FM assignment - I

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i)

→ $d^{(4)} = 8\%$

$$\delta = \ln\left(\frac{1}{1-d}\right)$$

$$= \ln\left(\frac{1}{1-\frac{d^{(p)}}{p}}\right)^p$$

$$= p\ln\left(\frac{1}{1-\frac{d^{(p)}}{p}}\right)$$

$$= 4\ln\left(\frac{1}{1-\frac{d^{(4)}}{4}}\right)$$

$$= 4\ln\left(\frac{1}{1-\frac{0.08}{4}}\right)$$

$$= 4\ln\left(\frac{1}{1-0.02}\right)$$

$$= 4\ln\left(\frac{1}{0.98}\right)$$

$\delta = 0.080810829$

$\therefore \boxed{\delta = 8.0810829\%}$

ii)

→ $d^{(4)} = 8\%$

$$1+i = \left(\frac{1}{1-\frac{d^{(4)}}{4}}\right)^4$$

$$i = \left(\frac{1}{1-0.02}\right)^4 - 1$$

$$i = \left(\frac{1}{0.98}\right)^4 - 1$$

$$= 1.08416 5785 - 1$$

$$\boxed{i = 0.084165785}$$

iii) $\frac{d^{(12)}}{12} = ?$

$\rightarrow \left(1 - \frac{d^{(4)}}{4}\right)^4 = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$

Raising both sides by $\frac{1}{12}$

$\therefore \left(1 - \frac{d^{(4)}}{4}\right)^{1/3} = 1 - \frac{d^{(12)}}{12}$

$\Rightarrow (1 - 0.02)^{1/3} = 1 - \frac{d^{(12)}}{12}$

$\Rightarrow \frac{d^{(12)}}{12} = 1 - 0.993288388379$

$\Rightarrow \frac{d^{(12)}}{12} = 0.006711611621$

$\Rightarrow \boxed{d^{(12)} = 8.0539339\%}$

2.

i)

$\rightarrow AV = 1000$

$\delta(t) = 0.004t + 0.0002t^2$

$AV = PV e^{\int_0^{10} \delta(t)}$

$PV = AV e^{-\int_0^{10} 0.004t + 0.002t^2}$

$= 1000 e^{-\left[0.002t^2 + \frac{0.002t^3}{3}\right]}$

$= 1000 e^{-\left[0.002(10)^2 + \frac{0.0002}{3}(10)^3\right]}$

$= 1000 e^{-[0.266667]}$

$= 1000 \times 0.7639283384$

$\boxed{PV = 763.9283384}$

ii)

$\rightarrow$ $AV = PV(1+i)^{10}$

$1000 = 765.9283384\,(1+i)^{10}$

$(1+i)^{10} = 1.305605172$

$1+i = (1.305605172)^{1/10}$

$i = 0.027025404$

$\therefore \boxed{i = 2.7025404\,\%}$

3.

i)

$\rightarrow$ $1+i = \dfrac{1}{1-d}$

$1-d = \dfrac{1}{1+i}$

$d = \dfrac{1+i-1}{1+i}$

$d = \dfrac{i}{1+i}$

$\boxed{d = iv}$

ii)

$\rightarrow$ a) $\dfrac{d^{(4)}}{4} = 2.25\,\% = 0.0225$

$d^{(2)} = ?$

$\left(1 - \dfrac{d^{(4)}}{4}\right)^4 = \left(1 - \dfrac{d^{(2)}}{2}\right)^2$

$(1 - 0.0225)^2 = 1 - \dfrac{d^{(2)}}{2}$

$\dfrac{d^{(2)}}{2} = 0.04449375$

$d^{(2)} = 0.0889875$

$\boxed{d^{(2)} = 8.89875\,\%}$

b) $d^{(1/2)} = 5\% = 0.05$
$d^{(2)} = ?$

$$\therefore \left(1 - \frac{d^{(1/2)}}{1/2}\right)^{1/2} = \left(1 - \frac{d^{(2)}}{2}\right)^{2}$$

$$\left(1 - (0.05 \times 2)\right)^{1/4} = 1 - \frac{d^{(2)}}{2}$$

$$\frac{d^{(2)}}{2} = 1 - \left(1 - (0.1)\right)^{1/4}$$

$$d^{(2)} = 2 \times 0.0259962537$$

$$\boxed{d^{(2)} = 5.19925075\%}$$

iii)

$\rightarrow$ $i = 5\%$ $n = 91$ days. $SD = ?$ annual.

let bill amt be ₹100

$$PV = AV(1+i)$$
$$= 100\,(1+i)^{91/365}$$
$$= 100\,(1+0.05)^{91/365}$$
$$PV = 101.2238407$$

$$\boxed{d = 4.8494619\%}$$

4.

