Calculus Assignment

1) $\lim_{h\to 0} \frac{2(-3+h)^2 - 18}{h}$

$\lim_{h\to 0} \frac{2(h^2 - 6h + 9) - 18}{h}$

$\lim_{h\to 0} \frac{2h^2 - 12h + \cancel{18} - \cancel{18}}{h}$

$\lim_{h\to 0} = \frac{2\cancel{h}(h-6)}{\cancel{h}}$, $h \to 0, \therefore h \neq 0$

$= 2(0-6)$

$= 2(-6)$

$= -12$ //

2) $g(x) = 2x \quad , x < 6$

$= x - 1$

$g(4) = 2(4) = 8$

$\lim_{x\to 4} g(4) = \lim_{x\to 4} 2x = 2(4) = 8$

$\therefore g(x)$ is continuous at $x = 4$

$\lim_{x\to 6^-} 2x = 2(6) = 12$

$\lim_{x\to 6^+} x - 1 = 6 - 1 = 5$

$\lim_{x\to 6^-} \neq \lim_{x\to 6^+}$

$\therefore g(x)$ is discontinuous at $x = 6$ //

3 $\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}}$

$\lim_{x \to 9} \frac{(x-9)(3+\sqrt{x})}{9-x}$

$\lim_{x \to 9} \frac{\cancel{(x-9)}(3+\sqrt{x})}{-\cancel{(x-9)}}$, $x \to 9, \therefore x-9 \neq 0$

$\lim_{x \to 9} -(3+\sqrt{x}) = -(3+\sqrt{9})$

$= -(3+3)$

$= -6$ or 0

4) $f(x) = \frac{4x+5}{9-3x}$

$f(-1) = \frac{4(-1)+5}{9-3(-1)} = \frac{-4+5}{9+3} = \frac{-1}{12}$

$\therefore f(x)$ is continuous at $x = -1$

$f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$

$\therefore f(x)$ is continuous at $x = 0$.

$f(3) = \frac{4(3)+5}{9-3(3)} = \frac{17}{0} = \infty$

$\therefore f(x)$ is discountinuous at $x = 3$

5) $\lim\limits_{x \to \infty} \dfrac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$\lim\limits_{x \to \infty} \dfrac{e^{x}(6e^{4} - e^{-2})}{e^{x}(8e^{4} - e^{2} + 3e^{-1})}$

$= \dfrac{327.45356}{430.4997825}$

$= 0.760635843$

6) $g(y) = y^2 + 5 \quad , y < -2$
$\qquad 1 - 3y \quad , y \geq -2$

a) $\lim\limits_{y \to 6} 1 - 3y = 1 - 3(6) = 1 - 18 = -17$

b) $\lim\limits_{y \to -2^-} y^2 + 5 = (-2)^2 + 5 = 4 + 5 = 9$

$\lim\limits_{y \to 2^+} 1 - 3y = 1 - 3(-2) = 1 + 6 = 7$

7) $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$g'(t) = (4t^2 - t)\dfrac{d(t^3 - 8t^2 + 12)}{dt} + t^3 - 8t^2 + 12 \dfrac{d(4t^2 - t)}{dt}$

$= (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$

$= (12t^4 - 64t^3 - 3t^3 + 16t^2)$
$\quad + (8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12)$

$= 80t^4 - 132t^3 + 24t^2 + 96t - 12$

8 $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

$R'(w) = \dfrac{(2w^2+1)\frac{d(3w+w^4)}{dw} - (3w+w^4)\frac{d(2w^2+1)}{dw}}{(2w^2+1)^2}$

