Assignment-1

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① The nominal rate of discount per annum convertible qauterly is 8%. Calculate

(i) equivalent force of Interest
(ii) equivalent effective rate of interest p.a.
(iii) equivalent nominal rate of discount p.a convertibly monthly.

Soln $d^{(4)} = 8\%$ p.a

$$d = 1 - \left[1 - \frac{d^{(4)}}{4}\right]^4$$

$$= 1 - [1 - 0.02]^4$$

$d = 7.76318\%$ p.a

(i) $\delta = -\ln(1-d)$
$\delta = -\ln(1 - 0.0776318)$
$\boxed{\delta = 8.08108\% \text{ p.a}}$

(ii) $i = \dfrac{d}{1-d}$

$\boxed{i = 8.4[illegible]7\% \text{ p.a}}$

(iii) $d^{(12)} = 12\left[1-(1-d)^{1/12}\right]$

$= 12\left[1-(1-0.0776318)^{1/12}\right]$

$\boxed{d^{(12)} = 8.0539\% \text{ p.a}}$

② The force of Interest $\delta(t) = 0.004t + 0.0002t^2$ for all t. Find

(i) Present Value of Sum of ruppee 1000 at end of 10 year from now.

(ii) Constant Annual effective rate of Interest over a 10 year period.

Sol^n $\delta(t) = 0.004t + 0.0002t^2$.

(i) $PV_{10} = 1000 \times v(0,10)$

$= 1000 \times e^{-\int_0^{10} 0.004t + 0.0002t^2}$

$= 1000 \times e^{-\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3}\right]_0^{10}}$

$= 1000 \times e^{-0.2666667}$

$\boxed{PV_{10} = 765.92834}$

(ii) $PV_{10} = 1000 \times v(0,10)$

$765.92834 = 1000 \times \frac{1}{A(0,10)}$

$\frac{1}{(1+i)^{10}} = 0.76592834$

$(1+i)^{10} = \frac{1}{0.76592834}$

$(1+i)^{10} = 1.305605169$

$\boxed{i = 2.7025\%}$

③ i) Explain the relationship d=iv by general reasoning where d is effective rate of discount and i is effective annual rate of interest.

ii) Calculate nominal rate of discount p.a convertible half yearly which is equivalent to

(a) An effective rate of discount of 2.25% p.a
(b) A Nominal rate of discount of 5% convertible 2 year

(iii) A 91-day government bill provide the investor with an annual effective rate of return of 5%. Calculate annual simple discount rate at which the bill is discounted.

Soln

(i) $1+i = \frac{1}{1-d}$

$1-d = \frac{1}{1+i}$

$d = 1 - \frac{1}{1+i} = \frac{1+i-i}{1+i} = \frac{i}{1+i} = iv$

(ii) a) $\frac{d^{(4)}}{4} = 2.25\%$ p.a

$$d^{(2)} = 2\left[1-(1-d)^{1/2}\right]$$

$$= 2\left[1-\left(1-1-\left(1-\frac{d^{(4)}}{4}\right)^4\right]^{1/2}\right]$$

$$= 2\left[1-\left[\left(\frac{d^{(4)}}{4}-1\right)^4\right]^{1/2}\right]$$

$$= 2\left[1-\left(\frac{d^{(4)}}{4}-1\right)^2\right]$$

$$\boxed{d^{(2)} = 8.89875\% \text{ p.a}}$$

b) $d^{(0.5)} = 5\%$

$$d^{(2)} = 2\left[1-(1+i)^{-1/2}\right]$$

$$d^{(2)} = 2\left[1-\left[1+\frac{0.05}{0.5}\right]^{-1/2}\right]$$

$$\boxed{d^{(2)} = 4.70918\% \text{ p.a}}$$

(iii) $i = 5\%$, $n = 91$ days

$$AY = 100(1.05)^{91/365}$$

$$= 101.2238407$$

$$101.2238407 = 100\left(1+\frac{91d}{365}\right)$$

$$\boxed{d = 4.8494619\%}$$

(4)(i) Explain the relationship $d = iv$ by general reasoning where d is effective rate of discounting and i is the effective annual rate of interest.

(ii) $i = 0.08$, Calculate

a) $d^{(12)}$

b) $i^{(365)}$

c) δ

d) $i^{(0.5)}$

Solⁿ (i) $d = iv$

$d = \frac{i}{1+i} = iv$

Thus, by discounting the annual rate of interest, we get the discount rate.

