Assignment

1. $d^{(4)} = 8\%$

(ii) $i = ?$

$$(1+i) = \left(1 - \frac{d^{(p)}}{p}\right)^{-p}$$

$$(1+i) = \frac{1}{\left(1 - \frac{0.08}{4}\right)^4}$$

$$(1+i) = \frac{1}{(1-0.02)^4}$$

$$1+i = \frac{1}{(0.98)^4}$$

$$1+i = \frac{1}{0.92236816}$$

$$1+i = 1.084165785$$

$$i = 8.416578\%$$

(i) $\delta = \log(1+i)$

$$\delta = \log(1.084165785)$$

$$\delta = 0.08081$$

(iii) $d^{(12)} = ?$

$$1 + i = \frac{1}{\left(1 - \frac{d^{(12)}}{12}\right)^{12}}$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = \frac{1}{1+i}$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = 0.92236816$$

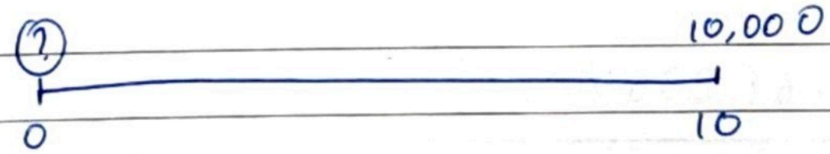
$$1 - \frac{d^{(12)}}{12} = 0.993288388$$

$$\frac{d^{(12)}}{12} = 0.006711612$$

$$d^{(12)} = 0.080539344$$

$$d^{(12)} = 8.0539344\%$$

2.



$$\delta(t) = 0.004t + 0.0002t^2$$

(i) $$A(0,10) = e^{\int_0^{10} 0.004t + 0.0002t^2 \, dt}$$

$$= e^{\left[\frac{4}{1000} \times \frac{t^2}{2} + \frac{2}{10^4} \times \frac{t^3}{3}\right]_0^{10}}$$

$$= e^{\left[\frac{4}{10^3} \times \frac{10^2}{2} + \frac{2}{10^4} \times \frac{10^3}{3} - (0+0)\right]}$$

$$= e^{[0.2 + 0.066]}$$

$$= e^{0.26666} = 1.30317158$$

$$PV_0 = C \times V(0,10) = C \times \frac{1}{A(0,10)}$$

$$= 1000 \times \frac{1}{1.30317158}$$

$$PV_0 = \underline{\underline{767.3587}}$$

(ii) $i = ?$

$$A(0,10) = 1.30317158$$

$$(1+i)^{10} = 1.30317158$$

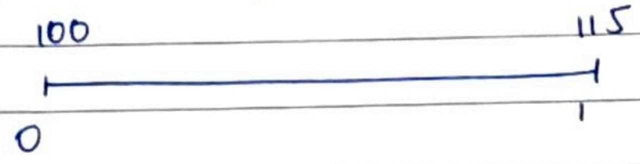
$$1+i = 1.02683381$$

$$i = 0.02683381$$

$$\underline{i = 2.68338100\,\%}$$

3. (i) Suppose if suraj invested ₹ 100 at $t=0$ at annual effective interest rate of 15% ; then

After 1 year, suraj will get Rs 115.



$$AV = 100\ A(0,1)$$
$$115 = 100\ A(0,1)$$

$$A(0,1) = 1.15$$

Now, accumulating factor in terms of (d) is

$$A(t_1, t_2) = \frac{1}{1-d}$$

$$A(0,1) = \frac{1}{1-d} = 1.15$$

$$\Rightarrow \frac{1}{1-d} = 1.15$$

$$d = 1 - \frac{1}{1.15} = \frac{1.15-1}{1.15} = \frac{0.15}{1.15}$$

$$d = 13.0435\%$$

$$1-d = \frac{1}{A(1)}$$

$$1-d = \frac{1}{1+i}$$

$$d = 1 - \frac{1}{1+i}$$

$$d = \frac{1+i-1}{1+i}$$

$$d = \frac{i}{1+i}$$

$$d = i \cdot \frac{1}{1+i}$$

$$\underline{\underline{d = i \cdot v}}$$

(ii) $d^{(2)} = ?$

a) $\frac{d^{(4)}}{4} = 2.25\%$

$$\left(1 - \frac{d^{(4)}}{4}\right)^4 = 1-d$$

$$(1 - 0.0225)^4 = 1-d$$

$$(0.9775)^4 = 1-d$$

$$0.912992194 = 1-d$$

$$d = 0.087007806$$

$$\underline{d = 8.70078\%}$$

Date:

