Financial Mathematics

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Assignment - 2

Q. 1 i. Loan Amount = 20000

Monthly Payments = 427.9

Time = 5 years

$$20000 = 427.9\,(12)\, a_{\overline{5}|}^{(12)} \;@\; i\%$$

$$\Rightarrow \quad 20000 = 427.9\,(12)\left(\frac{1-(1+i)^{-5}}{\left((1+i)^{1/12}-1\right)12}\right)$$

At $i = 10\%$, R.H.S = 20341

$i = 11\%$, R.H.S = 19916

$\therefore$ By interpolating, $i = 10.8\%$.

$\therefore$ APR = 10.8%.

ii. $$PV_1 = 427.9\,(12)\, a_{\overline{4}|}^{(12)} \;@\; i = 10.8\%.$$

$$= 427.9\,\cancel{(12)}\left(\frac{1-(1.108)^{-4}}{\left((1.108)^{1/12}-1\right)\cancel{12}}\right)$$

$$\cancel{= 427.9\,(41.18760914)} = 17622.03845$$

$$= 427.9\,(39.205369) = 16775.97728$$

$$16775.97728 = 274.49\,(12)\, a_{\overline{t}|}^{(12)} \;@\; 10.8\%.$$

$$\cancel{\frac{16}{}}\ \frac{16775.97728}{274.49} = \frac{\left(1-(1.108)^{-t}\right)\cancel{(12)}}{\left((1.108)^{1/12}-1\right)\cancel{12}}$$

$$1-(1.108)^{-t} = 0.52456676$$

$$-t = \cancel{-2.499}\ -7.2499$$

$$\boxed{t = \cancel{2.499}\ 7.2499}$$

iii. Total interest of initial loan => $427.9 \times \cancel{60}\ 48 - \cancel{20000}\ 16775.97728$
$= \cancel{5674}\ 3763.22$

∴ Total interest of restructed loan => $7.2499 \times 274.49 \times 12 - 16775.97728$
$= 7104.3233$

∴ They will pay ~~4430.3233~~ 3341.43 more interest.

Q.2 i. b. ~~Discounted~~ Payback Period
- Payback period is the number of years necessary to recover funds invested in the project without considering the interest, both on the borrowing and investing.

a. Discounted Payback Period
- Discounted Payback Period is the number of years after which the cumulative discounted cash inflows cover the initial investment.

ii. Unlike the NPV, neither the DPP nor the PP give any indication of how profitable a project is, as they ignore cash flows after the accumulated value of zero is reached.

The PP can give misleading results, as it does not take into account the time value of money.

Hence, NPV is a superior measure as compared to DPP or PP.

iii a. Project A

PV of outlay $= -170 - 20v - 10v^2$

PV of inlay $= 20v + 20v^2 + 200v^3$

$NPV = -170 - 20v - 10v^2 + 20v + 20v^2 + 200v^3$

$0 = -170 + 10v^2 + 200v^3$

IRR $\Rightarrow$ $0 = -170 + 10v^2 + 200v^3$

$170 = 10(1+i)^{-2} + 200(1+i)^{-3}$

At $i = 0.07$, LHS $= 171.99$

$i = 0.08$, LHS $= 167.33$

$\therefore$ By interpolating, $i = 7.4\%$.

Project B

PV of outlay $= -200$

PV of inlay $= 14a_{\overline{6|}} + 200v^6$

$NPV = -200 + 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200(1+i)^{-6}$

IRR $\Rightarrow$ $0 = -200 + 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200(1+i)^{-6}$

$200 = 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200(1+i)^{-6}$

At $i = 0.07$, LHS $= 200$ By interpolating

~~$i =$, LHS $=$~~ $i = 7\%$.

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b. $NPV_A = -170 + 10v^2 + 200v^3$

At $d = 6\%$

$NPV_A : -170 + 10(1-0.06)^2 + 200(1-0.06)^3$

$NPV_A = \underline{4.9528}$

$NPV_B = -200 + 14\ddot{a}_{\overline{6}|} + 200v^6$

$= -200 + 14\left(\frac{1-(0.94)^6}{0.06}\Big/(0.94)\right) + 200(0.94)^6$

$NPV_B = \underline{5.9958509}$

c. For Project A, the interest rate should be below 7.4%, if above this then the project is unprofitable.

For Project B, the interest rate should be below 7%, if above this then the project is unprofitable.

Other factors:

- Project A (No net income during FY) - Comparison time period
- Lower IRR for Project B for a longer period of both projects.

