# Calculus Assignment-2

Q1.] $\displaystyle\int_{\frac{1}{50}}^{2} \frac{e^{\frac{2}{w}}}{w^2}\, dw$

let $t = \frac{1}{w}$, $\therefore w = \frac{1}{t}$

when $w = \frac{1}{50}$, $t = 50$,
$w = 2$, $t = 1/2$

$t = \frac{1}{w}$.

$\frac{dt}{dw} = -\frac{1}{w^2} \Rightarrow \therefore dt = -\frac{1}{w^2} dw$

$= \displaystyle\int_{50}^{1/2} -e^{2t} \cdot dt$ $\quad \left[\because \frac{1}{w^2} dw = dt\right]$

$= -\left[\frac{e^{2t}}{2}\right]_{50}^{1/2}$

$= -\frac{1}{2}\left[e^{2t}\right]_{50}^{1/2}$

$= -\frac{1}{2}\left[e^{2 \cdot 1/2} - e^{2 \cdot 50}\right]$

$= -\frac{1}{2}\left[e - e^{100}\right]$

$= \frac{1}{2}\left[e^{100} - e\right] \Rightarrow \boxed{\frac{e}{2}\left[e^{99} - 1\right]}$ (Ans)

Q2.] $\int \frac{2t^3+1}{(t^4+2t)^3} \, dt.$

let $t^4+2t = x$

$\therefore \frac{dx}{dt} = 4t^3+2$

$\therefore dx = (4t^3+2)\,dt$

$= \cancel{\int \frac{2t^3+1}{}} \; \frac{1}{2} \int \frac{(4t^3+2)\,dt}{(t^4+2t)^3}$

substituting values-

$= \frac{1}{2} \int \frac{dx}{x^3}$

$= \frac{1}{2} \int x^{-3} \, dx$

$= \frac{1}{2} \; \frac{x^{-2}}{-2} + c$

$= -\frac{1}{4x^2} + c$

$= \boxed{-\frac{1}{4(t^4+2t)^2} + c}$

Q3] $f'(x) = x^4 + 3x - 9$

to find $f(x)$, we must integrate given function.-

$\therefore f(x) = \int f'(x)\,dx$

$\therefore f(x) = \int x^4 + 3x - 9\,dx$

$f(x) = \int x^4\,dx + 3\int x\,dx - 9\int dx$

$f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$

(Ans)- $\boxed{f(x) = \frac{2x^5 + 15x^2 - 90x + K}{10}}$ $[K = 10C]$

Q4] $f(x) = \begin{cases} 6 & , x > 1 \\ 3x^2 & , x \leq 1 \end{cases}$

$\int_{-2}^{3} f(x)\,dx$

$= \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$

$= 3\left[\frac{x^3}{3}\right]_{-2}^{1} + 6[x]_1^3$

$= [x^3]_{-2}^{1} + 6[x]_1^3$

$= [1^3 - (-2)^3] + 6[3-1] \Rightarrow (1+8) + 6(2)$

$= \boxed{21}$ (Ans)

Q5] $\int 3(8y-1)e^{4y^2-y}\,dy.$

let $4y^2 - y = x$

$\therefore \frac{dx}{dy} = 8y-1$

$\therefore dx = (8y-1)\,dy.$

substituting values ~~of~~ —

$\int 3e^{x}\,dx$

$= 3e^{x} + c$

(Ans) $= \boxed{3e^{4y^2-y} + c}$

Q6] $f''(x) = 15\sqrt{x} + 5x^3 + 6$ , $f(1) = \frac{-5}{4}$ , $f(4) = 404$

integrating given function —

$f'(x) = \int 15\sqrt{x}\,dx + \int 5x^3\,dx + 6\int dx$

$f'(x) = \cancel{15}^{5} \cdot \frac{x^{3/2}}{\cancel{3}/2} + \frac{5x^4}{4} + 6x + k$

integrating again —

$f(x) = \cancel{5}^{10}\int x^{3/2}\,dx + \frac{5}{4}\int x^4\,dx + 6\int x\,dx + k\int dx$

$f(x) = 2\cdot\cancel{10}\cdot\frac{x^{5/2}}{\cancel{5}} + \frac{5}{4}\cdot\frac{x^5}{5} + 6\frac{x^2}{2} + kx + c$