i)

$\rightarrow$ a) $i = 0.08$
$d^{(12)} = ?$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = \frac{1}{1+0.08}$$

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$$1-\frac{d^{(12)}}{12} = \left(\frac{1}{1.08}\right)^{1/12}$$

$$\frac{d^{(12)}}{12} = 1-0.99360710 2$$

$$d^{(12)} = 0.0767147761 4$$

$$\left[d^{(12)} = 7.671477614\%\right]$$

b) $i^{(365)} = ?$ $\quad i = 0.08$

$$\left(1+\frac{i^{(365)}}{365}\right)^{365} = 1+0.08$$

$$1+\frac{i^{(365)}}{365} = (1.08)^{1/365}$$

$$i^{(365)} = 0.07696915541$$

$$\left[i^{(365)} = 7.696915541\%\right]$$

c) $\delta = \ln(1+i)$

$$= \ln(1+0.08)$$

$$\delta = 0.07696104114$$

$$\left[\delta = 7.696104114\%\right]$$

d) $i^{(0.5)} = ?$ $\quad i = 0.08$

$$\left(1+\frac{i^{(0.5)}}{0.5}\right)^{0.5} = 1+0.08$$

$$1+\frac{i^{(0.5)}}{0.5} = (1.08)^{2}$$

$$i^{(0.5)} = 0.0832$$

$$\left[i^{(0.5)} = 8.32\%\right]$$

d is amount of money invested at the beginning and will be earned during the period when interest is paid at the start of the year (i). so d must be = to PV at the beginning of the year payment & interest paid at end

$\therefore d = iv$

5.

i) $d^{(4)} = 6\%$.

a) $i^{(2)} = ?$

$$\left(1 + \frac{i^{(2)}}{2}\right)^2 = \frac{1}{\left(1 - \frac{d^{(4)}}{4}\right)^4}$$

$$1 + \frac{i^{(2)}}{2} = \frac{1}{\left(1 - \frac{0.06}{4}\right)^2}$$

$$1 + \frac{i^{(2)}}{2} = 1.030688758$$

$$i^{(2)} = 0.06137751552$$

$$\left[i^{(2)} = 6.137751552\%\right].$$

b) $d^{(12)} = ?$ $d^{(4)} = 6\%$.

$$\left(1 - \frac{d^{(4)}}{4}\right)^4 = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

$$\left(1 - \frac{d^{(4)}}{4}\right)^{1/3} = 1 - \frac{d^{(12)}}{12}$$

$$\left(1 - \frac{0.06}{4}\right)^{1/3} = 1 - \frac{d^{(12)}}{12}$$

$$\frac{d^{(12)}}{\cancel{12}} = 12 \times 0.0053860786$$

$$d^{(12)} = 0.06030252528$$

$$\left[d^{(12)} = 6.030252528\%\right]$$

ii) a)

2,00,000 — 2,20,000

2005 — 2006

$n = 93$ days.

if paid on 15th apr 2006

$$\frac{220000}{200000} = 1 + i.$$

$i = 0.1$ or 10%.

$\therefore$ int paid for $n = \frac{10}{100} \times \frac{93}{365}$

$= 0.02547945205$.

Amt paid if contract is terminated =

$= 200000 + (20,0000 \times 0.0254$

$$\left[= 205905.8904\right].$$

6.

i) let the retail price of the toy be ₹100.

→ a) Cash payment = 30% below the recommended retail price

$= 100 - \left(\frac{30}{100} \times 100\right)$

$= 70$

b) Six month credit = 25% below

$= 75$

∴ the effective rate of discount is

$70 = 75(1-d)^{0.5}$

$\left(\frac{70}{75}\right)^2 = 1-d$

$d = ~~0.0~~ \; 0.1288889$

$\left[d = 12.88889\%\right]$

ii)

→ $d^{(12)} = ?$

$1-d = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$

$(1 - 0.1288889)^{1/12} = 1 - \frac{d^{(12)}}{12}$

$\left[d^{(12)} = 13.7195439\%\right]$

$i^{(2)} = ?$

$\left(1 + \frac{i^{(2)}}{2}\right)^2 = \frac{1}{1-d}$

$\frac{i^{(2)}}{2} = 0.0714285 7143$

$\left[i^{(2)} = 14.2857143\%\right]$

$\delta = \ln(1+i$

$= \ln\left(\frac{1}{1-d}\right)$

$= \ln\left(\frac{1}{1-0.01288889}\right)$

$= 0.137985743$

$\left[\delta = 13.7985743\%\right]$

7.

