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PROBABILITY & STATISTICS

1. (i)

x	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
4	-9.4	88.36
7	-6.4	40.96
9	-4.4	19.36
9	-4.4	19.36
11	-2.4	5.76
15	1.6	2.56
18	4.6	21.16
18	4.6	21.16
18	4.6	21.16
25	11.6	134.56
134	0	2627.84

$$\bar{x} = \frac{134}{10} = 13.4$$

$$\text{median} = \left[\frac{n+1}{2}\right]^{th} \text{term} = 5.5^{th} \text{ term}$$

$$\cancel{= 9 + \left[\frac{11-9}{2}\right] = 10}$$

$$= 11 + \left(\frac{15-11}{2}\right) = \underline{13}$$

$$\sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}} = \sqrt{\frac{2627.84}{10}}$$

$$\therefore \sigma = \underline{16.21}$$

mode = 18

$$Q_3 = \left[\frac{3(n+1)}{4}\right]^{th} \text{ term} = 8.25^{th} \text{ term}$$

$$= 8^{th} \text{ term} + 0.25(9^{th} - 8^{th} \text{ term})$$

$$= 18 + 0.25(0) = \underline{\underline{18}}.$$

$$Q_1 = \left[\frac{n+1}{4}\right]^{th} \text{ term} = 2.75^{th} \text{ term}$$

$$= 2\text{nd term} + 0.75(3\text{rd} - 2\text{nd term})$$

$$= 7 + 0.75(2) = \underline{\underline{8.5}}.$$

$$\therefore IQR = Q_3 - Q_1 = 18 - 8.5 = \underline{\underline{9.5}}.$$

(ii)

x	f	$x_i f_i$	cf	$\lvert x_i - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$	$f_i (x_i - \bar{x})^2$
0	5	0	5	2	10	20
1	11	11	16	1	11	11
2	26	52	42	0	0	0
3	15	45	57	1	15	15
4	3	12	60	2	6	12
	60	120				58

$$\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{120}{60} = \underline{\underline{2}}.$$

$$\text{median} = \left[\frac{n+1}{2}\right]^{th} \text{ term} = 30.5^{th} \text{ term} = \underline{\underline{2}}.$$

mode $= \underline{\underline{2}}.$

$$Q_3 = \left[\frac{3(n+1)}{4}\right]^{th} \text{ term} = 45.75^{th} \text{ term} = 3$$

$$Q_1 = \left(\frac{n+1}{4}\right)^{th} \text{ term} = 15.25^{th} \text{ term} = 1$$

$$IQR = 3 - 1 = \underline{\underline{2}}.$$

$$\sigma = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}} = \sqrt{\frac{58}{60}}$$

$$= 0.983$$

2. $\lambda = 0.5$

$\therefore X \sim P(\lambda = 0.5)$

(i) $P(X=0) = \dfrac{e^{-\lambda} \cdot \lambda^{x}}{x!}$

$$= \frac{e^{-0.5} \times 0.5^{0}}{0!} = e^{-0.5}$$

$\therefore P(X=0) = 0.60653$

(ii) $P(X=0) = 0.60653$

$\therefore 1 - P(X=0) = 0.39347$

0.39347 is the probability of getting one or more claim in one year.

$\therefore$ Total probability $= 3[P(X=0)]^2 \times [1 - P(X=0)]$

$= 3[(0.60653)^2 \times (0.39347)]$

$= 0.43424$

(iii) $P(\text{no claim for 2 years}) = [P(X=0)]^2$

$= (0.60653)^2$

$= 0.3679$

3. $\alpha = 2$

$\lambda = 1/2$

Range : $0 < x < \infty$

(i) $E(X) = \frac{\alpha}{\lambda} = \frac{2}{1/2}$

$\therefore E(X) = 4$

$V(X) = \frac{\alpha}{\lambda^2} = \frac{2}{(1/2)^2} = 8$

$\therefore V(X) = \sigma^2$

$\therefore \sigma = \sqrt{V(X)} = \sqrt{8} = \underline{\underline{2.82842}}$

(ii) According to me, the data is positively skewed.