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$f(x) = 4x^{5/2} + \frac{1}{4}x^5 + 3x^2 + kx + c.$

for $f(1) = -\frac{5}{4}$,

$-\frac{5}{4} = 4(1)^{5/2} + \frac{1}{4}(1)^5 + 3(1)^2 + k(1) + c$

$-\frac{5}{4} = 4 + \frac{1}{4} + 3 + k + c$

$\frac{\cancel{28}\ 16 + 1 + 12 + 4k + 4c}{4} = \frac{-5}{4}$

$29 + 4k + 4c = -5$

$4k + 4c = -34$ — (1)

for $f(4) = 404$ —

$\cancel{4(x)}\ 4(4)^{5/2} + \frac{1}{4}(4)^5 + 3(4)^2 + 4k + c = 404$

$128 + 256 + 48 + 4k + c = 404.$

$432 + 4k + c = 404$

$4k + c = -28$ — (2)

subtracting (1) from (2) —

$4k + c = -28$

$-4k - 4c = 32$

$-3c = 4$

$\therefore c = -\frac{3}{4}$

sub. value of c in (1) —

$4k - 3 = -28$

$k = -\frac{25}{4}$

Sub. value of k & c -

Ans- $\boxed{f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - \frac{25x}{4} - \frac{3}{4}}$

Q7.] $f(x+iy) + g(x-iy)$

diff. wrt x -

$$f'(x+iy) \cdot i\frac{dy}{dx} + g'(x-iy)\left(-i\frac{dy}{dx}\right)$$

$$= i\frac{dy}{dx}\left(f'(x+iy)\right) - i\frac{dy}{dx}\left(g'(x-iy)\right)$$

$$= \boxed{i\frac{dy}{dx}\left[f'(x+iy) - g'(x-iy)\right]}$$

Q8. $\dfrac{dr}{d\theta} = \dfrac{r^2}{\theta}$ , $r(1) = 2$

$$\frac{1}{r^2}\,dr = \frac{1}{\theta}\,d\theta$$

integrating both sides-

$$\int r^{-2}\,dr = \int \frac{1}{\theta}\,d\theta$$

$$\frac{r^{-1}}{-1} = \ln\theta + \ln c$$

$$\frac{1}{-r} = \ln\theta + \ln c$$

$$r = \cancel{\frac{1}{-\log\theta + \log c}} \quad \frac{1}{-\ln(c\theta)}$$

$r(1) = 2$ , $\therefore$ when $\theta = 1$ , $r = 2$

$$\cancel{2 = \frac{1}{\log(1)^{+} + c}} \qquad \cancel{2 = \frac{1}{-\ln\left(\frac{c}{\theta}\right)}}$$

$$\cancel{-\log 1 + c = \frac{1}{2}} \qquad 2 = \frac{1}{-\ln(c\theta)}$$

$$\cancel{\log(1)^{+} + c}$$

$$\therefore\ -\ln(c) = \frac{1}{2}$$

$\therefore$ eqn -

$$\boxed{r = -\ln\theta + \frac{1}{2}}$$

Q9.] $\frac{dv}{dt} = 9.8 - 0.196v$

$\frac{1}{9.8-0.196v} dv = dt.$

taking 0.196 common on LHS –

$\Rightarrow \frac{1}{50-v} dv = 0.196\, dt.$

$\frac{1}{v-50} dv = -0.196\, dt.$

integrating both sides –

$$\int \frac{1}{v-50} dv = -0.196 \int dt.$$

(Ans) – $\boxed{\log(v-50) = -0.196t + c}$

Q10.] $ty' + 2y = t^2 - t + 1$

$= t\frac{dy}{dt} + 2y = t^2 - t + 1$

dividing by t –

$\frac{dy}{dt} + \overset{P}{\frac{2}{t}} y = \overset{Q}{t - 1 + \frac{1}{t}}$

$\therefore IF = e^{\int \frac{2}{t} dt}$

$IF = \cancel{e^{2lo}}\ e^{2\ln t}$

$IF = t^2$

eqn –

$$\text{I.F.}\cdot y = \int \text{I.F.}\; Q$$

$$y\cdot t^2 = \int t^2\left(t - 1 + \frac{1}{t}\right)dt$$

$$y\cdot t^2 = \int (t^3 - t^2 + t)\,dt$$

$$yt^2 = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + C$$

$$y(1) = \frac{1}{2}, \quad \therefore t = 1,\ y = 1/2$$

$$\cancel{\frac{1}{2}}(1)^2 = \frac{1}{4} - \frac{1}{3} + \cancel{\frac{1}{2}} + C$$

$$\therefore C = \frac{1}{3} - \frac{1}{4}$$

$$C = \frac{1}{12}$$

∴ eqn –

$$\boxed{yt^2 = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + \frac{1}{12}}$$