i) $AV = PV\left(1+\frac{i^{(4)}}{4}\right)^4$

$\rightarrow 26000 = 20000\left(1+\frac{i^{(4)}}{4}\right)^{4\times 17/12_3}$

$\frac{26000}{20000} = \left(1+\frac{i^{(4)}}{4}\right)^{17/3}$

$\left(\frac{13}{10}\right)^{3/17} = 1+\frac{i^{(4)}}{4}$

$i^{(4)} = 0.1895525456$

$\left[i^{(4)} = 18.95525456\%\right]$

ii)

$\rightarrow \left(1-\frac{d^{(2)}}{2}\right)^2 = \frac{1}{\left(1+\frac{i^{(4)}}{4}\right)^4}$

$1-\frac{d^{(2)}}{2} = \frac{1}{1.047388136}$

$\frac{d^{(2)}}{2} = 0.0884417655$

$d^{(2)} = 0.1768823531$

$\left[d^{(2)} = 17.68823531\%\right]$

iii)

$\rightarrow 26000 = 20000\left(1+n(i)\right)$

$\frac{13}{10} = 1+\frac{17}{12}i$

$0.3 = \frac{17}{12}i$

$i = 0.2117647059 \Rightarrow \left[i = 21.17647059\%\right]$

iv) Interest rates keep getting lower as & when the frequency gets higher shows is the lowest numerical value & i is the highest.

$$d < d^{(12)} < i^{(4)} < i$$

The simple rate of interest has to be higher than the compound discount to maintain the same amount.

8.

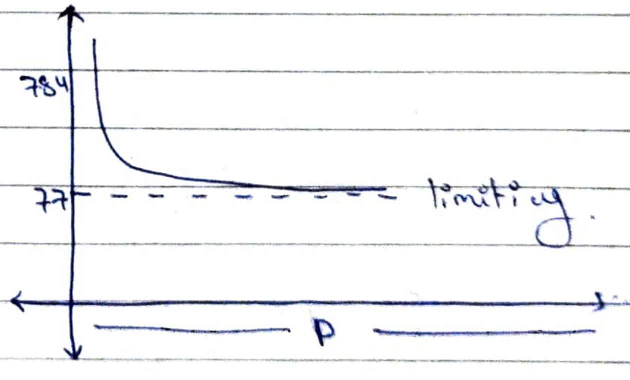
→ The relationship between $i^{(p)}$ and p can be represented as:

$$i^{(p)} = p\left[(1+i)^{1/p} - 1\right].$$

$i = 8\%$

$p = 1, 2, 3, 4, 5, 10, 50, 365$

p	$i^{(p)}$
1	0.08
2	0.07846096908
3	0.07795670402
4	0.07770618763
5	0.07755639199
10	0.07725789243
50	0.07702030156
365	0.07696915541



As p increases, the value of i decreases.
The change is less significant in going weekly, monthly, daily there is a limit to compounding.
The limiting case is when $p \to \infty$, there is instantaneous change and it keeps compounding till delta (δ).

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i)
→ Force of interest - Force of interest can be expressed a... There is a limiting factor to $i^{(p)}$ that limiting fa... is δ. It is the limiting interest rate where 'p' tends to infinity.

$$\lim_{p\to\infty} i^{(p)} = \delta$$

ii)
→ $i^{(2)} = 0.0775$

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = 1+i$$

$$\left(1+\frac{0.0775}{2}\right)^2 = 1+i$$

$$i = 0.079001563$$

$$\boxed{i = 7.9001563\%}$$

$$\delta = \ln\left(1+\frac{i^{(2)}}{2}\right)^2$$

$$= 2\ln\left(1+\frac{0.0775}{2}\right)$$

$$= 2\ln(1.03875)$$

$$\delta = 0.07603613438$$

$$\boxed{\delta = 7.603613438\%}$$

$d^{(12)} = ?$

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = \frac{1}{\left(1+\frac{i^{(2)}}{2}\right)^2}$$

$$\left(1-\frac{d^{(12)}}{12}\right) = \left(\frac{1}{1.03875}\right)^{1/6}$$

$$1-\frac{d^{(12)}}{12} = 0.993683687 8$$

$$d^{(12)} = 0.0757957468$$

$$\boxed{d^{(12)} = 7.5795748\%}$$

$i^{(1/2)} = ?$

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = \left(1+\frac{i^{(1/2)}}{1/2}\right)^{1/2}$$

$$\left(1+\frac{0.0775}{2}\right)^4 = 1+\frac{i^{(1/2)}}{1/2}$$

$$0.1642443719 = \frac{i^{(1/2)}}{1/2}$$

$$i^{(1/2)} = 0.08212218594$$

$$\boxed{i^{(1/2)} = 8.212218594\%}$$

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$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \le t \le 5 \\ 0.06 & \text{for } 5 \le t \le 10 \\ 0.04 & \text{for } t \ge 10 \end{cases}$

$v(t) = e^{-\delta}$ or $\frac{1}{A(t)}$.

