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Q.1 (i) Mean $= \frac{\sum x}{n} = \frac{4+7+9+9+11+15+17+17+18+25}{10}$

$= \frac{134}{10} = 13.4.$

Median $= \frac{15+11}{2} = 13$

Mode $=$ 13.

Variance $= \frac{\sum(x-\bar{x})^2}{n-1}$

SD $= \sqrt{var(x)} = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

$= \cancel{\sqrt{81+36+36+16+16+4+4+}}$

$= \sqrt{\frac{88.36+40.96+19.36+19.36+5.76+2.56+21.16+21.16+21.16+134.56}{9}}$

$= \sqrt{41.6}$

$= 6.449806$

Interquartile range is $\frac{3(n+1)}{4}^{th} - \left(\frac{n+1}{4}\right)^{th}$ term

$= 8.25^{th} - 2.75^{th}$ term.

$= 18 + \frac{(18-18)}{100} \times 25 - 7 + \frac{(9-7)}{100} \times 75$

$= 9.5$

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Q.1 i

(ii) No. of households

x_i	f_i	$x_i f_i$	$x_i - \bar{x}$	$f_i(x - \bar{x})^2$
0	5	0	-2	20
1	11	11	-1	11
2	26	52	0	0
3	15	45	1	15
4	3	12	2	12
	60	120		58

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{120}{60} = \boxed{2.}$$

$$\text{Median} = \left(\frac{60}{2}\right) \text{term} = 30^{th} \text{ term} = \boxed{2}$$

$$\text{Mode} = \boxed{2}$$

$$\sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}} = \sqrt{\frac{58}{60}} = 0.983192080$$

$$Q_1 = \frac{n}{4} = \frac{60}{4} = 15^{th} \text{ term} = 1$$

$$Q_3 = \frac{3n}{4} = \frac{180}{4} = 45^{th} \text{ term} = 3$$

Inter quartile range $= Q_3 - Q_1 = 2.$

Q.2. $\lambda = 0.5$

(i) $x = 0.$

$\therefore P(X \leq 0) = 0.60653$

(ii) $P(X \geq 1) = 1 - P(X \leq 1)$

$= 1 - 0.90980$

$= 0.090200$

$P(X \leq 0) = 0.60653.$

$\therefore$ Total probability $= 0.0902 + 2(0.60653)$

$= 1.30326$

Q.3.) $\alpha = 2$, $\lambda = 1/2$

$0 < x < \infty$

(i) $E(x) = \frac{\alpha}{\lambda} = \frac{2}{1/2} = 4$

$V(x) = \frac{\alpha}{\lambda^2} = \frac{2}{1/4} = 8$

S.D. $= \sqrt{V(x)} = \sqrt{8}$

$= 4\sqrt{2}$

$= 2.8284$

$= 2.$

(iii) $\cancel{F_x(x) = \int_0^8 (1/2)^2}$

$$F_x(x) = \int_0^\infty \frac{\lambda^\alpha e^{-\lambda x} x^{\alpha-1}}{\Gamma(\alpha)}$$

$$\Gamma(\alpha) = (2-1)! = 1!$$

$$= \int_0^\infty \frac{(1/2)^2 e^{-\frac{x}{2}} x^{2-1}}{1}$$

$$\therefore \frac{1}{4} \int_0^\infty (1\cancel{/2)^2} e^{-1/2 x} x^1$$

$\overline{-1/2}$ $\overline{-1/4}$

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$= \frac{1}{4} \times \left[e^{-1/2 x} \cdot (-2x + 4) \right]_0^\infty$

$= \frac{1}{4} \left[e^{-1/2 x} \times 2(x-2) \right]_0^\infty$

$= \frac{-1}{2} \left[e^0 \times (0-2) \right]$

$= \frac{-1}{2} (1 \times 0 - 2)$

$F_X(x) = 1$

Q.4. Total of 18 staff

$\frac{10}{18}$ ~~male~~ $P(M)$; $\frac{8}{18}$ $P(F)$

$\frac{6}{10}$ P(Male Acturies) ; $\frac{7}{8}$. P(Femal Acturi.)

P(2 female acturies / one of them if female) =

$$\frac{\frac{7}{8} \times \frac{6}{7}}{\frac{7}{8} \times \frac{6}{10} + \frac{7}{8} \times \frac{6}{7} + \frac{6}{10} \times \frac{7}{8}}$$

$$\frac{0.75}{\frac{21}{40} + 0.75 + \frac{21}{40}} = \frac{0.75}{1.275}$$

$= 0.5882358294$

Q5. Let A be Level 1
Let Q be Level 2
Let R be Level 3.