$$\left(1-\frac{d^{(2)}}{2}\right)^2 = 1-d$$

$$\left(1-\frac{d^{(2)}}{2}\right)^2 = 0.912992194$$

$$1-\frac{d^{(2)}}{2} = 0.95550625$$

$$\frac{d^{(2)}}{2} = 0.04449375$$

$$\underline{d^{(2)} = 0.0889875}$$

$$\underline{d^{(2)} = 8.89875\%}$$

b)

$$(1-d) = (1-2\times 0.05)$$

$$= 1-0.1$$

$$1-d = 0.9$$

$$\left(1-\frac{d^{(2)}}{2}\right)^{\cancel{1}4} = (1-d)$$

$$\left(1-\frac{d^{(2)}}{2}\right) = (0.9)^{1/4}$$

$$1-\frac{d^{(2)}}{2} = 0.974$$

$$\frac{d^{(2)}}{2} = 0.025996254$$

$$d^{(2)} = 0.051992507$$

$$\underline{\underline{d^{(2)} = 5.199\%}}$$

Date:

iii) $(1+i) = \left(1-\frac{d^{(p)}}{p}\right)^{-p}$

$i = 5\%$

$d = ?$

$\cancel{(1+i)}$

$\Rightarrow (1+0.05) = \left(1-\frac{91}{365}\times d\right)^{\cancel{-\frac{91}{365}}\ -\frac{365}{91}}$

$\Rightarrow (1.05)^{-\frac{91}{365}} = \left(1-\frac{91}{365}\times d\right)$

$\Rightarrow 0.987909561 = 1-\frac{91}{365}d$

$\Rightarrow 0.012090439 = \frac{91}{365}d$

$\Rightarrow d = 0.048494618$

$\Rightarrow d = 4.849461797\%$

4 i) d is the interest payable at the beginning of the year on a loan of 1 unit repayable at the end of the year.
The corresponding interest payable at the end of year on 1 unit of loan is i.

Thus, d must equal to the present value at the beginning of the year of the payment (i) payable at the end of the year i.e

$$\underline{\underline{d = iV}}.$$

4. (ii) $i = 0.08$

$$d = \frac{i}{1+i} = \frac{0.08}{1.08} = 0.074074$$

a) $$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = (1-d)$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = 0.925925922$$

