HOMEWORK

i)

$$\left(1-\frac{d^{4}}{4}\right)^{4} = e^{-\delta}$$

$$-\delta = \ln\left(1-\frac{0.08}{4}\right)^{4}$$

$$\delta = \ln(1-0.02)^{-4}$$

$$\delta = 8.081\%$$

ii)

$$e^{\delta} = 1+i$$

$$i = e^{\delta}-1$$

$$i = e^{0.08081}-1$$

$$i = 8.41648\%$$

iii)

$$\left(1-\frac{d^{12}}{12}\right)^{12} = e^{-\delta}$$

$$1-\frac{d^{12}}{12} = e^{-\delta/12}$$

$$d^{12} = \left[1-e^{-\delta/12}\right]12$$

$$d^{12} = 8.0539\%$$

$$\delta(t) = 0.004t + 0.0002t^{2}$$

$$PV_0 = 1000\left[e^{-\int_0^{10} 0.004t+0.0002t^{2}\,dt}\right]$$

$$PV_0 = 1000\,e^{-10[0.002t^{2} + 0.0002/3}$$

$$PV_0 = 1000\,e^{-10[0.2+0.2/3]}$$

$$PV_0 = 765.9283$$

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8) i) Since we know that $v = \frac{1}{1+i}$

$\Rightarrow$ $d = 1 - v \quad \ldots (v = 1-d)$

$\Rightarrow$ $d = 1 - \frac{1}{1+i}$

$\Rightarrow$ $d = \frac{1+i-1}{1+i}$

$\Rightarrow$ $d = i\left(\frac{1}{1+i}\right) \quad \ldots \left(\frac{1}{1+i} = v\right)$

$\Rightarrow$ $\boxed{d = iv}$

ii) a) $d^{(2)} = ?$

$\left(1 - \frac{d^2}{2}\right)^2 = (1-d)^4 \quad \ldots (d = 0.0225)$

$1 - \frac{d^2}{2} = (1-d)^2$

$d^2 = (1 - (1-d^2) \times 2$

$d^2 = 8.89875\%$

b. $d^2 = ?$

$\left(1 - \frac{d^2}{2}\right)^2 = \left(1 - \frac{d^{1/2}}{1/2}\right)^{1/2} \quad \ldots (d^{(1/2)} = 0.05)$

$1 - \frac{d^2}{2} = \left((1 - 0.05)2\right)^{1/4}$

$d^2 = \left[1 - (1-0.1)^{1/4}\right]$

$d^2 = 5.19925\%$

$(1+i)^{91/365} = \left(1 - \frac{91}{365} d\right)^{-1}$

$$\left(1 - \frac{91d}{365}\right) = (1+i)^{-91/365}$$

$$d = \left[1 - (1+i)^{-91/365}\right] \times \frac{365}{91}$$

$$d = 4.84946\%$$

7) i) Same as ③ → i.

ii) $i = 0.08$.

a) $$\left(1 - \frac{d^{12}}{12}\right)^{12} = (1+i)^{-1}$$

$$d^{12} = \left[1 - (1+i)^{-1/12}\right] \times 12$$

$$d^{12} = \left[1 - (1.08)^{-1/12}\right] 12$$

$$d^{12} = 7.67147$$

b) $$\left(1 + \frac{i^{(365)}}{365}\right)^{365} = 1+i$$

$$i^{365} = \left[(1+i)^{1/365} - 1\right] \times 365$$

$$i^{365} = 7.6969\%$$

c) $$e^{\delta} = 1+i$$

$$\delta = \ln(1+i) = \ln(1.08)$$

$$\delta = 7.6961\%$$

d) $$\left(1 + \frac{i^{(0.5)}}{0.5}\right)^{0.5} = 1+i$$

$i^{(0.5)} = \left[(1+i)^{1/0.5} - 1\right] \times 0.5$

$i^{(0.5)} = 8.2218\%$

(ii) $d^{(4)} = 10\% = 0.1000$

(a) $\left(1+\frac{i^{(2)}}{2}\right)^{2} = \left(1-\frac{d^{(4)}}{4}\right)^{-4}$