$P(A) = 40\%$.
$P(Q) = 50\%$.
$P(R) = 10\%$.

$P(A/L) = 20\%$.
$P(Q/L) = 40\%$.
$P(R/L) = 10\%$.

$$P(L/Q) = \frac{P(Q) \times P(Q/L)}{P(Q) \times P(Q/L) + P(A) \times P(A/L) + P(R)\, P(R/L)}$$

$$= \frac{0.5 \times 0.4}{0.5 \times 0.4 + 0.4 \times 0.2 + 0.1 \times 0.1}$$

$= 0.689655172$.

Q.6. $X \sim U(1,2)$

$a = 1, \quad b = 2$

$f_X(x) = \frac{1}{2-1} = \frac{1}{1} = 1$

$y = \frac{3}{6-x}$

$6x - x = \frac{3}{y}$

$x = 6 - \frac{3}{y}$

$= w^{-1}(y)$

$F_Y(y) = f_X(w^{-1}(y)) \left| \frac{d}{dy}(w^{-1}(y)) \right|$

$= f_X\left(6 - \frac{3}{y}\right) \cdot \frac{3}{y^2}$

$= 1 \times \frac{3}{y^2} = \frac{3}{y^2}$

$\frac{dx}{dy} = \frac{3}{y^2}$

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Q.(1) . $F(3)$

$= 0.05 + 0.15 + 0.35$

$= 0.55$

(ii) $E(X) = \sum x \, P(X=x)$

$= 0.05 + 0.3 + 0.35(3) + 0.4(4) + 0.05(5).$

$= 3.25$

$E(X)^2 = (0.05)(1)^2 + 0.15(2)^2 + 0.35(3)^2 + 0.4(4)^2 + 0.05(5)^2$

$= 0.05 + 0.60 + 3.15 + 6.4 + 1.25.$

$= 11.45$

$E(X)^2 - (E(X))^2$

$11.45 - (3.25)^2 = 0.8875.$

Skewness =

$E(X)^3 \quad 0.05 + 0.35(3)^3 + 0.15(2)^3 + 0.4(4)^3 + 0.05(5)^3.$

Skew $= E(X)^3 - E(X)^3.$

$= 8.221875$

Q. 8. $n = 15$, $x = 3$, $p = 0.2$

$q = 0.8$.

$P(X<3) = P(0) + P(1) + P(2)$

$= {}^{15}C_0 \times (0.2)^0 \times (0.8)^{15} +$

${}^{15}C_1 (0.2)^1 \times (0.8)^{14} +$

${}^{15}C_2 (0.2)^2 \times (0.8)^{13}$.

$= 0.03517 + \left(15 \times 0.2 \times 0.04398\right) +$

$105 \times 0.04 \times 0.05497$

$P(X<3) = 0.397994$.

Q. 9. $n = 7$; $p = 0.01$, $q = 0.99$

$r = 3$.

Negative Binomial

$^{6}C_{2} \; p^{3} \; q^{4}$

$$\frac{6!}{4! \times 2!} \times (0.01)^3 \times (0.99)^4 = 0.000014907.$$

Q.10. $\lambda = \frac{2}{5} = 0.4$

$P(X > 2) = 1 - P(X < 2)$

$= 1 - 0.99207$

~~= 0.00793~~

$= 0.00793$

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11. $n = 10$; $p = 0.5$, $q = 0.5$, $r = 8$

$$\text{Probabil} = \frac{10!}{2!\,8!} \times (0.5)^8 \times (0.5)^2$$

$$= \frac{5 \cancel{10} \times 9}{2} \times (0.5)^8 \times (0.5)^2$$

$$= 0.043945313$$

Q.12. $\lambda = 10$; $\alpha = 1$
$n = 6.1$ days.

$P(T < 0.1) = P(0.1)$
$= 1 - e^{-\lambda t}$
$= 1 - e^{-1}$
$= 0.63212$

(13.) $x \sim U(75, 120)$

$f_x(x) = \frac{1}{b-a} = \frac{1}{120-75} = \frac{1}{45}$

$= 0.22222$

$P(80 < x < 100) = f_x(100) - f_x(80)$

$= \frac{100}{45} - \frac{80}{45}$

$= \frac{20}{45} = 0.44444$

14. $X \sim \text{Gamma}(5, 0.4)$ $\lambda = 0.4$, $\alpha = 5$

$P(X < 22.89)$.

$2\lambda X \sim \chi^2_{10}$

$0.8X \sim \chi^2_{10}$

$\therefore$ $P(X < 22.89) = P(0.8X < 18.312)$

$= P(\chi^2_{10} < 18.312)$

$= 0.9499296$