$$1 - \frac{d^{(12)}}{12} = 0.993607102$$

$$\frac{d^{(12)}}{12} = 0.006392898$$

$$\underline{d^{(12)} = 0.076714776}$$

$$\underline{d^{(12)} = 7.671477606\%}$$

b) $i^{(365)}$

$$\left(1 + \frac{i^{(365)}}{365}\right)^{365} = 1+i$$

$$1 + \frac{i^{365}}{365} = \cancel{0.993104082}\ 1.000210874$$

$$\frac{i^{365}}{365} = \cancel{0.006895918}\ 0.000210874$$

$i^{365} = 0.076969010$

$i^{365} = 7.696901\%$

c. $\delta = \log(1+i)$

$\delta = \log(1.08)$

$\delta = 0.07696104$

d. $i^{(0.5)}$

$$\left(1+\frac{i^{(0.5)}}{0.5}\right)^{0.5} = 1+i$$

$$\left(1+\frac{i^{(0.5)}}{0.5}\right)^{1/2} = 1.08$$

$$1+\frac{i^{(0.5)}}{0.5} = 1.1664$$

$$\frac{i^{(0.5)}}{0.5} = 0.1664$$

$$i^{0.5} = 0.0832$$

$$i^{(0.5)} = 8.32\%$$

5 i) $d^{(4)} = 6\%$

a) $i^{(2)} = ?$

$$(1+i) = \frac{1}{\left(1-\frac{0.06}{4}\right)^4}$$

$$(1+i) = \frac{1}{(0.985)^4}$$

$$1+i = \frac{1}{0.941336551}$$

$$1+i = 1.062319315$$

$$i = 6.2319315\%$$

$$(1+i) = \left(1+\frac{i^{(2)}}{2}\right)^2$$

$$1.062319315 = \left(1+\frac{i^{(2)}}{2}\right)^2$$

$$1.03068758 = 1+\frac{i^{(2)}}{2}$$

$$\frac{i^{(2)}}{2} = 0.03068758$$

$$i^{(2)} = 6.13775151\%$$

b) $\cancel{\frac{d^{(12)}}{12}}$ $d^{(12)} = ?$

$$(1+i) = \frac{1}{\left(1-\frac{d^{(12)}}{12}\right)^{12}}$$

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = \frac{1}{1+i}$$

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = \frac{1}{1.062319315}$$

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = 0.941336551$$

$$1-\frac{d^{(12)}}{12} = 0.99497479$$

$$\frac{d^{(12)}}{12} = 0.00502521$$

$$d^{(12)} = 6.030252492\%$$

(11) Original term of loan = 365 day.

Settlement date is actual date of repayment 17^{th} July 2005

Paid loan term → 272 days.

$$\Rightarrow \frac{272}{365} \times 200000$$

$$\Rightarrow 149041.0959.$$

Sum paid to settle the loan = 220000 − 149041.0959

= 70958.9041

6 i) Assume that the retail price of the toy is 100.

Cash Price → 70
75 in 6 months time.

$$\frac{70}{75} = (1-d)^{\frac{1}{2}}$$

$$0.93333 = (1-d)^{1/2}$$

$$(1-d) = 0.8711111$$

$$d = 0.12889$$

$$d = 12.89\%.$$

ii) $d^{(12)} = ?$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = (1-d)$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = 1 - 0.1289$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = 0.8711$$

$$1 - \frac{d^{(12)}}{12} = 0.988565996$$

$$\frac{d^{(12)}}{12} = 0.011434004$$

$$d^{(12)} = 0.137208$$

$$\underline{d^{(12)} = 13.72\%}$$

$i^{(2)} = ?$

$$d = \frac{i}{1+i}$$

$$\Rightarrow i = \frac{d}{1-d}$$

$$\Rightarrow i = \frac{0.1289}{0.8711} = \underline{0.147973826}$$

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = (1+i)$$

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = 1.147973826$$

$$1+\frac{i^{(2)}}{2} = \cancel{0.384673}\ 1.071435404$$

$$\frac{i^{(2)}}{2} = \cancel{0.615326338}\ 0.071435404$$

$$i^{(2)} = 0.142870809$$

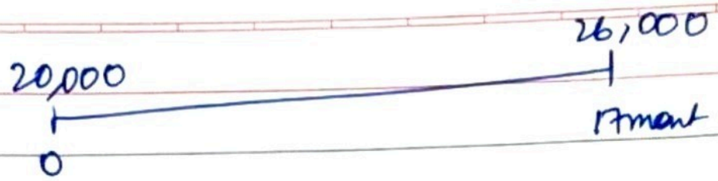
$$\underline{\underline{i^{(2)} = 14.28\%}}$$

$$\delta = ?$$

$$\delta = \log(1+i)$$

$$\delta = \log(1.147973826)$$

$$\underline{\underline{\delta = 0.1380}}$$



7.

