1) Let X be the amount of a home insurance claim
Let Y be the amount of a car insurance

$X \sim N(800, 100^2)$ $\quad\quad$ $Y \sim N(1200, 300^2)$

$$P((Y_1+Y_2+Y_3) > (X_1+X_2+X_3+X_4) + 800) = P\left(\sum_{i=1}^{3} Y_i - \sum_{i=1}^{4} X_i > 800\right) \quad \text{(i)}$$

$$\sum_{i=1}^{3} Y_i - \sum_{i=1}^{4} X_i \sim N\left(3 \times 1200 - 4 \times 800,\ 3 \times 300^2 + 4 \times 100^2\right)$$

$$\sum_{i=1}^{3} Y_i - \sum_{j=1}^{4} X_j \sim N(400, 310000)$$

$$P\left(\sum_{i=1}^{3} Y_i - \sum_{j=1}^{4} X_j > 800\right) = P\left(Z > \frac{800-400}{\sqrt{310000}}\right) = P(Z > 0.71842)$$

$$= 1 - P(Z \leq 0.71842)$$

$$P(Z < 0.71842) = P(Z < 0.71) + \frac{P(Z < 0.72) - P(Z < 0.71)}{0.72 - 0.71} \times (0.71842 - 0.71)$$

(linear interpolam)

$$P(Z < 0.71842)\ 0.76115 + 0.00260 = 0.76375$$

$$P\left(\sum_{i=1}^{4} Y_i - \sum_{j=1}^{4} X_j > 800\right) = 1 - 0.76375 = 0.23625.$$

Q.2) $M_Y(t) = M_N(\ln M_X(t))$

$P(N=n) = {}^{n+2}C_n\, 0.9^3\, 0.1^n$

(i)
$$= M_N\left(\ln\left(1-\frac{t}{\lambda}\right)^{-\alpha}\right)$$

$p = 0.9$
$q = 0.1$
$k = 3$

$$= M_N\left(\ln\left(1-\frac{t}{\cancel{6}\,2}\right)^{-6}\right)$$

$$= M_N\left(-6\ln\left(1-\frac{t}{2}\right)\right)$$

$$= \left(\frac{0.9\,e^{\ln\left(1-\frac{t}{2}\right)^{-6}}}{1-0.1\,e^{\ln\left(1-\frac{t}{2}\right)^{-6}}}\right)^3 \cdots\cdots \left[M_N(t) = \left(\frac{pe^t}{1-qe^t}\right)^k\right]$$

$$M_Y(t) = \left(\frac{0.9 * \left(1-\frac{t}{2}\right)^{-6}}{1-0.1\left(1-\frac{t}{2}\right)^{-6}}\right)^3$$

(ii) $E(N) = \frac{k}{p} = \frac{10}{3}$ $\qquad E(X) = \frac{\alpha}{\lambda} = 3$

$V(N) = \frac{kq}{p^2} = \frac{10}{27}$ $\qquad V(X) = \frac{\alpha}{\lambda^2} = \frac{3}{2000}$

$$V(Y) = E(N)V(X) + V(N)(E(X))^2$$

$$= \frac{1\cancel{0}}{\cancel{3}} \times \frac{\cancel{3}}{200\cancel{0}} + \frac{10}{\cancel{27}\,3} \times \cancel{9} = \frac{1}{200} + \frac{10}{3}$$

$V(Y) = 3.3338$

$\cancel{6(iii)}$ $S(Y) = \sqrt{V(Y)} = \cancel{1.87}\ 1.82711$

Q.3) $f(x) = \lambda e^{-\lambda(x-\alpha)}$

(i) $MGF = E(e^{tx}) = \int_0^\infty e^{tx} \lambda e^{-\lambda(x-\alpha)} dx$

$$= \lambda \int_0^\infty e^{tx - \lambda x + \lambda\alpha} dx$$

$$= \cancel{\lambda \int} \lambda e^{\lambda\alpha} \int_0^\infty e^{x(t-\lambda)} dx$$