$1+\frac{i^{(2)}}{2} = \left(1-\frac{d^{(4)}}{4}\right)^{-2}$

$i^{(2)} = \left[\left(1-\frac{d^{(4)}}{4}\right)^{-2} - 1\right] \times 2$

$i^{(2)} = 6.618175\%$

(b) $\left(1+\frac{d^{(12)}}{12}\right)^{12} = \left(1-\frac{d^{(4)}}{4}\right)^{4}$

$d^{(12)} = \left[1-\left(1-\frac{d^{4}}{4}\right)^{1/3}\right] \times 12$

$d^{(12)} = 6.03025\%$

$\frac{220000}{200000} = 1+i$

$i = 0.1 = 10\%$

Time period between 15th April, 2005 to 17th July, 2005 = 3 month

Town population 17th July, 2005 $= \left[(1+i)^{3/12}\right] \times 200000$

$= [1.1]^{3/12} \times 200{,}000$

$= 204822.7378.$

6) i) $x = R(1-0.3)$ ...(cash payment) Retail Price - R

$y = R(1-0.25)^2$ .... (Six months Credit)

$$\frac{x}{y} = \frac{\cancel{R}(1-0.3)}{\cancel{R}(1-0.25)^2} = 1-d$$

$$\frac{0.7}{(0.75)^2} = 1-d$$

$$d = \frac{0.75-0.7}{0.75}$$

$$d = \frac{0.05}{0.75} = \frac{5}{75} = 6.6667\%$$

ii) $d^{(12)} = [1-(1-d)^{1/12}] \times 12$

$d^{12} = 6.8798\%$

$i^{(2)} = [(1-d)^{-1/2} - 1] \times 2.$

$i^{(2)} = 7.02003\%$

$\delta = -\ln(1-d)$

$\delta = 6.8996\%$

7) PV = 20,000

AV = 26 000

i) $$\left(1+\frac{i^{(4)}}{4}\right)^{4 \times (17/12)} = \frac{26000}{20,000} = 1.3.$$

$$\left(1+\frac{i^{(4)}}{4}\right) = (1.3)^{3/17}$$

$$\left(1+\frac{i^{(4)}}{4}\right) = (1.3)^{3/17}$$

$$i^{4} = \left[(1.3)^{3/17} - 1\right] \times 4$$

$$i^{(4)} = 18.95525\%$$

ii) $$\left(1-\frac{d^{(2)}}{2}\right)^{2} = \left(1+\frac{i^{(4)}}{4}\right)^{-4}$$

$$\left(1-\frac{d^{(2)}}{2}\right) = \left(1+\frac{i^{(4)}}{4}\right)^{-2}$$

$$d^{(2)} = \left[1-\left(1+\frac{i^{(4)}}{4}\right)^{-2}\right] \times 2$$

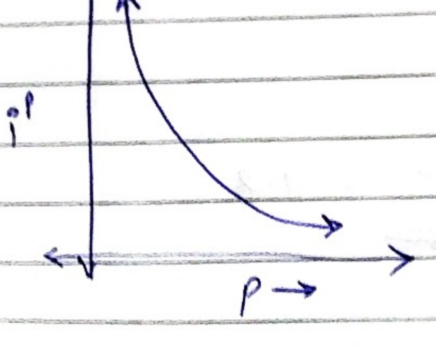
$$d^{2} = 17.68828\%$$

iii) $$\frac{26\,000}{2000} = \left(1+\frac{17}{12}i\right)$$

$$\frac{17}{12} i = 1.3 - 1$$

$$i = \frac{(0.3)(12)}{17}$$

$$i = 21.17647\%$$

8) 

On increasing the value of p, the nominal rate of interest ($i^{(p)}$) tends to get closer to the value of force of interest.