(iii) $F_X(x) = \int_0^x f_X(x)\, dx.$

$= \int_0^x \frac{(0.5)^2}{\lceil 2} x^{(2-1)} \cdot e^{-x/2}$

$= (0.5)^2 \int_0^x x \cdot e^{-x/2}\, dx.$

$= (0.5)^2 \left[x \int e^{-x/2}\, dx - \int \frac{dx}{dx}\, dx \int e^{-x/2}\, dx \right]_0^x$

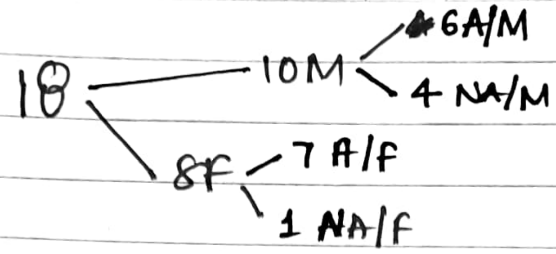
$= (0.5)^2 \left\{ \left[x \cdot \frac{e^{-x/2}}{-1/2} - (1)\left(\frac{e^{-x/2}}{(-1/2x)(-1/2)} \right) \right\} \right._0^x$

$= (0.5)^2 \left[\cancel{-2ex} -2xe^{-x/2} - 4e^{-x/2} \right]_0^x$

$= (0.5)^2 \left[\left(-2xe^{-x/2} - 4e^{-x/2} \right) - (-4) \right]$

$= 1 - \frac{0.5x}{\sqrt{e^x}} - \frac{1}{\sqrt{e^x}}$

4.



5. $L1 \xrightarrow{1D} 40\%$

$L2 \xrightarrow{2D} 50\%$

$L3 \xrightarrow{3D} 10\%$

Probability of parcel being late :

~~P(Late / 1) =~~

$P(1 / Late) = 20\%$

$P(2 / Late) = 40\%$

$P(3 / Late) = 10\%$

$$P(Late / 2) = \frac{P(2 / Late) \cdot P(2)}{(0.2)(0.4) + (0.4)(0.5) + (0.1)(0.1)}$$

$$= \frac{0.2}{0.29} = \boxed{\frac{20}{29}}$$

6. ~~X ~ (U~~ $X \sim U(1,2)$

$$Y = \frac{3}{6-X}$$

To determine PDF of Y.

$$\therefore Y = \frac{3}{6-X} \Rightarrow 6-X = \frac{3}{Y}$$

$$\therefore X = 6 - \frac{3}{Y} = \frac{6Y-3}{Y}$$

$$P(Y \le y) = F_Y(y)$$

$$\therefore F_Y(y) = P\left(\frac{3}{6-X} \le y\right) = P\left(X \ge \frac{6Y-3}{Y}\right)$$

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now,

$\cancel{f_X(y)}$ $F_X(x) = \int_{LL}^{x} f_X(x)\, dx$

$X = 6 - \frac{3}{Y}$

$\therefore \frac{dx}{dy} = \frac{-(-3)}{Y^2} = \frac{3}{Y^2}.$

$\therefore f_Y(y) = \frac{3}{y^2}$.

7.

X	$P(X=x)$	fx	$\cancel{fx^2}$	fx^2
1	0.05	0.05	~~0.0025~~	0.05
2	0.15	0.3	~~0.045~~	0.6
3	0.35	1.05	~~0.3675~~	3.15
4	0.4	1.6	~~0.64~~	6.4
5	0.05	0.25	~~0.0125~~	1.25
		3.25	~~1.0675~~	11.45

(i) $F(3) = P(X=1) + P(X=2) + P(X=3)$

$= 0.05 + 0.15 + 0.35$

$\therefore F(3) = 0.55$

(ii) $E(X) = \Sigma fx = 3.25$

(iii) $V(X) = E(X^2) - (E(X))^2$

$= 11.45 - (3.25)^2 = 0.8875$

(iv) Coefficient of Skewness = $\frac{3(\text{Mean} - \text{Median})}{\sigma}$

Median = 3

$\sigma = \sqrt{V(X)} = \sqrt{0.8875} = 0.94207$

$\therefore$ Co-efficient of Skewness = $\frac{3(3.25 - 3)}{0.94207}$

$= \underline{\underline{0.796119}}$

8. $p = 0.2$

$n = 15$

$P(X < 3$ exceed $1000)$

$\therefore \quad X \sim Bin(15, 0.2)$

x can take values $0, 1, 2, \dots 1000$

$\therefore P(X < 3) = P(X=0) + P(X=1) + P(X=2)$

$= [{}^{15}C_0 \cdot (0.2)^0 \cdot (0.8)^{15}] + [{}^{15}C_1 (0.2)^1 (0.8)^{14}]$

$+ [{}^{15}C_2 (0.2)^2 (0.8)^{13}]$

$= 0.035184 + 0.131941 + 0.230897$

$\therefore P(X<3) = \underline{\underline{0.39802}}$

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~~9~~ ~~$p = 0.01$~~ $\therefore X \sim Bin(3, 0.01)$

~~$n = 3$~~

~~$P(X=7) = {}^nC_x \cdot p^x \cdot q^{n-x}$~~

~~$= {}^3C_7$~~

9. $p = 0.01$

$x = 3$

$n = 7$

~~$X \sim Bin(7, 0.01)$~~

1 2 3 4 5 6 7 ← wins third price

$\therefore$ X follows negative binomial.