$$= \lambda e^{\lambda\alpha} \left( \frac{e^{x(t-\lambda)}}{t-\lambda} \right)_0^\infty$$

$$= \lambda e^{\lambda\alpha} \left( 0 - \frac{1}{t-\lambda} \right) \qquad \lambda > t$$

$$= \cancel{\frac{\lambda e^{\lambda\alpha}}{t-\lambda} (t-\lambda-1)}$$

$$\boxed{MGF = \frac{\lambda e^{\lambda\alpha}}{\lambda - t}}$$

(ii) $C_x(t) = \cancel{M_x(t)} \ln M_x(t) = \ln\left( \frac{\lambda e^{\lambda\alpha}}{\lambda - t} \right)$

$\cancel{M_x'(t)} =$

(ii) $M_x(t) = \dfrac{e^{\lambda\alpha}}{1-\frac{t}{\lambda}}$

$$M_x'(t) = e^{\lambda\alpha}\left(1-\frac{t}{\lambda}\right)^{-2} \times (-1) \times \left(-\frac{1}{\lambda}\right) = \frac{e^{\lambda\alpha}}{\lambda}\left(1-\frac{t}{\lambda}\right)^{-2}$$

$$M_x'(0) = \frac{e^{\lambda\alpha}}{\lambda}$$

$$M_x''(t) = \frac{-2e^{\lambda\alpha}}{\lambda}\left(1-\frac{t}{\lambda}\right)^{-3} \times \left(-\frac{1}{\lambda}\right)$$

$$M_x''(0) = E(x^2) = \cancel{2e\lambda}\ \frac{2e^{\lambda\alpha}}{\lambda^2}$$

$$Var(x) = E(x^2) - (E(x))^2 = \frac{2e^{\lambda\alpha}}{\lambda^2} - \frac{e^{2\lambda\alpha}}{\lambda^2} = \cancel{\frac{e^{\lambda\alpha}(2-e^{2})}{}}$$

$$Var(x) = \frac{e^{\lambda\alpha}(2-e^{\lambda\alpha})}{\lambda^2}$$

$$mean = E(x) = \frac{e^{\lambda\alpha}}{\lambda}$$

4) $G_V(t) = \left(\frac{p}{1-qt}\right)^k$

$G_V(t) = p^k (1-qt)^{-k}$

~~PGF~~ $G_V(t) = E(t^x)$

$G_X(t) = E(t^x) = t^0 P(X=0) + t^1 (P X=1) + t^2 P(X=2) + t^3 P(X=3) + t^4 P(X=4) \cdots$

$G_V(t) = E(t^V) = t^0 P(V=0) + t^1 P(V=1) + t^2 P(V=2) + t^3 P(V=3) + t^4 P(V=4) + \cdots$

$G_V'(0) = P(V=1)$

$G_V''(0) = P(V=2)$

$G_V'''(0) = P(V=3)$

$G_V''''(0) = P(V=4)$

$G_V(t) = p^k (1-qt)^{-k}$

$G_V'(t) = p^k \left[-k(1-qt)^{-k-1}(-q)\right] = p^k qk \left[(1-qt)^{-(k+1)}\right]$

$G_V''(t) = p^k qk \left[-(k+1)(1-qt)^{-(k+2)}(-q)\right]$

$= p^k q^2 k(k+1)(1-qt)^{-(k+2)}$

$G_V'''(t) = p^k q^3 k(k+1)(k+2)(1-qt)^{-(k+3)}$

$G_V''''(t) = p^k q^4 k(k+1)(k+2)(k+3)(1-qt)^{-(k+4)}$

$\boxed{G_V''''(0) = P(V=4) = p^k q^4 k(k+1)(k+2)(k+3)}$

6) $f(x,y) = ax(x+y) \; ; \quad 0 < x < 5$

$10 < y < 20$

$$E(Y/x) = \int y \cdot \frac{f(x,y)}{f(x)}\, dy$$

$$f(x) = \int_{10}^{20} ax^2 + axy\, dy = \cancel{a}\; 10ax^2 + \frac{axy^2}{2}\Bigg|_{10}^{20}$$

