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Assignment - 1
Unit 1 & 2
Statistics and Risk Modelling - 4

Ans 1
(i) Let T_i be the time until bond default.

Joint probability

$$P[T_1 \le 1, T_2 \le 1, T_3 \le 1) = C[u_1, u_2, u_3$$

where $u_i = P[T_i < 1] = 0.1$ for $i = 1, 2, 3$

Using Gumbel copula with $\alpha = 2$,

$$P[T_1 \le 1, T_2 \le 1, T_3 \le 1) = C[u_1, u_2, u_3$$

~~$= \exp\{((-\ln u_1)^2 + (-\ln u_2)^2 + (-\ln u_3$~~

$$= \exp\{-((-\ln u_1)^2 + (-\ln u_2)^2 + (-\ln u_3)^2)^{1/2}\}$$

$$= \exp\{-(3(-\ln 0.1)^2)^{1/2}\}$$

$$= 0.0185 \text{ or } 1.85\%$$

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ii) Suitability of Gumbel Copula

a) Gumbel copula exhibits (non-zero) upper-tail dependence, the degree of which can be varied by adjusting the single parameter.

b) However, the Gumbel copula is appropriate if we believe that the three investments are likely to behave similarly as the term approaches five years but not at early durations.

c) This is unlikely to be the case though. If one bond defaults early on, then it may be indicative of problems in the industry sector or the economy ~~sector~~ and so other investments may also likely to default early on.

d) If we believe performance of investments issued by companies within the industry are much more closely associated (eg subject to the same systemic and operational risk factors), then a copula that exhibits both lower and upper tail dependence such as Student's T copula, may be more appropriate.

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Q2

i) Three parameters of the GEV distribution are:-

a) a location parameter, α

b) a scale parameter, $\beta > 0$

c) a shape parameter, γ

ii) These parameters ~~determine~~ α and β just rescale (shift and stretch) the distribution.

They are analogous to (but do not usually correspond to) the mean and standard deviation.

The parameter γ determines the overall shape of the distribution (analogous to skewness) and its sign (positive, negative or zero) results in three different shaped distributions.

iii. α = mean, β = standard deviation, γ = skewness

Q3.

i) Four ways of measuring the tail weight of distn :-
- existence of moments
- limiting density ratios
- hazard rate
- mean residual life.

ii) $$\lim_{x \to \infty} \frac{f_{\alpha=0.2}(x)}{f_{\alpha=0.3}(x)} = \lim_{x \to \infty} \left\{ \frac{0.2\lambda^{0.2} / (\lambda+x)^{1.2}}{0.3\lambda^{0.3} / (\lambda+x)^{1.3}} \right\}$$

$$= \frac{2}{3} \times \frac{1}{\lambda^{0.1}} \lim_{x \to \infty} (\lambda+x)^{0.1}$$

$$= \infty$$

The distribution with $\alpha = 0.2$ has a much thicker tail.

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Q4. Given $_{30}p_{50} = 0.6$ for male & $_{30}p_{50} = 0.65$ for female

$\therefore$ $_{30}q_{50} = 0.4$ for male & $_{30}q_{50} = 0.35$ for females

Let x be ~~future~~ future lifetime of male and Y be future lifetime of female. Then,
$P(X \le 30) = 0.4$ and $P(Y \le 30) = 0.35$

i) Gumbel copula with $\alpha = 3.5$
$u = 0.4$ and $v = 0.35$

$$C[u,v] = \exp\left\{-\left((-\ln u)^{\alpha} + (-\ln v)^{\alpha}\right)^{1/\alpha}\right\}$$

$$= \exp\left\{-\left((-\ln 0.4)^{3.5} + (-\ln 0.35)^{3.5}\right)^{1/3.5}\right\}$$

$= 0.2996$

ii) Clayton copula $\alpha = 3.5$

$$C[u,v] = \left(u^{-\alpha} + v^{-\alpha} - 1\right)^{-1/\alpha}$$

$$= \left(0.4^{-3.5} + 0.35^{-3.5} - 1\right)^{-1/3.5}$$

$= 0.30595$

iii) Frank copula
$\alpha = 3.5$

$$C[u,v] = \left(-1/\alpha\right) \ln\left\{1 + \frac{(e^{-\alpha u} - 1)(e^{-\alpha v} - 1)}{e^{-\alpha} - 1}\right\}$$

$$= \frac{-1}{3.5} \ln \left\{ 1 + \frac{\left(e^{-3.5\times0.4} - 1\right)\left(e^{-3.5\times0.35} - 1\right)}{e^{-3.5} - 1} \right\}$$

$$= 0.2273$$

iv) Clayton copula gives the highest probability of claims because it exhibits lower tail dependence. This means that if one life does not survive for long then there is a high probability that the other life will also not survive for long.
If deaths are independent then the probability of paying the benefit is 0.14.
However, two lives covered are related, so we would expect the probability of paying the benefit to be higher than under the assumption of independence. Hence Clayton is more appropriate.

Q5.

i) Mean residual life of two year old phone

$$= \frac{\int_x^\infty \{1 - F(y)\}\, dy}{1 - F(x)}$$

$$= \frac{\int_x^\infty e^{-\lambda y}\, dy}{e^{-\lambda x}}$$

$$= \cancel{Z}\ \frac{1}{\lambda} = 8 \text{ years}$$

ii. Expected life time of new phone $= \frac{1}{\lambda} = 8$ years

expected lifetime of a two year old phone is also 8 years.

This is due to memoryless property of exponential distn.
Hence the exponential distn is not appropriate for measuring Mean residual life of a product.