(ii) $i = 0.08$

(a) $d^{(12)} = 12\left[1 - (1.08)^{-1/12}\right]$

$\boxed{d^{(12)} = 7.67148\%}$

(b) $i^{(365)} = 365\left[(1.08)^{1/365} - 1\right]$

$\boxed{i^{(365)} = 7.69691\%}$

(c) $\delta = \ln(1+i)$

$\delta = \ln(1.08)$

$$\boxed{\delta = 7.6961\%}$$

d) $i^{(0.5)} = 0.5\left[(1.08)^{\frac{1}{0.5}} - 1\right]$

$$\boxed{i^{(0.5)} = 8.32\%}$$

(5) i) The rate of discount p.a convertible qauterly is 6%.. Calculate.

(a) The equivalent rate of interest p.a convertible half yearly.
(b) The equivalent rate of discount p.a convertible monthly.

ii) On 15^{th} April, 2005 Amit borrowed Rs 2,00,000 to be repaid after 1 year later by single payment of Rs. 2,00,000. Amit repayed loan early by 17^{th} july, 2005.

(a) Find the sum paid by Amit to terminate the contract assuming that the interest is reduced proportionaly for early settlement.

Sol^n

(i) $d^{(4)} = 6\%$

$d = 1 - \left[1 - \frac{d^{(4)}}{4}\right]^4 = 5.86635\%$

(a) $i = \dfrac{d}{1-d}$

$= 6.23193\%$

$i^{(2)} = 2\left[(1+i)^{1/2} - 1\right]$

$i^{(2)} = 2\left[(1.0623193)^{1/2} - 1\right]$

$\boxed{i^{(2)} = 6.13775\%}$

(b) $d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$

$= 12\left[1 - (1 - 0.0586635)^{1/12}\right]$

$\boxed{d^{(12)} = 6.03025\%}$

(ii) $A V = 2{,}00{,}000 \ A(0,1)$

$2{,}20{,}000 = 2{,}00{,}000 \ (1+i)$

$1.1 = 1 + i$

$i = 10\%$

$(1+i)^n = \dfrac{A.V}{P.V}$

$A V = 2{,}00{,}000 \ (1.1)^{93/365}$

$\boxed{A.V = 2{,}00{,}470.5755}$

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Q6) i) The manufacter of a certain toy sells to retairs on either of the following terms:

(a) Cash payment : 30% below the recommended retail price
(b) Six Month Credit : 25% below the recommended retail price.

Find the effective annual rate of discount offered by the manufacter to the retailers, who pay cash as opposed to those retailers who accept the credit terms.

ii) Find

$d^{(12)}$, $i^{(2)}$ and δ.

Soln

i) let the retail price of toy be -100

a) 30% below the recommanded return cash payment

$= 100 - \frac{30}{100} \times 100$

$= 100 - 30 = 70$

b) 25% bellow recommanded retun cash payment

$= 100 - \frac{25}{100} \times 100$

$= 100 - 25 = 75$

$$\frac{70}{75} = (1-d)^{0.5}$$

$$\left(\frac{70}{75}\right)^2 = 1-d$$

$$\boxed{d = 12.88889\%}$$

(ii) $(1-d) = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$

a) $(1-0.1288889)^{1/12} = 1 - \frac{d^{(12)}}{12}$

$$\boxed{d^{(12)} = 13.7195439\%}$$

b) $i^{(2)} = ?$

$$\left(1 + \frac{i^{(2)}}{2}\right)^2 = \frac{1}{1-d}$$

$$1 + \frac{i^{(2)}}{2} = \left(\frac{1}{1-0.1288889}\right)^{1/2}$$

$$\boxed{i^{(2)} = 14.2857142\%}$$

c) $\delta = -\ln(1-d)$

$= -\ln(1-0.1288889)$

$$\boxed{\delta = 13.7985756\%}$$

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(7) If an investment fund offer to increase INR 20,000 to INR 26,000 in 17 month, Calculate the following:

(i) The nominal rate of interest p.a convertible quarterly
(ii) The nominal rat of discount p.a convertible half yearly
(iii) Simple rate of interest p.a
(iv) comment on the result.