Q.3 PV of annuity 1 = $200\,\ddot{a}_{\overline{22}|}\left(v^{3/12}\right)$

$$= (1.08)^{-1/4}\,(200)(10.2007)\frac{(0.08)}{(0.074074)}$$

$$= 2161.365534$$

PV of annuity 2 = $320\,\ddot{a}^{(4)}_{\overline{16.25}|}\,v^{2/12}$

$$= 320\left(\frac{1-(1.08)^{-16.25}}{0.08}\right)(1.049519)(1.08)^{-1/6}$$

$$= 2996.048870$$

PV of annuity 3 = $\left(15\,\ddot{a}_{\overline{226}|} \text{ @ } \frac{d^{(12)}}{12} = 0.006392917\right)$

$$= 15\left(\frac{1-(1.08)^{-226/12}}{0.006392917}\right)$$

$$= 1795.651419$$

Total PV = 6953.065823

Revised annuity = $6953.065823 = x\,\ddot{a}^{(2)}_{\overline{21.5}|}$ @ $d^{(2)}$

$$6953.065823 = x\left(\frac{1-(1.08)^{-21.5}}{0.08}\right)(1.059615)$$

$$x = 649.0135657$$

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Q.4 To Derive : $(I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}$

Proof

$PV = 1 + 2v + 3v^2 + \ldots\ldots + nv^{n-1}$ —— ①

$(1-d)(PV) = v + 2v^2 + 3v^3 + \ldots\ldots + (n-1)v^{n-1} + nv^n$ —— ②

① - ②

$PV - (1-d)PV = 1 + v + v^2 + v^3 + \ldots\ldots + v^{n-1} - nv^n$

$d(PV) = \ddot{a}_{\overline{n}|} - nv^n$

$$\boxed{(I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}}$$

Q.5 For Property Fund

i. $1+i = \frac{\cancel{13.1}}{12.4} \times \frac{\cancel{14.8}}{\cancel{13.1}} \times \frac{\cancel{15.8}}{\cancel{14.8}} \times \frac{16.4}{\cancel{15.8}}$

TWRR = i = 32.2580645 %

For Equity fund

$1+i = \frac{\cancel{9.2}}{12.1} \times \frac{\cancel{10.3}}{\cancel{9.2}} \times \frac{\cancel{13.1}}{\cancel{10.3}} \times \frac{15.5}{\cancel{13.1}}$

TWRR = i = 28.0991736 %

ii.

a. $12.4(1+i) + 13.1(1+i)^{3/4} + 14.8(1+i)^{1/2} + 15.8(1+i)^{1/4} = 4(16.4)$

$i = 29.32\%$

iii.

a. $(12.1)(1+i) + 9.2(1+i)^{3/4} + 10.3(1+i)^{1/2} + 13.1(1+i)^{1/4} = 4 \times 15.5$

$i = 67.31\%$

Q.6 i. $160000 = x\, a_{\overline{10}|} @ 8\%$

$x = \frac{160000}{6.7101} = 23844.65201$

ii. $PV_4 = 23844.65201\, a_{\overline{6}|} @ 8\%$

$= 23844.65201(4.6229)$

$= 110231.4422$

$110231.4422 = x'\, a_{\overline{6}|} @ 10\%$

$x' = \frac{110231.4422}{4.3553} = 25309.72428$

iii. $PV_7 = 25309.72428\, a_{\overline{3}|} @ 10\%$

$= 25309.72428(2.4869)$

$= 62942.75331$

$62942.75331 = X'' a_{\overline{3}|}$ @ 9%.

$X'' = \frac{62942.75331}{2.5313} = \underline{24865.78174}$

Q.8 i. PV of outlay $= 2.7 + 0.2 a_{\overline{3}|}$

PV of inlay $= \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3$

$0 = NPV = \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3 - 2.7 - 0.2a_{\overline{3}|}$

$\therefore 0 = \frac{1-(v^{10})}{-\ln(v)}(0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3 - 2.7 - 0.2\left(\frac{1-v^3}{v^{-1}-1}\right)$

$\underline{i = 9.3\%}$.

ii. $NPV = 0.1v^3 + 0.85\bar{a}_{\overline{10}|}v^3 - 2.7 - 0.2a_{\overline{3}|}$ @ 10%.

$= 0.1(1.1)^{-3} + 0.85(1.1)^{-3}\left(\frac{1-(1.1)^{-10}}{\ln(1.1)}\right) - 2.7 - 0.2\left(\frac{1-(1.1)^{-3}}{0.1}\right)$

$= \underline{1.0089}$

Q.9 i. $(1+i)^3 = \frac{33}{30.5} \times \frac{\cancel{37.5}}{33+6} \times \frac{41.05-4.5}{\cancel{37.5}} \times \frac{\cancel{45}}{41.05} \times \frac{45.6}{\cancel{45}} \times \frac{47}{45.6-2.5}$

$(1+i)^3 = 1.228313$

$\underline{i = 7.0951\%}$ = TWRR

ii. $30.5(1+i)^3 + 6(1+i)^{2.25} + 4.5(1+i)^{1.5} - 2.5(1+i)^{0.5} = 47$

At $i = 7\%$, RHS = 46.74
$i = 7.5\%$, RHS = 47.37

By intepolation, $i = 7.208\% = MWRR$

ii. Linked rate of return

2011 =>
$$30.5(1+i) + 6(1+i)^{0.25} = 38.5$$
$$i = 6.266\%$$

2012 =>
$$38.5(1+i) + 4.5(1+i)^{0.5} = 45$$
$$i = 4.92\%$$

2013 =>
$$45(1+i) + 2.5(1+i)^{0.5} = 47$$
$$i = 10.276\%$$

$$(1+i)^3 = 1.06266 \times 1.0492 \times 1.10276$$
$$i = 7.13\%$$

iii. TWRR eliminates the effect of the cashflow amount and timing, and therefore gives a fairer basis for assessing the investment perfomance for the fund.
MWRR is sensitive to the amount and timing of the net cashflows.