$= \dfrac{(2w^2+1)(3+4w^3) - (3w+w^4)(4w)}{(2w^2+1)^2}$

$= \dfrac{6w^2 + 8w^5 + 3 + 4w^3 - 12w^2 - 4w^5}{(2w^2-1)^2}$

$= \dfrac{4w^5 + 4w^3 - 6w^2 + 3}{(2w^2-1)}$ //

$= \cancel{4w^3(w^2+1) - 3(2w^2-1)}$

9) $f(x) = 2e^x - 8^x$

$f'(x) = 2e^x - 8^x \log 8$ //

10) $y = z^5 - e^2 \ln(z)$

$\dfrac{dy}{dz} = 5z^4 - \left[e^2 \dfrac{d}{dz}\ln(z) + e^2 \ln(z)\right]$

$= 5z^4 - \dfrac{e^2}{z} + e^2 \ln(z)$

//

c) $f(t) = \ln(1+t^2)$

$f'(t) = \frac{1}{1+t^2} \cdot \frac{d(1+t^2)}{dt}$

$= \frac{2t}{1+t^2}$

$f''(x) = \frac{(1+t^2)(2) - 2t(2t)}{(1+t^2)^2}$

$= \frac{2+2t^2 - 4t^2}{(1+t^2)^2}$

$= \cancel{2-2t^2} \; \frac{2-2t^2}{(1+t^2)^2}$

$= \frac{2(1-t^2)}{(1+t^2)^2}$

13) $x^3 - 3x^2 - 9x + 12 = f(x)$

$f'(x) = 3x^2 - 6x - 9$

$0 = 3(x^2 - 2x - 3)$

$x^2 - 2x - 3 = 0$

$x^2 - 3x + x - 3 = 0$

$x(x-3) + 1(x-3) = 0$

$(x+1)(x-3) = 0$

$\therefore x = -1$ or 3

$f''(x) = 6x - 6$

$f''(-1) = 6(-1) - 6$

$= -6 - 6$

$= -12$

$f''(3) = 6(3) - 6$

$= 18 - 6$

$= 12$

maxima at $x = -1$.

minima at $x = 3$.

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$\therefore$ maximum value $= f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= 17$

Minimum value $= f(3) = (3)^3 - 3(3)^2 - 9(3) + 12$

$= -27 + 12$

$= -15$

(14) $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

$0 = 12(x^2 - 3x + 2)$

$0 = x^2 - 3x + 2$

$x^2 - 2x - x + 2 = 0$

$x(x-2) - 1(x-2) = 0$

$(x-1)(x-2) = 0$

$\therefore x = 1$ or 2

$f''(x) = 24x - 36$

$f''(1) = 24(1) - 36$

$= -12$

$f''(2) = 24(2) - 36$

$= 48 - 36$

$= 12$

$\therefore$ maximum value $= f(1) = 4(1)^3 - 18(1)^2 + 24(1) - 7$

$= 4 - 18 + 24 - 7$

$= 3$

minimum value $= f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$

$= 32 - 72 + 48 - 7$

$= 1$

15 $f(x) = x^2(x+3)$

$= x^3 + 3x^2$

$f'(x) = 3x^2 + 6x$

$3x(x+2) = 0$

$x + 2 = 0$

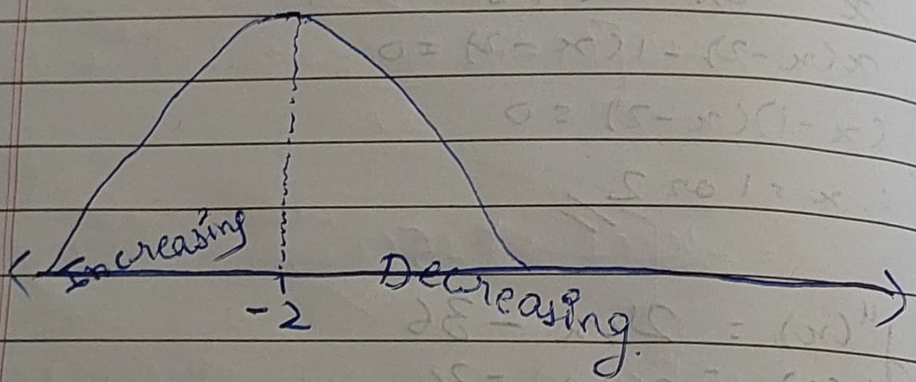
$x = -2$

$f''(x) = 6x + 6$

$f''(2) = 6(-2) + 6$

$= -12 + 6$

$= -6$



16 $f(x) = 3x^2 - 6x$

$f'(x) = 6x - 6$

$6(x-1) = 0$

$x - 1 = 0$

$x = 1$

$f''(x) = 6$

$f''(1) = 6$