Sol^n

(i) $26{,}000 = 20{,}000(1+i)^{17/12}$

$(1+i)^{17/12} = 1.3$

$i = 0.20346$

$i^{(4)} = 4\left[(1+i)^{1/4} - 1\right]$

$i^{(4)} = 4\left[(1.20346)^{1/4} - 1\right]$

$\boxed{i^{(4)} = 18.9555\%\ p.a}$

(ii) $20{,}000 = \dfrac{26{,}000}{(1+i)^{17/12}}$

$i = 0.20346$

$d = \dfrac{0.20346}{1.20346}$

$d = 0.16906$

$d^{(2)} = 2\left[1 - (1-d)^{1/2}\right]$

$d^{(2)} = 2\left[1 - (1 - 0.16906)^{1/2}\right]$

$$\boxed{d^{(2)} = 17.68845\%}$$

(iii) $26{,}000 = 20{,}000\left[1 + \frac{17 i}{12}\right]$

$$\frac{17}{12}(i) = 0.3$$

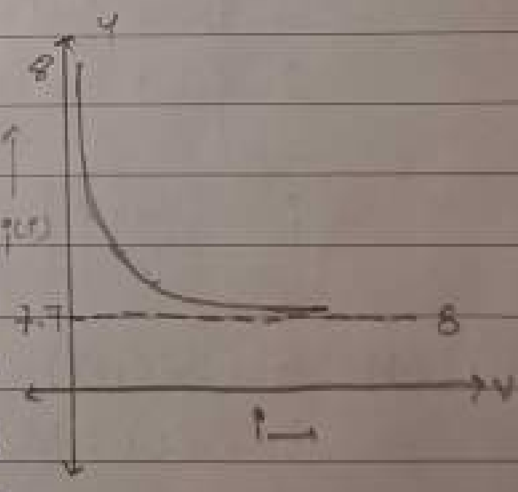
$$\boxed{i = 21.17647\%}$$

(iv) As a result, the ~~int~~ simple interest would be greater than compound interest.

(5) Explain with the help of graph, for an effective interest rate of 8% p.a, the relation btwn equivalent nominal rate of interest convertible pthyly $[i^{(p)}]$ and p.

Soln $i = 8$. $i^{(p)} = p\left[(1+i)^{1/p} - 1\right]$

p	$i^{(p)}$
1	0.08
2	0.07846
4	0.07770
5	0.07755
10	0.07725



→ As p increas the vale of i decrease.
– The limiting case in when $p \to \infty$, there is an instataneous chart and it keeps compounded till δ.

① (i) Define Force of Interest

(ii) If $i^{(2)} = 0.0775$

Calculate $i, \delta, d^{(12)}, i^{(0.5)}$

Solⁿ (i) "The measure of the interest at individual moment of time is called the force of Interest"

→ It is represented by 'δ'

(ii) $i^{(2)} = 0.0775$

(a) $i = \left[1 + \frac{i^{(p)}}{p}\right]^{p} - 1$

$i = \left[1 + \frac{0.0775}{2}\right]^{2} - 1$

$\boxed{i = 0.079}$

(b) $\delta = \ln(1+i)$

$\delta = \ln(1.079)$

$\boxed{\delta = 0.076035}$

(c) $d^{(12)} = 12\left[1-(1+i)^{-1/12}\right]$

$d^{(12)} = 12\left[1-(1.079)^{-1/12}\right]$

$\boxed{d^{(12)} = 0.07579}$

(d) $i^{(0.5)} = 0.5\left[(1+i)^{1/0.5}-1\right]$

$i^{(0.5)} = 0.5\left[(1.079)^{2}-1\right]$

$\boxed{i^{(0.5)} = 0.08212}$

(10) $\delta(t)$ is the force of interest p.a at time t years is defined as

$$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \le t \le 5 \\ 0.06 & \text{for } 5 \le t \le 10 \\ 0.04 & \text{for } t \ge 10 \end{cases}$$

Using $\delta(t)$, find the expression for $v(t)$, the present value of 1 due at t.

Sol^n

$$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \le t \le 5 \\ 0.06 & \text{for } 5 \le t \le 10 \\ 0.04 & \text{for } t \ge 10 \end{cases}$$

$$v(t) = e^{-\delta}$$

$$= e^{-\int_0^5 0.08\,dt} \times e^{-\int_5^{10} 0.06\,dt} \times e^{-\int_{10}^{t} 0.04\,dt}$$

$$= e^{-[0.08t]_0^5} \times e^{-[0.06t]_5^{10}} \times e^{-[0.04t]_{10}^{t}}$$

$$= e^{-[0.08(5)]} \times e^{-[0.06(10) - 0.06(5)]} \times e^{-[0.04(t) - 0.04(10)]}$$

$$= e^{-0.4} \times e^{-[0.6 - 0.3]} \times e^{-[0.04t - 0.4]}$$

$$= e^{-0.4} \times e^{-0.3} \times e^{-0.04t + 0.4}$$

$$= e^{[\cancel{(-0.4)} + (-0.3) + (-0.04t) + \cancel{(0.4)}]}$$

$$= e^{-[0.3 + 0.04t]}$$

$$\boxed{v(t) = e^{-[0.3 + 0.04t]}}$$

Hence, this is required expression for v(t).