$$= 10ax\,(x+15)$$

$$E(Y/x) = \int_{10}^{20} \frac{y \cdot \cancel{ax}(x+y)}{10\cancel{ax}(x+15)}\, dy$$

$$= \frac{1}{10} \int_{10}^{20} \frac{yx + y^2}{(x+15)}\, dy$$

$$= \frac{1}{10(x+15)} \left[ \frac{y^2 x}{2} + \frac{y^3}{3} \right]_{10}^{20}$$

$$= \frac{1}{10(x+15)} \left( \frac{300x}{2} + \frac{7000}{3} \right)$$

$P(Y > \frac{1}{3}x + 10)$

$$P(Y > \tfrac{1}{3}x + 10, x) = \int_{10}^{35/3} \int_{0}^{5} ax^2 + axy \, dx \, dy$$

$$= \int_{10}^{35/3} \left[\frac{ax^3}{3} + \frac{ax^2 y}{2}\right]_0^5 dy$$

$$= \int_{10}^{35/3} \frac{a \times 125}{3} + \frac{25ay}{2} \, dy$$

$$= \frac{125}{3} a \left(\frac{35-30}{3}\right) + \frac{25}{2} a \times \frac{1}{2}\left(\frac{1225}{9} - 100\right)$$

$$= \frac{125 \times 5}{9} a + \frac{25}{4} a \left(\frac{325}{9}\right)$$

$$= \cancel{825} \quad \cancel{25000a+} \quad \frac{10625}{36} a$$

$E(X)=3 \quad E(Y)=4 \quad corr(X,Y)=-0.3$ 8) $X \sim Gamma(3,2)$

6) $X \sim Poi(\lambda)$, $Y \sim Poi(\mu)$

~~X-Y~~?

Let $Z = X - Y$.

$$m_Z(t) = E(e^{Zt}) = E(e^{Xt-Yt}) = E(e^{Xt} \cdot e^{-Yt})$$

$$= m_X(t) \cdot m_Y(-t)$$

$$= e^{\lambda(e^t-1)} \cdot e^{\mu(e^{-t}-1)}$$

$$= e^{\lambda e^t - \lambda + \mu e^{-t} - \mu}$$

$$= e^{\lambda(e^t-1)+\mu(e^{-t}-1)}$$

Thus, X-Y does not follows $Poi(\lambda-\mu)$

$$\cancel{= \frac{m_X(t)}{m_Y(t)}}$$

$$\cancel{= \frac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}}}$$

$$\cancel{= e^{(\lambda-\mu)(e^t-1)}}$$

Let $Z = X + Y$.

$$m_Z(t) = E(e^{Zt}) = E(e^{Xt} \cdot e^{Yt}) = m_X(t) \cdot m_Y(t) = e^{(\lambda+\mu)(e^t-1)}$$

On comparing and by uniqueness property

$$X+Y \sim Poi(\lambda+\mu)$$

7) (i) $var(Y/X=2) = E(Y^2/X=2) - (E(Y/X=2))^2$

$= 1\times 0 + 4\times 0.3 - (2\times 0.3)^2$

$= 4(0.3 - 0.3\times 0.3)$

$= 4\times 0.3\times 0.7$

$= 0.84$

, $var(X/Y=2) = E(X^2/Y=2) - (E(X/Y=2)^2)$

$= (1^2\times 0 + 2^2\times 0.3 + 3^2\times 0.1 + 4^2\times 0.2) - (1\times 0 + 2\times 0.3 + 3\times 0.1 + 4\times 0.2)$

$= (0 + 1.2 + 0.9 + 3.2) - (1.7)$

$= 5.3 - 1.7$

$= 3.6.$

(ii) $f(U,V) = \frac{48}{67}(2UV - U^2)$ , $0<U<1$, $\frac{U}{2}<V<2$

$E\cancel{(U/V=v) = \int_{v/2}^{2} f(u,v)\,dv}$

$f_V(v) = \int_0^1 f(u,v)\,du = \int_0^1 \frac{48\times 2uv}{67} - \frac{48u^2}{67}\,du$

