SRM G

Q2] i) The three parameters of GEV are:

a) A location parameter α

b) A scale parameter β, this is always greater than 0.

c) A shape parameter γ.

ii) The first two parameters rescale the distribution & determine shift & stretch of the distribution. The third parameter determines overall shape of distribution & its sign.

iii) Analogous to following in standard distribution

1. Mean
2. Standard ~~skewness~~ deviation
3. skewness

Q1]

i) Let T_i be the time until default of bond i where $i = 1, 2, 3$.
We want to calculate the joint probability:
$P(T_1 <= 1, T_2 <= 1, T_3 \leq 1) = C[u1, u2, u3]$
where $u_i = P[T_i <= 1] = 0.1$ for $i = 1, 2, 3$.
$\alpha = 2$
$P(T_1 <= 1, T_2 <= 1, T_3 <= 1) = C[u1, u2, u3]$
$= \exp(-((-\ln(u1))^2 + (-\ln(u2))^2 + (-\ln(u3))^2)^{1/2})$
$= 0.0185$ or 1.85%.

ii) The Gumbel copula exhibits upper tail dependence, it exhibits no lower tail dependence.
The Gumbel copula is suitable if we believe that three investments are likely to behave similarly as term approaches five years.
If one bond defaults it can be an indication of problems in the industry sector or economy so other investments are also likely to default early on.
If the investments issued by companies within the industry are much more closely associated, then a copula that exhibits both lower & upper tail dependence may be more appropriate.

Q3] i) 4 ways of measuring the tail weight of a distribution:
1. Existence of moments
2. Limiting density ratio
3. Hazard rate
4. Mean residual

ii) $\alpha_1 = 0.2$ $\quad \alpha_2 = 0.3$

$$= \lim_{x\to\infty} \frac{f_{\alpha=0.2}(x)}{f_{\alpha=0.3}(x)} = \lim_{x\to\infty} \left(\frac{0.2\lambda^{0.2}}{(\lambda+x)^{1.2}} \Big/ \frac{0.3\lambda^{0.3}}{(\lambda+x)^{1.3}} \right)$$

$$= \frac{2}{3} \times \frac{1}{\lambda^{0.1}} \lim_{x\to\infty} (\lambda+x)^{0.1}$$

$$= \infty$$

$\therefore$ distribution with $\alpha = 2$ has a thicker tail.

Q4] i) $_{30}p_{50} = 0.6 \to$ Male

$_{30}p_{50} = 0.65 \to$ female

$_{30}q_{50} = 0.4 \to$ Male $\quad$ $_{30}q_{50} = 0.35 \to$ female

X R.V. representing future lifetime of Male

Y R.V. representing future lifetime of Female

$P(X \le 30) = 0.4$ & $P(Y \le 30) = 0.35$

We need $P(X \le 30, Y \le 30)$

i) Gumbel copula $\alpha = 3.5$

$u = 0.4$ & $v = 0.35$

$$C[u,v] = \exp\left(-(-\ln u)^\alpha + (-\ln v)^\alpha\right)^{1/\alpha}]$$

$$C[u,v] = 0.2996$$

ii) $\alpha = 3.5$ Clayton

$$C[u,v] = (u^{-\alpha} + v^{-\alpha} - 1)^{-1/\alpha}$$

$$C[u,v] = 0.30595$$

iii) $\alpha = 3.5$ Frank

$$C[u,v] = \left(-\frac{1}{\alpha}\right) \ln\left[\frac{1 + (e^{-\alpha u} - 1)(e^{-\alpha v} - 1)}{(e^{-\alpha} - 1)}\right]$$

$$C[u,v] = 0.2273$$

iv) Clayton copula gives highest probability of claims because it exhibits lower tail dependence. If one life does not survive for long then there is high prob. other life will also not survive for long.

Q5] i) Mean residual life of 2 year old phone

$$= \frac{\int_x^\infty (1 - F(y))\,dy}{1 - F(x)}$$

$$= \int_x^\infty e^{-\lambda y}\,dy \Big/ e^{-\lambda x}$$

$$= 1/\lambda = 8 \text{ years.}$$

ii) The expected lifetime of a new phone $= \frac{1}{\lambda} = 8$ years
& expected lifetime of a two year old phone is also 8 yrs.
This is due to memory loss property of exponential dist.
Hence, exponential dist is not appropriate for measuring Mean residual life of a product.