i) $i^{(4)} = ?$

$$AV = \cancel{A}\, C \times \cancel{ACC}, \left(1 + \frac{i^{(4)}}{4}\right)^{4 \times \frac{17}{12}}$$

$$26000 = 20000 \times \left(1 + \frac{i^{(4)}}{4}\right)^{\frac{17}{3}}$$

$$\left(1 + \frac{i^{(4)}}{4}\right)^{\frac{17}{3}} = 1.3$$

$$1 + \frac{i^{(4)}}{4} = 1.047388136$$

$$\frac{i^{(4)}}{4} = 0.047388136$$

$$i^{(4)} = 0.189552546$$

$$\underline{\underline{i^{(4)} = 18.96\%}}$$

ii) $d^{(2)} = ?$

$$26000 = 20,000 \times \frac{1}{\left(1 - \frac{d^{(2)}}{2}\right)^{2 \times \frac{17}{12}}}$$

$$26000 \times \left(1 - \frac{d^{(2)}}{2}\right)^{\frac{17}{6}} = 20000$$

$$\left(1 - \frac{d^{(2)}}{2}\right)^{\frac{17}{6}} = 0.769230769$$

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$$1 - \frac{d^{(2)}}{2} = 0.911558823$$

$$\frac{d^{(2)}}{2} = 0.08844117$$

$$d^{(2)} = 0.176882353$$

$$d^{(2)} = 17.69\%$$

iii) $AV = C\, AC(0, \frac{17}{12})$

$$26,000 = 20,000 \times \left(1 + \frac{17}{12} \times i\right)$$

$$1 + \frac{17}{12} \times i = 1.3$$

$$\frac{17}{12} \times i = 0.3$$

$$i = 0.211764706$$

$$i = 21.18\%$$

iv) For small period of time, simple interest would be beneficial than compounding.

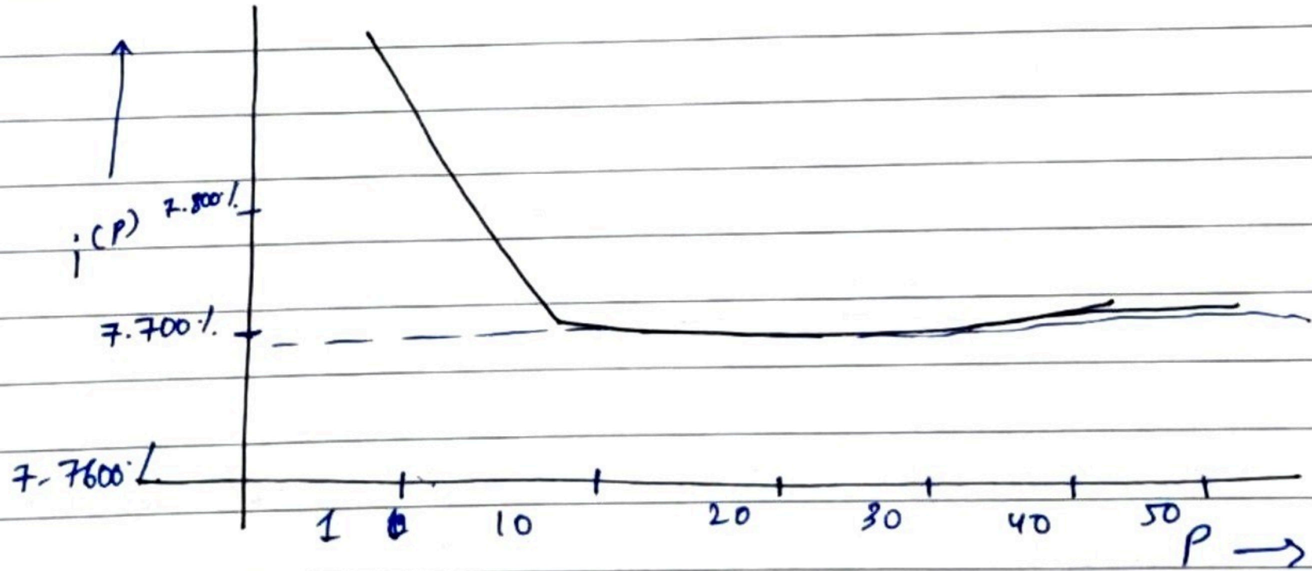
8. The relationship between i \$ $i^{(p)}$

$$\left(1+\frac{i^{(p)}}{p}\right)^p = (1+i)$$

$$\delta = \log(1+i)$$

$$\delta = \log(1.08) = \underline{\underline{7.696\%}}$$

The force of interest will be the lowest value which will be attained by $i^{(p)}$



