

HomeWork -1: Lagrange's Interpolation

x	7	8	9	10
y=f(x)	3	1	1	9

Using Lagrange's interpolation formula,

$$\begin{aligned}
 f(9.5) &= \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} \times 3 \\
 &+ \frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)} \times 1 \\
 &+ \frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)} \times 1 \\
 &+ \frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)} \times 9
 \end{aligned}$$

$$= \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} \times 3$$

$$= 0.1875 + (-0.8125) + 0.9375 + 2.8125$$

$$= 3.625$$

Thus, the function $y=f(x)$ at $x=9.5$ gives 3.625.

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2.	x:	3	7	9	10
	y:	168	120	72	63

Putting $x=6$ in Lagrange's formula,

$$y_6 = \frac{(6-7)(6-9)(6-10)}{(8-7)(8-9)(8-10)} 168 + \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} 120$$

$$+ \frac{(6-3)(6-9)(6-10)}{(9-3)(9-7)(9-10)} 72 + \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} 63$$

$$= 12 + 180 - 72 + 27 = 147$$

Thus the best estimate of function at position 6 is 147.

3.

x:	1	2	7	8
y:	4	5	5	4

Putting $x=6$ in Lagrange's formula.

$$f(6) = \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)} \times 4 + \frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)} \times 5$$

$$+ \frac{(6-1)(6-2)(6-8)}{(7-1)(7-2)(7-8)} \times 5 + \frac{(6-1)(6-2)(6-7)}{(8-1)(8-2)(8-7)} \times 4$$

$$= \frac{(4)(-1)(-2)(4)}{(-1)(-6)(-7)} + \frac{(5)(-1)(-2)(5)}{(1)(-5)(-6)}$$

$$+ \frac{(5)(4)(-2)(5)}{(6)(5)(-1)} + \frac{(5)(4)(-1)(4)}{(7)(6)(1)}$$

$$= \left(\frac{-32}{42} \right) + \frac{50}{30} + \left(\frac{-40}{6} \right) + \left(\frac{-80}{42} \right)$$

$$= 5.666667$$

$$f(6) = 5.666667$$

4.

Using Lagrange's interpolation,

$$f(5) = \frac{(5-2)(5-3)(5-7)}{(1-2)(1-3)(1-7)} \times 2 + \frac{(5-1)(5-3)(5-7)}{(2-1)(2-3)(2-7)} \times 4$$

$$+ \frac{(5-1)(5-2)(5-7)}{(3-1)(3-2)(3-7)} \times 8 + \frac{(5-1)(5-2)(5-3)}{(7-1)(7-2)(7-3)} \times 128$$

$$f(5) = 38.8$$

$$5. \quad \begin{array}{cccc} x = & 10 & 20 & 14 & 16 \\ y = f(x) = & 0.25 & 0.2 & 0.15 & 0.10 \end{array}$$

$$f(15) = \frac{(15-20)(15-14)(15-16)}{(10-20)(10-14)(10-16)} \times 0.25$$

$$+ \frac{(15-10)(15-14)(15-16)}{(20-10)(20-14)(20-16)} \times 0.2$$

$$+ \frac{(15-10)(15-20)(15-16)}{(14-10)(14-20)(14-16)} \times 0.15$$

$$+ \frac{(15-10)(15-20)(15-14)}{(16-10)(16-20)(16-14)} \times 0.10$$

$$= \frac{(-5)(1)(-1)}{(-10)(-4)(-6)} \times 0.25 + \frac{(5)(1)(-1)}{(10)(6)(4)} \times 0.2$$

$$+ \frac{(5)(-5)(-1)}{(4)(-6)(-2)} \times 0.15 + \frac{(5)(-5)(1)}{(6)(-4)(2)} \times 0.10$$

$$= -0.005208333 + (-0.0041667) + 0.078125 + 0.05208334$$

$$= 0.120833306$$

$$f(15) \sim 0.12$$