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Q.10 i. $PV = 180000\, a_{\overline{15}|} + 20000\,(Ia)_{\overline{15}|}$ @ 12%.

$= 180000\,(6.8109) + 20000\,(40.7313)$

$= \underline{2040588}$

$\therefore$ Total cost of house $= \dfrac{2040588}{0.8} = 2550735$

ii. $PV_9 = 340000\, a_{\overline{7}|} + 20000\,(Ia)_{\overline{7}|}$ @ 12%.

$= 340000(4.5638) + 20000\left(\dfrac{5.1114 - 7.\times 0.45235}{0.12}\right)$

$= 1875850.33$

Year	Loan	Payment	Interest	Capital Repayment
9	1875850.33	360000	225102.04	134897.96
10	1740952.37	380000	208914.28	171085.72
11	1569866.65			

iii. $1569866.65 = X\, a_{\overline{5}|}$ @ 10%.

$X = \dfrac{1569866.65}{3.790786} = 414126.87$

Total installments less loan outstandings at the end of 10^{th} year $= 630133.35$

Extra Payment made $= 5(414126.87) + 0.01(1569866.65) - 1569866.65$

$= \underline{516466.35}$

Q.11 i. TWRR : $(1+i)^2 = \frac{500}{460} \times \frac{550-50}{500+40} \times \frac{600}{550} \times \frac{X}{600}$

$$1.44 = \frac{500}{460} \times \frac{500}{540} \times \frac{X}{550}$$

$$\underline{X = 786.9312}$$

ii. MWRR ⇒ $460(1+i)^2 + 40(1+i)^{21/12} - 50(1+i) = 786.9312$

$$\underline{i = 30.909\%}$$

Q.12 i. Discounted Payback Period (DPP) is the number of years after which the cumulative discounted cash inflows cover the initial investment.

ii. a. $-800(1+i)^t - 50(1+i)^{t-1} + 100\bar{s}_{\overline{t-2|}} = 0$ @ $i = 7\%$

$$-800(1.07)^t - 50(1.07)^{t-1} + 100\left(\frac{(1.07)^{t-2} - 1}{\ln(1.07)}\right) = 0$$

$$\therefore t = \underline{17.767}$$

b. $100\bar{s}_{\overline{4.233|}}$ @ 6%

$$= 100\left(\frac{(1.06)^{4.233} - 1}{\ln(1.06)}\right) = 480$$

iii. $-800(1+i)^t - 50(1+i)^{t-1} + 102.971\, s_{\overline{t-2|}}$ @ 7% = 0

⇒ $t = 18$

$-800(1+i)^{18} - 50(1+i)^{17} + 102.971 s_{\overline{16|}}$ @ 7% = 9.7714

⇒ $9.7714(1+i)^4 + 100\bar{s}_{\overline{4|}}$ @ 6%

$= \underline{\underline{462.79}}$ = Accumulated amount after 22 years

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Q.13 i. $988000 = X(12)\, a^{(12)}_{\overline{25}|}$ @ 7%.

$$988000 = 12X\left(\frac{1-(1.07)^{-25}}{\left((1.07)^{1/12}-1\right)12}\right)$$

$X = 6848$, Monthly repayment = $\underline{\underline{6848}}$.

ii. $X a^{(12)}_{\overline{34/3}|} = 6848\left(a^{(12)}_{\overline{34/3}|} @ 7\%\right) = 648600$

iii. PV ~~on~~ after 10^{th} September's payment = $X\, a^{(12)}_{\overline{83/6}|}$ @ 7%.

PV after 10^{th} October's payment = $X\, a^{(12)}_{\overline{13.75}|}$ @ 7%.

Capital repaid ⇒ $X a^{(12)}_{\overline{13.75}|} - X a^{(12)}_{\overline{83/6}|}$

$$= X\left(\frac{v^{13.75} - v^{83/6}}{i^{(12)}}\right) @ 7\%.$$

$= \underline{26.86}$

iv. Capital repayment in these 12 instalments ⇒

$$= X\left(a^{(12)}_{\overline{22/3}|} - a^{(12)}_{\overline{19/3}|}\right) @ 7\%$$

$= 516.2$

∴ Total interest in these 12 installments ⇒

~~12X - 516.20 = 305.56~~

$12X - 51620 = 30556$