$\therefore X \sim Bin(7, 0.01)$

$P(X=3) = {}^{n-1}C_{x-1}\, p^x \cdot q^{n-x}$

$= {}^6C_2 \cdot (0.01)^3 \cdot (0.99)^4$

$\therefore P(X=3) = 0.000014409$

10. $\lambda = 2$ $\qquad X \sim \text{Poisson}(2)$

$P(X > 2) = 1 - P(X \leq 2)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \left[\frac{e^{-2} \cdot 2^0}{0!} + \frac{e^{-2} \cdot 2^1}{1!} + \frac{e^{-2} \cdot 2^2}{2!}\right]$

$= 1 - (0.135335 + 0.27067 + 0.27067)$

$= 1 - 0.676676\,1$

$\therefore P(X > 2) = 0.32332$

11. $P(\text{boys}) = 0.5$

Let x be success of having a boy.

$\therefore p = 0.5$

both events are indipendent.

$\therefore P(\text{having next 3 boys}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$\therefore P(\text{having next 3 boys}) = \frac{1}{8}$

12. $\lambda = 10$

$\therefore X \sim \text{Poi}(10)$

but, we have find time,

$\therefore X \sim \text{Exp}(\lambda = 10)$

$\therefore P(X < 0.1) = 1 - e^{-\lambda x}$

$= 1 - e^{-10(0.1)}$

$\therefore P(X < 0.1) = 0.63212$

13. $X \sim U(75, 120)$

To find $P(80 < X < 100)$.

$\therefore$ ~~$P(X <$~~ $P(80 < X < 100) = F_X(100) - F_X(80)$

$$= \frac{100-75}{120-75} - \frac{80-75}{120-75}$$

$$= \frac{25-14}{45}$$

$$= 0.2222$$

14. $X \sim \text{Gamma}(5, 0.4)$

$\therefore \alpha = 5 \quad ; \quad \lambda = 0.4$

$P(X < 22.89)$

multiplying by 2λ

$\therefore P(2\lambda X < 22.89(2\lambda))$

$= P(2\lambda X < 18.312)$

$= P(\chi^2_{2\alpha} < 18.312)$

$= P(\chi^2_{10} < 18.312)$

using interpolation,

0.5 $\begin{cases} 18 = 0.9450 \\ 18.5 = 0.9529 \end{cases}$ 0.0079

$\therefore 1 - \frac{0.079}{0.5}$

$\therefore 0.312 - \frac{0.079}{0.05}(0.312) = 0.049296$

$\therefore P(\chi^2_{10} < 18.312) = 0.9450 + 0.049296$

$= 0.994296$

15. (i) $\dfrac{k}{(x+5)^4}$, $x > 0$.

we know that,

$$\int_{-\infty}^{\infty} f_x(x)\,dx = 1.$$

$$\therefore \int_0^{\infty} k \div (x+5)^4 = 1.$$

$$k \int_0^{\infty} (x+5)^{-4} = 1.$$

$$k \left[\frac{(x+5)^{-3}}{-3} \right]_0^{\infty} = 1$$

$$\frac{-k}{3} \left[(x+5)^{-3} \right]_0^{\infty} = 1$$

$$k \left[\frac{(0) + 5^{-3}}{-3} \right] = 1$$

$$\frac{k}{375} = 1$$

$$\therefore k = 375$$

(ii) $$F(10) = \int_0^{10} \frac{375}{(x+5)^4}\,dx$$

$$= 375 \int_0^{10} (x+5)^{-4}\,dx.$$

$$= \cancel{375 \int_0^{10}}\ 375 \left[\frac{(x+5)^{-3}}{-3} \right]_0^{10}$$

$$= \cancel{125}\ -125\,(15^{-3} - 5^{-3})$$

$$\therefore F(10) = 0.96296$$

(iii) $E(X) = \int_0^\infty x \cdot \frac{375}{(x+5)^4} \, dx$

$= 375 \int_0^\infty \frac{x}{(x+5)^4} \, dx$

put $x+5 = t \Rightarrow x = t-5$

$\frac{dt}{dx} = 1$

$\therefore dt = dx$

~~37~~ at $x = 0$, $t = 0$

at $x = \infty$, $t = \infty$

$\therefore E(X) = \int_0^\infty (t-5)(t)^{-4} \, dt$

$= \int_0^\infty \frac{1}{t^3} - \frac{5}{t^4} \, dt$

$= \int_0^\infty t^{-3} - 5t^{-4} \, dt$

$= \left[\frac{t^{-2}}{-2} - \left(\frac{5t^{-3}}{-15} \right) \right]_0^\infty$

$= \left[\frac{-1}{2t^2} \right]_0^\infty -$