Q11.] $y' = \frac{xy^3}{\sqrt{1+x^2}}$, $y(0) = -1$

$$\frac{dy}{dx} = \frac{x\,y^3}{\sqrt{1+x^2}}$$

$$\frac{1}{y^3}\,dy = \frac{x}{\sqrt{1+x^2}}\,dx$$

Integrating –

$$\int \frac{1}{y^3}\,dy = \int \frac{x}{\sqrt{1+x^2}}\,dx$$

$$\frac{y^{-2}}{-2} = \frac{1}{2}\int \frac{2x}{\sqrt{1+x^2}}\,dx$$

$$\frac{-1}{2y^2} = \frac{1}{2}\left(2\sqrt{1+x^2}\right) + c$$

$$\frac{-1}{2y^2} = \sqrt{1+x^2} + c$$

when $x=0$, $y=-1$

$$\frac{-1}{2(-1)^2} = \sqrt{1} + c$$

$$-\frac{1}{2} = c \pm 1$$

$\therefore$ $c = -\frac{1}{2} + 1$ or $c = -\frac{1}{2} - 1$

$c = 1/2$ or $c = -3/2$

$\therefore$ eqn $\Rightarrow$ $\boxed{\frac{-1}{2y^2} = \sqrt{x^2+1} + 1/2 \text{ or } -3/2}$

Q12.] $f(x) = x^3 - 10x^2 + 6$, $x = 3$

Taylor Series-

$$x^3 - 10x^2 + 6 = f(3) + 1 \cdot f'(3)(x-3) + \frac{1}{2} f''(3)(x-3)^2 + \frac{1}{6} f'''(3)(x-3)^3 + \frac{1}{24} f''''(3)(x-3)^4$$

$$x^3 - 10x^2 + 6 = -57 - 33(x-3) - (x-3)^2 + (x-3)^3$$

Q13.] $f(x) = (1+x)^{\mu}$

Maclaurin Series-

$$(1+x)^{\mu} = 1 + \mu x + \frac{\mu(\mu-1)x^2}{2!} + \frac{\mu(\mu-1)(\mu-2)x^3}{3!}$$

$$+ \frac{\mu(\mu-1)(\mu-2)(\mu-3)x^4}{4!} + \dots$$

Q14.] $f(x) = \sqrt{1+x}$, $f(0) = 1$

$f'(x) = \frac{1}{2\sqrt{1+x}}$, $f'(0) = \frac{1}{2}$

$f''(x) = \frac{-1}{\quad}$, $f''(0) = \frac{1}{4}$

$f'''(x) =$ $\quad$ $f'''(0) = \frac{1}{8}$

Maclaurin Series-

$$\sqrt{1+x} = 1 + \frac{1}{2}x + \frac{1}{2^2 \cdot 2!}x^2 + \frac{1 \cdot 3}{2^3\, 3!}x^3 + \frac{1 \cdot 3 \cdot 5}{2^4 \cdot 4!}x^4 + \dots$$

Q15.] $f(x) = e^{-6x}$ , $x = -4$

Taylor series-

$$f(x) = f(4) + f'(4)\frac{(x+4)}{1} + \frac{f''(4)(x+4)^2}{2!} + \frac{f'''(4)(x+4)^3}{3!} + \dots$$

$$e^{-6x} = e^{24} - 6e^{24}(x+4) + 18e^{24}(x+4)^2 - 36e^{24}(x+4)^3 + 54e^{24}(x+4)^4 - \frac{324}{5}e^{24}(x+4)^5$$

Q16.] $f(x) = \ln(3+4x)$ , $x = 0$

Taylor Series-

$$\ln(3+4x) = \ln(3) + \frac{4}{3}x - \frac{8}{9}x^2 + \frac{64}{81}x^3 - \frac{64}{81}x^4 + \frac{1024}{1215}x^5 + \dots$$