$= \frac{1}{A(0,5), A(5,10), A(0,t)}$

$= e^{-\int_0^5 0.08\,dt} \times e^{-\int_5^{10} 0.06\,dt} \times e^{-\int_{10}^{t} 0.04\,dt}$

$=$

$= e^{-(0.08t)_0^5} \times e^{-(0.06t)_5^{10}} \times e^{-(0.04t)_{10}^{t}}$

$= e^{-0.08(5)} \times e^{-\{0.06(10)-0.06(5)\}} \times e^{-[0.04t-0.4]}$

$= e^{-0.4} \times e^{-0.3} \times e^{-0.04t+0.4}$

$= e^{-[0.3+0.04t]}$

$\left[v(t) = e^{-(0.3+0.04t)}\right]$.

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a)

$PV = AV\,(v(t))$

$= C(1-d)^n$

$= 50000\,(1-0.18)^{9/12}$

$= 50000\,(0.82)$.

$\{PV = 43085.42623\}$

$i\delta = 6\%$ $\quad d^{(12)} = ?$

$\delta = \ln\left(\frac{1}{1-d}\right)$

$0.06 = \ln\left(\frac{1}{1-\frac{d^{(12)}}{12}}\right)^{12}$

$e^{0.06} = \left(\frac{1}{1-\frac{d^{(12)}}{12}}\right)^{12}$

$1-\frac{d^{(12)}}{12} = \left(\frac{1}{e^{0.06}}\right)^{1/12}$

$\frac{d^{(12)}}{12} = 0.004987520807$

$d^{(12)} = 0.05985024969$

$\{d^{(12)} = 5.985024969\%\}$

b)

$\rightarrow$ $d^{(4)} = 6\% = 0.06$.

$i^{(12)} = ?$

$$1+i = \frac{1}{1-d}$$

$$1+\frac{i^{(12)}}{12} = \left(\frac{1}{1-\frac{0.06}{4}}\right)^{1/3}$$

$$\frac{i^{(12)}}{12} = 0.005057072\,1$$

$$i^{(12)} = 0.0606070\,8865.$$

$$\left[i^{(12)} = 6.0607\,08865\%\right]$$

c)

$\rightarrow$ $i, d, i^{(4)}, \delta$.

$d < \delta < i^{(4)} < i$.

The order is purely based on the payment of interest & is payable at the beigning of the year., immediate payment. hence the numeric percentage would be less smaller than i which is interest payable at end. similarly δ and $i^{(4)}$ is also less than i as δ (force of interest) at exact due to force of interest and $i^{(4)}$ is nominal ~~r~~ interest convertible quaterly

d)

$\rightarrow$ Same ques as 4.i).

12.

→ Accumulated value $= 7049 \times A(0,2) \times A(2,4\frac{1}{2}) \times A(4\frac{1}{2},6)$

$$= 7049 \times \left(1+\frac{i^{(12)}}{12}\right)^{24} \times (1+10i) \times \frac{1}{\left(1-\frac{d^{(2)}}{12}\right)^{3/2 \times 12}}$$

$$= 7049 \times \left(1+\frac{0.06}{12}\right)^{24} \times (1+0.15) \times \left(\frac{1}{1-\frac{0.06}{12}}\right)^{18}$$

$$= 7049 \times (1.127159776) \times (1.15) \times 1.090421325$$

$$\boxed{AV = 9999.893613}$$

13.

→ let the initial investment be x Rs.

$AV = 2(x) = 2x$.

i) x at 0, $2x$ at 1. $i = 10\%$.

$2x = x(1+i)^n$

$2 = 1+(0.01)^n$

$2 = (1.01)^n$

taking log on both the sides.

$n \ln(1.01) = \ln 2$.

$\boxed{n = 7.272540902 \text{ years}}$

ii) $d^{(12)} = 10\%$.

$\cancel{x} = 2\cancel{x}(1-d)^n$.

$\frac{1}{2} = \left(1-\frac{d^{(12)}}{12}\right)^{12n}$.

taking log

$\ln \frac{1}{2} = 12n \ln\left(1-\frac{0.1}{12}\right)$

$12n = 82.83060471$.

$\boxed{n = 6.902550392 \text{ years}}$