$= \left(\frac{48\times 2u^2 v}{67\times 2}\right)_0^1 - \left(\frac{48u^3}{67\times 3}\right)_0^1$

$= \frac{48}{67}\left(v - \frac{1}{3}\right)$

$= \frac{48}{3\times 67}(3v-1)$

$E(U/V=v) = \int u \cdot \frac{f(u,v)}{f_V(v)}\,du$

$= \int u \cdot \frac{\cancel{\frac{48}{67}}(\cancel{2u}\, u(2v-u)}{\cancel{\frac{48}{3\times 67}}(3v-1)}\,du$

$E(U/V=v) = \int_0^1 \frac{3u^2(2v-u)}{3v-1}\,du = \frac{3}{3v-1}\left[\frac{2vu^3}{3} - \frac{u^4}{4}\right]_0^1 = \frac{3}{3v-1} * \left[\frac{2v}{3} - \frac{1}{4}\right] = \frac{\cancel{3}}{3v-1}\left[\frac{8v-3}{4\times\cancel{3}}\right] = \frac{8v-3}{12v-4}.$

8) $X \sim Gamma(3,2)$

$E(Y/X=x) = 3x+1$ $\qquad E(X) = \frac{3}{2} = 1.5$

$Var(Y/X=x) = 2x^2+5$

(i) $E(Y) = E\left(E(Y/X=x)\right) = E(3x+1) = 3E(X)+E(1)$

$= 3(1.5)+1$

$E(Y) = 5.5$

(ii) $Var(Y) = E(Var(Y/X=x)) + Var(E(Y/X=x))$

$= E(2x^2+5) + Var(3x+1)$

$= 2E(X^2)+5+9Var(X)+0$

$= 2\left(\frac{3}{4}+\frac{9}{4}\right)+5+9*\frac{3}{4}$

$= 2\left(\frac{\cancel{12}^{3}}{\cancel{4}}\right)+5+\frac{27}{4}$

$= \frac{44+27}{4}$

$\boxed{Var(Y) = \frac{71}{4}}$

9) $E(X)=3 \quad E(Y)=4 \quad corr(x,y)=-0.3$

$V(X)=4 \quad V(Y)=1$

$Z=X+Y$

$E(Z)=E(X)+E(Y)=7$

$V(Z)=V(X)+V(Y)+2cov(X,Y)$

$= \cancel{7}\ 9+1+2\times\sqrt{V(X)V(Y)}\,corr(x,y)$

$= 10+2\times 2\times -0.3$

$= 10+\cancel{(-1.2)}\ (-1.2)$

$= \cancel{\phi}\ \underline{\underline{8.8}}$

(i) $cov(X,Z)=cov(X,X+Y)$

$=cov(X,X)+cov(X,Y)$

$= var(X)+2\times(-0.3)$

$= \cancel{9-0.9}\ 4-0.6$

$= \cancel{8.1}\ 3.14.$

(ii) variance $(Z) = 8.8.$

10) $f(x,y) = kx^{-\alpha} e^{-y/\beta}$

$$1 = \int_y \int_x f(x,y)\,dx\,dy$$

$$1 = k\left(\int_1^\infty \int_1^\infty k\,x^{-\alpha} e^{-y/\beta}\,dx\,dy\right) = k\left(\int_1^\infty \left[k\frac{x^{-\alpha+1}}{1-\alpha}\right]_1^\infty e^{-y/\beta}\,dy\right)$$

$$1 = k\left(\int_1^\infty \frac{\cancel{k}}{\cancel{k}\,\alpha-1} e^{-y/\beta}\,dy\right)$$

$$1 = \frac{k}{\alpha-1}\left[\frac{e^{-y/\beta}}{-y/\beta}\right]_1^\infty = \frac{k}{\alpha-1}\left[0 + \frac{e^{-1/\beta}}{1/\beta}\right] = \frac{k\beta\,(e^{-1/\beta})}{\alpha-1}$$

$$\frac{\alpha-1}{\beta\,(e^{-1/\beta})} = k$$