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(17) a) A 9-month loan, repayable by a single payment of 50,000 is issued at a rate of commercial discount of 18% p.a. what was intial lent to the borrower.

b) (i) if $\delta = 6\%$. find $d^{(12)}$

(ii) if $^{(4)}d = 6\%$. find $i^{(12)(4)}$

c) Arrange the following qauntities in ascending order of numerical value, giving brif reasoning for order assuming that they all correspond to same effective interest rate. i, d, $i^{(4)}$, δ.

d) Explain in words why the relationship $d = iv$ holds true.

Sol^n a) $d = 18\%$ p.a

$i = 0.21951$

$AV = 50{,}000 v^{9/12}$

$PV = \dfrac{50{,}000}{(1+i)^{3/4}}$

$\boxed{PV = 43085.48439}$

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b) (i) $\delta = 6\%$

$$(1-d) = e^{-\delta}$$

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = e^{-0.06}$$

$$\left(1-\frac{d^{(12)}}{12}\right) = 0.995012479$$

$$\frac{d^{(12)}}{12} = 0.004987521$$

$$\boxed{d^{(12)} = 5.985025\%}$$

(ii) $d^{(4)} = 6\%$

$$d = 1 - \left[1 - \frac{0.06}{4}\right]^4$$

$$d = 0.058663449$$

$$i = \frac{d}{1-d} = 0.062319315$$

$$i^{(12)} = 12\left[(1+i)^{1/12} - 1\right]$$

$$= 12\left[(1.062319315)^{1/12} - 1\right]$$

$$\boxed{i^{(12)} = 6.0607089\%}$$

$$i^{(4)} = 4\left[(1+i)^{1/4} - 1\right]$$

$$= 4\left[(1.062319315)^{1/4} - 1\right]$$

$$\boxed{i^{(4)} = 6.0913705\%}$$

c) Ascending order

~~[illegible]~~ d

$d < \delta < i^{(4)} < i$

d) In the expression

$d = iv$

d = discount rate
i = interest rate

Here, when the interest rate is discount we the get discount rate.

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(12) A deposit of 7,049 is accumalated at the following rates of interest: 6% per annum nominal convertible monthly for first 2 years followed by 1.5% per qauter simple for next 2.5 years followed by rate of discount of 6% per annum convertible monthly for next 1.5 years. Calculate Accumalated Value after 6 years.

Soln Amount at time of investment = 7049

$i^{(12)} = 6\%$ p a for 2 years
$i^{(4)} = 2.5\%$ p q for 2.5 years.
$d^{(12)} = 6\%$ p a for 1.5 years.

$$AV_6 = 7049 \times A(0,2) \times A(2,4.5) \times A(4.5,6)$$

$$A(0,2) = \left[1 + \frac{i^{(12)}}{12}\right]^{24}$$

$$= 1.127159776$$

$$A(2,4.5) = \left[1 + \frac{i^{(4)}}{4}\right]^{10}$$

$$= [1.05]^{10}$$

$$A(2,4.5) = 1.160540825$$

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$$A(4.5, 6) = \frac{1}{(1-d)} = \frac{1}{\left[1 - \frac{d^{(12)}}{12}\right]^{18}}$$

$$A(4.5, 6) = 1.094421325$$

$$AV_6 = 7049 \times A(0,2) \times A(2, 4\tfrac{1}{2}) \times A(4\tfrac{1}{2}, 6)$$

$$= 7049 \times 1.127159776 \times 1.160540825 \times 1.094421325$$

$$\boxed{AV_6 = 10091.55199}$$

(3) Calculate the time in years for an investment to double at :

i) An effective rate of interest of 10% p.a
ii) A nominal rate of 10% p.a convertible monthly.

Sol^n

(i) let the Amount invested at start is x

$$2\cancel{x} = \cancel{x}(1.1)^t$$
$$t = \frac{\ln(2)}{\ln(1.1)}$$

$$\boxed{t = 7.27254 \text{ years}}$$

(ii) $d^{(12)} = 10\%$ p.a

$$d = 1 - \left[1 - \frac{0.1}{12}\right]^{12}$$

$$d = 0.09554$$

$$i = 0.10564$$

$$\ln(2) = t \ln(1.10564)$$

$$t = \frac{\ln(2)}{\ln(1.10564)}$$

$$\boxed{t = 6.90218 \text{ years}}$$

——x——x——x——