x^6 - 3x^5 + 3x^4 - x^3) + 6(x^4 - 2x^3 + x^2) + 4(x^2 - x) + 1$

$= x^8 - 4x^7 + 6x^6 - 4x^5 + x^4 + 4x^6 - 12x^5 + 12x^4 - 4x^3 + 6x^4 - 12x^3 + 6x^2 + 4x^2 - 4x + 1$

$\therefore (1 - x + x^2)^4 = x^8 - 4x^7 + 10x^6 - 16x^5 + 19x^4 - 16x^3 + 10x^2 - 4x + 1$

Q15 $e^{-x} = 3\log(x)$

$e^{-x} - 3\log(x) = 0$

x	1	2
y	0.368	-0.768

$x_1 = 1, \quad y_1 = 0.368$

$x_2 = 2, \quad y_2 = -0.768$

$x_3 = \frac{x_1 + x_2}{2} = \frac{1+2}{2} = \frac{3}{2} = 1.5$

~~$y_3 = e^{-}$~~

$y_3 = e^{-1.5} - 3\log(1.5) = -0.305$

~~x_4~~

$x_4 = \frac{x_1 + x_3}{2} = \frac{1+1.5}{2} = \frac{2.5}{2} = 1.25$

$y_4 = e^{-1.25} - 3\log(1.25) = -0.004.$

$\therefore \boxed{x = x_4 = 1.25.}$