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q) i) Force of interest refers to a nominal interest rate or a discount rate compounded infinite number of times per time period

ii) $i^{(2)} = 0.0775$

$$\left(1 + \frac{i^{(2)}}{2}\right)^2 = 1+i$$

$$i = \left(1 + \frac{0.0775}{2}\right)^2 - 1$$

$$i = 7.90015\%$$

$$\delta = \ln(1+i)$$
$$\delta = \ln(1.07900015$$
$$\delta = 7.60361\%$$

$$\left(1 + 2i^{(1/2)}\right)^{1/2} = 1+i$$
$$1 + 2i^{(1/2)} = (1+i)^2$$
$$2i^{(1/2)} = (1+i)^2 - 1$$
$$i^{1/2} = \left[\frac{(1+i)^2 - 1}{2}\right]$$
$$i^{1/2} = 8.21221\%$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = (1+i)^{-1}$$

$$d^{12} = \left(1 - (1+i)^{-1/12}\right) \times 12.$$

$$d^{12} = 7.579568\%$$

$$\delta(t) = \begin{cases} 0.08 & , 0 \leq t \leq 5 \\ 0.06 & 5 \leq t \leq 10 \\ 0.04 & t \geq 10 \end{cases}$$

when $0 \leq t \leq 5$

$$v(t) = -\int_0^t 0.08\, ds$$

$$= e^{-[0.08s]_0^t}$$

$$= e^{-0.08t} \cdot v(t)$$

when $5 \leq t \leq 10$

$$v(t) = e^{-0.08(5)} e^{-\int_5^t 0.06 ds}$$

$$= e^{-0.4} \left(e^{-[0.06s]_5^t}\right)$$

$$= e^{-0.4} \left(e^{-0.06t+0.3}\right)$$

$$= e^{-0.4} \left(e^{-0.06t+0.3}\right)$$

$$v(t) = e^{-0.1-0.06t}$$

when $t \geq 10$

$$v(t) = e^{-0.1-0.06(10)} \; e^{-\int_{10}^t 0.04 ds}$$

$$= e^{-0.1-0.6} \; e^{-(0.04s)_{10}^t}$$

$$= e^{-0.7} \; e^{-0.04t+0.4}$$

$$v(t) = e^{-0.3-0.04t}$$

11) a $T = 9$ month loan

$C = 50,000$

$d = 0.18$ p.a

$$PV_0 = C(1-d)^{T/12}$$

$$PV_0 = 50,000 \cdot (1-0.18)^{9/12}$$

$$PV_0 = 43085.42623$$

5) $\delta = 6\%$

$$\left(1 - \frac{d^{12}}{12}\right)^{12} = e^{-\delta}$$

$$d^{12} = \left[1 - e^{-\delta/12}\right] \times 12$$

$$d^{12} = 5.98502$$

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iii) $d^4 = 6\%$

$$\left(1 + \frac{i^{(12)}}{12}\right)^{12} = \left(1 - \frac{d^4}{4}\right)^{-4}$$

$$1 + \frac{i^{(12)}}{12} = \left(1 - \frac{d^4}{4}\right)^{-1/3}$$

$$i^{(12)} = \left[\left(1 - \frac{d^{(4)}}{4}\right)^{-1/3} - 1\right] \times 12.$$

$$i^{12} = 6.060708\%$$

c) $d < \delta < i^{(4)} < i$

12) a. $AV_6 = 7049 \left(1 + \frac{i^{(12)}}{12}\right)^{24} \left(1 + 10(0.015)\right)\left(1 - \frac{d}{12}\right.$

.... $\left(i^{(12)} = 0.06\right)$, $\left(d^{12} = 0.06\right)$

$$AV_6 = 7049 (1.127156)(1.15)(1.09442)$$

$$AV_6 = 9999.883762.$$

13) i) $\cancel{AV_2}$ $AV = 2x \quad C = x$

$$AV = C(1 + 0.1)^t$$

$$2x = x(1.1)^t$$

$$\ln(2) = t \ln(1.1)$$

$$t = \frac{\ln(2)}{\ln(1.1)}$$

$t = 7.2725$ years

ii) $$\left(1 - \frac{0.1}{12}\right)^{-12t} = 2$$

$$\ln 2 = -12t \ln\left(1 - \frac{0.1}{12}\right)$$

$$t = \frac{\ln(2)}{-12 \ln\left(1 - \frac{0.1}{12}\right)} = 6.90255$$