9 i) <u>Force of interest</u> → The force of interest can be defined as the nominal rate of interest per unit time at time (t) convertible momenthy.

ii) $i^{(2)} = 0.0775$

$i = ?$

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = (1+i)$$

$$(1+0.03875)^2 = 1+i$$

$$1+i = 1.079001563$$

$$\underline{i = 7.9002\%}$$

$\cancel{d^{(12)} = ?}$ $\qquad \delta = ?$

$\cancel{1-d}$ $\qquad \delta = \log(1+i)$

$$\delta = \log(1.079001563)$$

$$\delta = 0.076036135$$

$$\underline{\delta = 7.6036\%}$$

Date:

$$d^{(12)} = 12\left(1 - v^{(\frac{1}{12})}\right)$$

$$d^{(12)} = 12\left(1 - \frac{1}{(1+i)^{1/12}}\right)$$

$$d^{(12)} = 12\left(1 - \left(\frac{1}{1.079}\right)^{1/12}\right)$$

$$= 12\left(1 - \frac{1}{1.00635634}\right)$$

$$= 12\left(1 - 0.993683808\right)$$

$$d^{(12)} = 0.075794306$$

$$d^{(12)} = 7.5796\%$$

$$i^{(1/2)} = ?$$

$$(1+i) = \left(1 + \frac{i^{(\frac{1}{2})}}{\frac{1}{2}}\right)^{1/2}$$

$$1.079001563 = \left(1 + 2\,i^{(\frac{1}{2})}\right)^{1/2}$$

$$1.164244373 = 1 + 2\,i^{(\frac{1}{2})}$$

$$2\,i^{(\frac{1}{2})} = 0.164244373$$

$$i^{(\frac{1}{2})} = 0.082122186$$

$$i^{(\frac{1}{2})} = 8.2122\%$$

10.

$$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \le t \le 5 \\ 0.06 & \text{for } 5 \le t \le 10 \\ 0.04 & \text{for } t \ge 10 \end{cases}$$

$$v(t) = e^{\left[-\int_0^t \delta(s)\,ds\right]}$$

$$\int_0^t \delta(s)\,ds = \begin{cases} 0.08t & 0 \le t \le 5 \\ 0.1 + 0.06t & 5 \le t \le 10 \\ 0.3 + 0.04t & t \ge 10 \end{cases}$$

$$v(t) = \begin{cases} e^{-0.08t} & 0 \le t \le 5 \\ e^{-(0.1+0.06t)} & 5 \le t \le 10 \\ e^{-(0.3+0.04t)} & t \ge 10 \end{cases}$$

11. a) $t = \frac{9}{12}$ $A = 50{,}000$ $d = 18\%$

$PV_0 = ?$

$$50{,}000 = P + \left(P \times \frac{\overset{3}{\cancel{18}}}{100} \times \frac{9}{\underset{2}{\cancel{12}}}\right)$$

$$50{,}000 = P + 0.135P$$

$$1.135P = 50000$$

$$P = 44052.86344$$

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b) i) $\delta = 6\%$ $\quad d^{(12)} = ?$

$\delta = 0.06$

$\delta = \log(1+i)$

$0.06 = \log(1+i)$

$1+i = 1.0614067$

$i = 0.0614067$

$i = 6.14\%$

$$(1+i) = \frac{1}{\left(1-\frac{d^{(12)}}{12}\right)^{12}}$$

$$1.0614067 = \frac{1}{\left(1-\frac{d^{(12)}}{12}\right)^{12}}$$

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = 0.942145928$$

$$1-\frac{d^{(12)}}{12} = 0.995046$$

$$\frac{d^{(12)}}{12} = 0.004953947$$

$$d^{(12)} = 0.059447366$$

$$d^{(12)} = 5.944736613\%$$

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ii) $d^{(4)} = 6\% = 0.06$

$i^{(12)} = ?$

$$(1+i) = \frac{1}{\left(1-\frac{d^{(4)}}{4}\right)^4}$$

$$\cancel{\left(1-\frac{d^{(4)}}{4}\right)^4 = 1}$$

$$1+i = \frac{1}{\left(1-\frac{0.06}{4}\right)^4}$$

$$1+i = \frac{1}{(1-0.015)^4}$$

$$1+i = \frac{1}{(0.985)^4}$$

$$1+i = \frac{1}{0.941336551}$$

$$1+i = 1.0623219315.$$

$$i = 0.06232$$

$$\underline{i = 6.232\%}$$

$$(1+i) = \left(1+\frac{i^{(12)}}{12}\right)^{12}$$

$$1.0623219315 = \left(1+\frac{i^{(12)}}{12}\right)^{12}$$

$$1+\frac{i^{(12)}}{12} = 1.00505059|$$

$$\frac{i^{(12)}}{12} = 0.00505059|$$

$$i^{(12)} = 0.06060708 8$$

$$i^{(12)} = 6.0607\%$$

c) $i, d, i^{(4)}, \delta$

Ascending order

$$d < d^{(4)} < \delta < i^{(4)} < i$$

The order is determined ~~through~~ on when the interest is paid.

d is interest paid in the starting of the period, which requires small payment.

i is interest paid at the end.

$\delta \rightarrow$ Payment throughout.

d) let 1 unit be the initial capital, and (d) is the interest paid at the starting of the period.

So, d must be equal to present value (PV) at the beginning of the year and (i) interest paid at the end of the year.

e.g $d = iv$.

12.

$i^{(12)} = 6\%$

$i = \frac{i^{(12)}}{12} = \frac{6\%}{12} = \underline{\underline{0.5\%}}$

$A(0,2)$ (Monthly) $= (1+i)^n = (1+0.005)^{24}$

$= \underline{1.1272}$

~~A(2,4)~~ $AV_2 = C \times A(0,2)$

$AV_2 = 7049 \times 1.1272$

$AV_2 = 7945.6328$

Accumulated for next 2.5 years (working inquaten)

$= (1 + 10 \times 1.5\%)$

$= \underline{\underline{1.15}}$

$AV_{4.5} = 7945.6328 \times 1.15$

$AV_{4.5} = \underline{9137.4772}$

$d^{(12)} = 6\%$

$$1+i = \frac{1}{\left(1-\frac{d^{(12)}}{12}\right)^{12}}$$

$$1+i = \frac{1}{\left(1-\frac{0.06}{12}\right)^{12}}$$

$$1+i = \frac{1}{(1-0.005)^{12}}$$

$$1+i = \frac{1}{0.941622807}$$

$$1+i = 1.0671996367$$

$$i = 0.0672$$

$$\underline{\underline{i = 6.72\%}}$$

Accumulated factor for next 1.5 year (In year) $= (1+6.72\%)^{1.5}$

$= 1.102474936.$

$AV(6) = 7049 \times 1.1272 \times 1.15 \times 1.1024$

$= \underline{\underline{10073.15}}$.

13. i) $(1+i)^n = 2$

Taking log

$n \log(1+1) = \log 2$

$$n = \frac{\log 2}{\log(1.1)}$$

$$n = \frac{0.69314718)}{0.095310180}$$

$n = 7.272$ years

ii) $d^{(12)} = 10\% = \underline{\underline{0.1}}$

$$1+i = \frac{1}{\left(1-\frac{d^{(12)}}{12}\right)^{12}}$$

$$1+i = \frac{1}{\left(1-\frac{0.1}{12}\right)^{12}}$$

$$1+i = \frac{1}{(1-0.008333)^{12}}$$

$$1+i = \frac{1}{(0.9916666)^{12}}$$

$$1+i = \frac{1}{0.90445}$$

$$1+i = 1.105632090$$

$$i = 0.105632090$$

$$\underline{\underline{i = 10.563\%}}$$

$$n = \frac{\log 2}{\log(1.10563)} = \frac{0.6931}{0.1003}$$

$$\underline{\underline{n = 6.902 \text{ years}}}$$